Representations of bisected posets and reflection functors

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Abstract

We construct a complete set of reflection functors for the representations of posets and prove that they really have the usual properties. In particular, when the poset is of finite representation type, all its indecomposable representations can be obtained from some "trivial" ones via reflections. To define such reflection functors, a wider class of matrix problem is introduced, called "representations of bisected posets".

Representations of partially ordered sets ("posets") introduced by Nazarova and Roiter [7] were, maybe, the first example of matrix problems, whose theory was circumstantially elaborated. Later on they played an eminent role in the whole theory of matrix problems as well as of representations of algebras. So it is not surprising that it was also the first case, when one tried to introduce something like Bernstein–Gelfand–Ponomarev's reflection functors. Namely, one could not define all of them though in [2] the author managed to introduce some substitutes for one of them and the product of all others. That was enough to consider the "Coxeter transformation" and to obtain almost all necessary results for the posets of finite representation type. Nevertheless, some unsatisfaction remained, as one would like to get all reflections, not only their composition. Some hint was contained in Ovsienko's paper [9], where all reflections were introduced for another class of matrix problems. The thing was that, though starting with usual representations of

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algebras, Ovsienko had to extend the considered class of problems in such way that the sought construction became possible. The resulting class was far from being too evident as it contained both algebras and "boxes".

The aim of this paper is to present an analogous construction for the representations of posets. To do it, we also need to widen the frames. The resulting class of matrix problems, which we call "representations of bisected posets" ("bisposets"), is rather unusual as it contains non-free (even non-semi-free) boxes. Remark that the boxes arising in [9] could also be non-free, but this "non-freedom" was more or less imaginary. Namely, no generator ("arrow") of those boxes could be involved both into the relations and into the differential of the box, i.e. "additional transformations" of representations. On the contrary, in our case the interaction between relations and differential becomes nearly indispensable. Nevertheless, we are able to establish practically all "usual" properties for the new reflection functors as well as to exhibit their applicability in the simplest case of the posets of finite representation type.

Remind that another approach to the reflection functors (for quivers) led to the tilting theory. One may hope that a more detailed study of the new reflection functors would lead to some extended tilting theory applicable both to algebras and to boxes and showing their interrelations at least in the "simply connected case" (whatever the last term could mean).

Of course, we use freely the language and technique of boxes (=bocses=BOCS's) referring to [4] or [10] as recommended sources. For the readers of the English translation of [2] we should also remark that its translator called indecomposable representations "irreducible". Fortunately, irreducible representations themselves do not occur there, so no mixture is possible.

1 Bisposets and their representations

Throughout this paper we fix some field $\mathbf{k}$ and all algebras, categories etc., which we are considering, are algebras, categories etc. over this field.

A *bisected poset* (or *bisposet*) is, by definition, a poset $\mathbf{S}$ together with its bisection: $\mathbf{S} = \mathbf{S}^- \sqcup \mathbf{S}^+$, such that $j \in \mathbf{S}^+$ and $i < j$ implies that $i \in \mathbf{S}^+$ too. Choose a new symbol 0 not lying in $\mathbf{S}$ and put $\hat{\mathbf{S}} = \mathbf{S} \sqcup \{\,0\,\}$. For each bisposet $\mathbf{S}$ define the corresponding box $\mathcal{A}(\mathsf{S}) = (\mathsf{A}, \mathsf{V})$ in the following way: