On inverse spectral theory for self–adjoint extensions: mixed types of spectra

Sergio Albeverio[1] Johannes Brasche[2]
Hagen Neidhardt[3]

August 5, 1996

[1]Institut für Mathematik, Ruhr–Universität Bochum, Universitätsstr. 150, 44801 Bochum, Germany; BiBoS; SFB 237 Bochum–Essen–Düsseldorf; CERFIM Locarno

[2]Fakultät für Mathematik, Universität Bielefeld, Postfach 10 01 31, 33501 Bielefeld, Germany

[3]Fachbereich Mathematik, Universität Potsdam, Postfach 60 15 53, 14415 Potsdam, Germany

Abstract

Let H be a symmetric operator in a separable Hilbert space $\mathcal{H}$. Suppose that H has some gap J. We shall investigate the question about what spectral properties the self–adjoint extensions of H can have inside the gap J and provide methods how to construct self–adjoint extensions of H with prescribed spectral properties inside J.

Under some weak assumptions about the operator H which are satisfied, e. g., provided the deficiency indices of H are infinite and the operator $(H-\lambda)^{-1}$ is compact for one regular point λ of H, we shall show that for every (auxiliary) self–adjoint operator M' in the Hilbert space $\mathcal{H}$ and every open subset J_0 of the gap J of H there exists a self–adjoint extension $\tilde{H}$ of H such that inside J the self–adjoint extension $\tilde{H}$ of H has the same absolutely continuous and the same point spectrum as the given operator M' and the singular continuous spectrum of $\tilde{H}$ in J equals the closure of J_0 in J. Moreover we shall present a method how to construct such a self–adjoint extension $\tilde{H}$. Via our methods it is possible to construct new kinds of self–adjoint realizations of the Laplacian on a bounded domain Ω in $\mathbb{R}^d$, $d > 1$, with spectral properties very different from the spectral properties of the self–adjoint realizations known before.

Mathematics Subject Classification (1991). 47A10, 47A60, 47B25, 47E05, 47F05

Key Words. Inverse spectral theory, self–adjoint extension, Laplace operator