ON A PROBLEM OF H. BASS FOR CHEVALLEY GROUPS OF TYPE E_7

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Abstract. This paper is devoted to the study of the surjective stability of the K_1–functor for Chevalley groups of type E_7. This is a particular case of the stability problem for Chevalley groups, which was posed by H. Bass. In the paper, surjective stability of the K_1–functor is proved under natural conditions on the dimension of the maximal spectrum of a ring and, independently, under a special condition which is meaningful from the point of view of equations defining a Chevalley group.

Introduction

The problem of K_1–functor stability for semisimple algebraic groups was mentioned by H. Bass in the book, "Algebraic K–theory"[Ba2, p.278]. He proposed to formulate stability hypotheses in terms of relations between the dimension of the maximal spectrum of a ring and the rank of a maximal split torus of a group.

In the framework of Chevalley groups, this problem was studied by M. Stein [S1],[S2] who developed an approach that considers, in the same way, both classical and exceptional cases. Despite this, however, the problem is still open. It consists of two, in fact, independent tasks. These are surjective and injective stability questions.

Traditionally, injective stability was considered as a more difficult question. However, recently it has been completely investigated. N.A. Vavilov has shown that *injective stability of K_1–functor for all regular embeddings of root systems holds under natural (depending on embedding) conditions* on **the stable rank of a ring**. This result is based on the Suslin–Tulenbaev proof [ST] of the Dennis–Vasershtein decomposition [D1],[D2],[V2].

For surjective stability the situation is as follows. It is known [Ba1],[V1],[BMS] that for the embeddings $A_{n-1} \to A_n$, $C_{n-1} \to C_n$, surjective stability holds under stable rank conditions. However, the cases of the orthogonal groups $B_{n-1} \to B_n$, $D_{n-1} \to D_n$ require stronger conditions on a ring [S2],[V2]. These are either the absolute stable rank condition ASR_n [S2], or some special condition V_n, introduced by L.Vaserstein [V2] for orthogonal and unitary groups, or just the condition on the maximal spectrum of a ring $\mathrm{dimMax}(R)$. Absolute stable rank condition turns out to be sufficient also for some other cases of Chevalley groups embeddings ($A_1 \to G_2$, $B_3, C_3 \to F_4$, $D_5 \to E_6$, [S1],[S2], [P1],[P2]).