All Ramsey numbers $r(K_3, G)$ for connected graphs of order 7 and 8

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Abstract

In this paper we will give all Ramsey numbers $r(K_3, G)$ for connected graphs G of order 7 and 8. For smaller orders and not connected graphs of order up to 8, the number of graphs with given Ramsey number is listed for all possible values.

Introduction

All graphs are assumed to be finite undirected simple graphs. The Ramsey number $r(G_1, G_2)$ of two graphs G_1 and G_2 is defined as the minimum n so that every graph on n vertices contains an isomorphic copy of G_1 as a (weak) subgraph or its complement contains an isomorphic copy of G_2. The existence of this number follows directly from the fundamental theorem of Ramsey (cf. [1]). With $|G|$ we denote the number of vertices in G.

In the literature several collections of Ramsey numbers for specific sets of graphs can be found. All numbers $r(G_1, G_2)$ with $|G_1| \leq 4$ and $|G_2| \leq 4$ were already listed by Chvatal and Harary [4],[5]. Clancy [6] extended these results to $|G_1| \leq 4$ and $|G_2| \leq 5$ except for 5 entries. The last missing case $G_1 = K_4$ and $G_2 = K_5$ was solved by McKay and Radziszowski in 1995 [13].

Special attention was paid to the case $G_1 = K_3$: In 1980 Faudree, Rousseau and Schelp [7] determined all numbers $R(K_3, G)$ with $|G| = 6$. Schelten and Schiermeyer [17] determined the Ramsey numbers $R(K_3, G)$ – in the sequel called triangle Ramsey numbers – for 812 connected graphs (out of 853) of order 7. Xia Jin [11] computed the triangle Ramsey numbers for all connected graphs of order 7. Unfortunately there were some errors in his thesis, so that this problem remained open to some extend. The triangle Ramsey numbers for the most important cases $G = K_n$ or $G = K_n - e$ have been determined by various authors for all $n \leq 9$, see [9],[8], [10], [12], [15]. Further information on small Ramsey numbers can be found in [16].

In the following we will give all Ramsey numbers $R(K_3, G)$ for connected graphs G of order up to 8. The results will be given in the form of maximal and minimal elements of the subgraph chains belonging to a certain number.

The algorithm

The triangle Ramsey numbers were obtained with the aid of a computer. All numbers given except $R(K_3, K_8)$ have been computed, giving an independent check for those numbers already known before.
The basic tool was a computer program developed together with Stephan Brandt and Thomas Harmuth [2] to generate *maximal triangle free* graphs. A maximal triangle free graph (in short MTF) is defined as a triangle free graph so that the insertion of any additional edge gives a triangle.
The lists generated by that program were sufficient to determine the triangle Ramsey numbers because of the following lemma, which follows directly from the definition:

Lemma 1

For every graph G and number n we have $R(K_3, G) \leq r$ if and only if the complement of every MTF with r vertices contains a copy of G as a weak subgraph.

Table 1 gives the numbers of MTFs with up to 20 vertices. Up to 17 vertices the lists were checked by a completely independent generation program for MTFs and up to 14 vertices they were additionally checked by generating all triangle free graphs using the graph generator *makeg* [14] and filtering the output for graphs with diameter 2.

vertex number	number of graphs	vertex number	number of graphs
3	1	12	147
4	2	13	392
5	3	14	1 274
6	4	15	5 036
7	6	16	25 617
8	10	17	164 796
9	16	18	1 337 848
10	31	19	13 734 745
11	61	20	178 587 364

Table 1: Numbers of MTFs

Since the number of MTFs is not too large for small vertex numbers, for small triangle Ramsey numbers one could simply apply Lemma 1. Obviously no connected graph of order n can be