Approximation of arbitrary Dirichlet processes by Markov chains

Zhi-Ming Ma [1),2)] Michael Röckner [2)]

Tu–Sheng Zhang [2), 3)]

Abstract

We prove that any Hunt process on a Hausdorff topological space associated with a Dirichlet form can be approximated by a Markov chain in a canonical way. This also gives a new and "more explicit" proof for the existence of Hunt processes associated with strictly quasi-regular Dirichlet forms on general state spaces.

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[1)]Institute of Applied Mathematics, Academia Sinica, Beijing 100080, China
[2)]Fakultät für Mathematik, Universität Bielefeld, Postfach 100131, 33501 Bielefeld, Germany
[3)]Faculty of Engineering, HSH, Skåregt 103, 5500 Haugesund, Norway

1 Introduction

In the last few years the theory of *Dirichlet forms* on general (topological) state spaces has been used to construct and analyze a number of fundamental processes on infinite-dimensional "manifold-like" state spaces which so far could not be constructed by other means. Among these some of the most important are: solutions to infinite-dimensional stochastic differential equations with very singular drifts such as the stochastic quantization of (infinite volume) *time zero* and *space-time quantum fields* in Euclidean field theory (see e.g. [AR91], [RZ92], [AMR94]); diffusions on loop spaces (see [DR92], [ALR93], [ElM96]); a class of interacting Fleming-Viot processes (see [ORS95, in particular Subsections 5.2, 5.3] and [OR94]); infinite particle systems with very singular interactions (see e.g. [Os95], [Y95]); stochastic dynamics associated with Gibbs states (see e.g. [AKoR95]). We also refer to the survey article [R96]. All these processes are diffusions (i.e., have continuous sample paths almost surely) and all except for the one in [ALR93] are conservative, hence, in particular, they are Hunt processes.

In [AMR95a] (see also [MR92, Chap. V, Sect. 2]) Dirichlet forms (not necessarily symmetric) on general state spaces which are associated with Hunt processes have been characterized completely through an analytic property which is checkable in examples and is called *strict quasi-regularity*. However, the construction of the Hunt process was quite indirect using "Kolmogorov's scheme" and a number of (partly rather technical) tools from potential theory and the general theory of Markov processes. It has been an open question for quite some time whether the method of constructing Markov processes based on the Yoshida approximation (for the transition semigroup) and tightness arguments (cf. the beautiful exposition in [EK86]) can be extended to this case. This would be desirable, since, in addition, this would yield an approximation of the Hunt processes by Markov chains in a canonical way and would make the limit process more explicit and provide another tool for its analysis.

We recall, however, that this approximation method was, so far, only developed under some additional assumptions on the state space (i.e., it was assumed to be a locally compact separable metric space) and on the underlying transition semigroup (e.g. it was assumed to be Feller). It should be emphasized that these conditions are not even fulfilled in the classical case of regular Dirichlet forms on locally compact sepaerable metric state spacees