Dobrushin's uniqueness for quantum lattice systems with nonlocal interaction

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Abstract

Based on Dobrushin's fundamental criterion, we prove uniqueness of Euclidean Gibbs states for a certain class of quantum lattice systems with unbounded spins, nonharmonic pair potentials and infinite radius of interaction. The necessary estimates on Dobrushin's coefficients are obtained from the Log-Sobolev inequality which holds for the one-point conditional distributions on the infinite dimensional single spin (= loop) spaces.

Dedication

This paper is dedicated to the memory of Ronald Dobrushin, to whom we are forever grateful, for all he has given us, by his inspired work, his warm friendship, his being in many ways a true teacher for us.

1 Temperature loop space representation for quantum Gibbs states

Let $\mathbb{Z}^d$, $d \in \mathbb{N}$, be the integer lattice with the Euclidean distance $|k-j|$, $k, j \in \mathbb{Z}^d \subset \mathbb{R}^d$. By $S(\mathbb{Z}^d)$ and $S'(\mathbb{Z}^d)$ we denote the mutually dual spaces of fastly decreasing resp. slowly increasing sequences over $\mathbb{Z}^d$, i.e.,

$$S(\mathbb{Z}^d) := \left\{ x = (x_k)_{k\in\mathbb{Z}^d} \in \mathbb{R}^{\mathbb{Z}^d} \mid \forall\, p \in \mathbb{Z}_+, \sum_{k\in\mathbb{Z}^d} (1+|x|)^p x_k^2 < \infty \right\},$$

$$S'(\mathbb{Z}^d) := \left\{ x = (x_k)_{k\in\mathbb{Z}^d} \in \mathbb{R}^{\mathbb{Z}^d} \mid \exists\, p \in \mathbb{Z}_+, \sum_{k\in\mathbb{Z}^d} (1+|x|)^{-p} x_k^2 < \infty \right\}.$$

Note that $S'(\mathbb{Z}^d)$ obviously is a sub-algebra of $\mathbb{R}^{\mathbb{Z}^d}$. We consider a translation invariant system of interacting quantum anharmonic oscillators with the heuristic Hamiltonian

$$H := -\frac{1}{2m}\sum_{k\in\mathbb{Z}^d}\Delta_k + \frac{a^2}{2}\sum_{k\in\mathbb{Z}^d} x_k^2 + \sum_{\{k,j\}\subset\mathbb{Z}^d} J(|k-j|)U(x_k - x_j) + \sum_{k\in\mathbb{Z}^d} V(x_k), \tag{1.1}$$

where $m > 0$ (= physical mass). The two-particle interactions $U(x_k - x_j)$ have intensities

$$J : \mathbb{R}_+ \to \mathbb{R}_+,\ J(0) = 0,\ (J(|k|))_{k\in\mathbb{Z}^d} \in S(\mathbb{Z}^d),\ \widetilde{J} := \sum_{k\in\mathbb{Z}^d} J(|k|), \tag{1.2}$$

and $U \in C^2(\mathbb{R}^1)$ is a convex function such that $\exists b_-, b_+ > 0$: $\forall q \in \mathbb{R}^1$

$$b_-^2 \le U''(q) \le b_+^2,\ 0 = U(0) \le U(q) = U(-q) \le b_+^2 q^2/2\ . \tag{1.3}$$

The harmonic self-interaction is defined by $a^2 x_k^2/2$ with a constant $a^2 > 0$. The anharmonic self-interaction potential has the form

$$V(q) = V_0(q) + W(q)\ . \tag{1.4}$$

Here $V_0 \in C^2(\mathbb{R}^1)$ is a convex function with polynomially bounded derivatives, that is, $\exists\ b,\ C > 0,\ M \in \mathbb{N}$: $\forall q \in \mathbb{R}^1$

$$V_0''(q) \ge b^2\ ,\ |V_0^{(l)}(q)| \le C(1 + |q|)^M,\ l = 0, 1, 2. \tag{1.5}$$

The potential W describes the presence of (possible) wells in the self-interaction V and is given by a bounded function

$$W \in C_b(\mathbb{R}^1). \tag{1.6}$$

We set $\delta(W) := \sup_{\mathbb{R}^1} W - \inf_{\mathbb{R}^1} W$.

Models of the above type in the classical case where the single spin spaces are just $\mathbb{R}^1$ have been studied in [BH-K].

A fundamental problem of equilibrium quantum statistical physics is the study of temperature (i.e., Gibbs) states. We will take the Euclidean approach (see e.g. the early work [AH-K] and its further developments in [KL], [GK]) to give a rigorous meaning to the Gibbs state G_β of the lattice system (1.1) at inverse temperature $\beta > 0$, i.e., we construct (via a path space representation) the corresponding Euclidean Gibbs measure ν_β on the "loop lattice" Ω_β defined as follows:

Let S_β be the circle of length β. The spaces $C(S_\beta)$ and $\mathcal{H}_\beta := L_2(S_\beta)$ consist of all continuous resp. (relative to Lebesgue measure) square integrable functions $\omega : S_\beta \to \mathbb{R}^1$, equipped with the sup-norm $\|\cdot\|_{C(S_\beta)}$ resp. L_2-norm $\|\cdot\|_\beta = (\cdot,\cdot)_\beta^{1/2}$. For the corresponding Borel σ-algebras we have:

$$\mathcal{B}(C(S_\beta)) = \mathcal{B}(\mathcal{H}_\beta) \cap C(S_\beta).$$

As the configuration space we introduce the temperature loop space ("loop lattice")

$$\Omega_\beta := C(S_\beta)^{\mathbb{Z}^d} = \{\omega = (\omega_k)_{k\in\mathbb{Z}^d} | \omega : S_\beta \to \mathbb{R}^{\mathbb{Z}^d},\ \omega_k \in C(S_\beta)\}\ ,$$

endowed with the product topology and with the Borel σ-algebra $\mathcal{B}(\Omega_\beta)$ (= σ-algebra generated by the cylinder sets

$$\{\omega \in \Omega_\beta | (\omega_k)_{k\in\Lambda} \in B_\Lambda\} \in \mathcal{B}(C(S_\beta)^\Lambda),\ ,\ \Lambda \subset \mathbb{Z}^d \text{ with } |\Lambda| < \infty).$$