Ergodicity for the stochastic dynamics of quasi–invariant measures with applications to Gibbs states

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Abstract

The convex set $\mathcal{M}^a$ of quasi–invariant measures on a locally convex space E with given "shift"–Radon-Nikodym derivatives (i.e., cocycles) $a = (a_{tk})_{k\in K_0, t\in\mathbb{R}}$ is analyzed. The extreme points of $\mathcal{M}^a$ are characterized and proved to be non-empty. A specification (of lattice type) is constructed so that $\mathcal{M}^a$ coincides with the set of the corresponding Gibbs states. As a consequence, via a well-known method due to Dynkin–Föllmer a unique representation of an arbitrary element in $\mathcal{M}^a$ in terms of extreme ones is derived. Furthermore, the corresponding classical Dirichlet forms $(\mathcal{E}_\nu, D(\mathcal{E}_\nu))$ and their associated semigroups $(T_t^\nu)_{t>0}$ on $L^2(E;\nu)$ are discussed. Under a mild positivity condition it is shown that $\nu \in \mathcal{M}^a$ is extreme if and only if $(\mathcal{E}_\nu, D(\mathcal{E}_\nu))$ is irreducible or equivalently, $(T_t^\nu)_{t>0}$ is ergodic. This implies time–ergodicity of associated diffusions. Applications to Gibbs states of classical and quantum lattice models as well as those occuring in Euclidean quantum field theory are presented. In particular, it is proved that the stochastic quantization of a Guerra–Rosen–Simon Gibbs state on $\mathcal{D}'(\mathbb{R}^2)$ in *infinite volume* with polynomial interaction is ergodic, if the Gibbs state is extreme (i.e., is a pure phase) which solves a long-standing open problem.

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AMS Subject Classification (1991)
Primary: 82 B 26, 60 G 60, 31 C 25
Secondary: 81 T 08, 60 J 60, 47 D 07, 81 S 20, 82 B 31, 46 G 12. 28 C 20
Keywords and phrases: quasi–invariant measures, Gibbs states, extremality, irreducibility of Dirichlet forms, ergodicity of operator semigroups and of Markov processes / diffusions, ergodicity of measures w.r.t. shifts, classical and quantum lattice models, Euclidean quantum field theory.

Contents