APPROXIMATION OF QUADRATIC FORMS OF INDEPENDENT RANDOM VECTORS BY ACCOMPANYING LAWS

V. Bentkus[1] F. Götze[1] A. Yu. Zaitsev[2]

University of Bielefeld[1]
St. Petersburg Branch of Steklov Mathematical Institute[2]

April 1996

Abstract. Let $X, X_1, X_2, \dots$ be i.i.d. random vectors taking values in $\mathbf{R}^d$. Assume that $\mathbf{E}\, X = 0$, $\mathbf{E}\, |X|^{8/3} < \infty$ and that X is not concentrated in a proper subspace of $\mathbf{R}^d$. Let $Y, Y_1, Y_2, \dots$ denote i.i.d. random vectors with common distribution which is accompanying to that of X. We compare the distributions of the non-degenerate quadratic forms $Q[S_N]$ and $Q[T_N]$ of the normalized sums $S_N = N^{-1/2}\,(X_1 + \dots + X_N)$ and $T_N = N^{-1/2}\,(Y_1 + \dots + Y_N)$ and prove that

$$\sup_x \Big|\, \mathbf{P}\,\big\{\, Q[S_N - a] < x \,\big\} - \mathbf{P}\,\big\{\, Q[T_N - a] < x \,\big\} \Big| = O\big((1 + |a|^4)\, N^{-1} \big), \qquad a \in \mathbf{R}^d,$$

provided that $9 \leqslant d \leqslant \infty$. The constant in this bound depends on $\mathbf{E}\, |X|^{8/3}$, on Q and on the covariance operator of X. We also show the optimality of the bound $O\big(N^{-1}\big)$.

1. Introduction

In the limit theory for sums of independent identically distributed random variables and vectors main efforts were devoted to study the rate of approximation in the Central Limit Theorem (CLT) and for problems with stable limit distributions. Compared to the general structure of limit distributions of sums these are still rather special situations. Moreover, in many problems the accuracy of normal approximation is too slow and one needs to enlarge the class of approximating distributions. A natural approximation for the distributions of sums of a large number of i.i.d.

1991 *Mathematics Subject Classification.* Primary 60F05.

Key words and phrases. Compound Poisson approximation, accompanying laws, convergence rates, multidimensional spaces, Hilbert spaces, quadratic forms, ellipsoids, hyperboloids.

[1]Research supported by the SFB 343 in Bielefeld.

[2]Research supported by the Alexander von Humboldt Foundation, by the Russian Foundation of Fundamental Researches, grant 96-01-00672, and by the SFB 343 in Bielefeld.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TeX

random vectors is given by infinitely divisible (in particular, accompanying) distributions.

The problem of the estimation of the rate of infinitely divisible approximations for the distribution functions of sums of N i.i.d. one-dimensional random variables as $N \to \infty$ was stated by Kolmogorov (1953). Prohorov (1955) showed that these distribution functions are approximable by the set of infinitely divisible laws in the sense of the uniform distance between distribution functions. Kolmogorov (1956) showed that the rate of this approximation is uniform with respect to the class of all one-dimensional distributions and proved a rate of the form $c\,N^{-1/5}$, where c is an absolute constant. The best possible rate $c\,N^{-2/3}$ was obtained by Arak (1981, 1982). For accompanying approximating distributions, the optimal bound $c\,N^{-1/3}$ was proved by Le Cam (1965). This bound was sharpened up to $c\,N^{-1/2}$ for symmetrically distributed summands by Zaitsev (1981) (see also Ibragimov and Presman (1973)) and up to $c\,N^{-1}$ by Arak (1980) for summands with non-negative characteristic functions, see also Čekanavičius (1989). A more detailed review of one-dimensional results mentioned above and detailed proofs can be found in the monograph of Arak and Zaitsev (1988).

The majority of the corresponding multidimensional results used the uniform distance between distribution functions. Presman (1973) proved an analog of Le Cam's result with a constant depending on the dimension $d < \infty$ only. When the distribution of the summands is symmetric or even has a non-negative characteristic function, the corresponding analogs of the one-dimensional results of Zaitsev (1981) and Arak (1980) were obtained by Zaitsev (1988, 1992). These results can be easily extended to the comparison of distributions on convex polytopes (see Zaitsev (1994)). The constants in these bounds depend on certain characteristics of the polytopes what do not allow to obtain estimates for arbitrary convex sets and balls by passing to the limit.

In spite of the fact that the results described above give the definitive answer to the question about the asymptotic behaviour of the bounds which are uniform with respect to the class of all one-dimensional or d-dimensional distributions, these inequalities do not describe the asymptotics of the rate of infinitely divisible approximation of N-fold convolutions for fixed distribution of summands. The analog of the result of Prohorov (1955) mentioned above for finite dimensional convex sets was obtained by Bakštys and Paulauskas (1986, 1987). For the approximation by accompanying laws they also proved an estimate for the Prohorov distance between suitably normalized vectors in the infinite dimensional case which is of order $O(N^{-1/6})$ in the case of Hilbert space valued vectors. Nagaev (1990) proved that if $4 \leqslant d \leqslant \infty$, the norm of the summands has finite second moment and their distribution is non-degenerate, then the rate of approximation by accompanying laws for balls is $O(N^{-1/2})$. From the recent estimate in the CLT of Bentkus and Götze (1995) it follows that this rate of approximation is $O(N^{-1})$ for arbitrary ellipsoids and hyperboloids when $9 \leqslant d \leqslant \infty$