A New Upper Bound for the Isoperimetric Number of deBruijn Networks

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Abstract

C. Delorme and J.-P. Tillich found an upper bound and a lower bound for the isoperimetric number $i^n(d)$ of deBruijn Networks over the alphabet $\{0, 1, \ldots, d-1\}$ using eigenvalue techniques (see [DT 96]). We improve their upper bound for $i^n(d)$ and give constructions for the sets of vertices of the de-Bruijn Network, which lead to our bound.

1 Introduction

The ***d*-ary deBruijn Network $\{0, 1, \ldots, d-1\}^n_{dB}$** is an undirected graph without loops and without multiple edges. The set of vertices V^n consists of all n-tuples over the alphabet $\{0, 1, \ldots, d-1\}$. Let $u, v \in V^n$ with $u \neq v$. Assume that $v = [v_0, \ldots, v_{n-1}]$.

The vertices u and v are connected by an edge, if

$$u \in \{[b, v_0, \ldots, v_{n-2}], [v_1, \ldots, v_{n-1}, b] \mid b \in \{0, 1, \ldots, d-1\}\}.$$

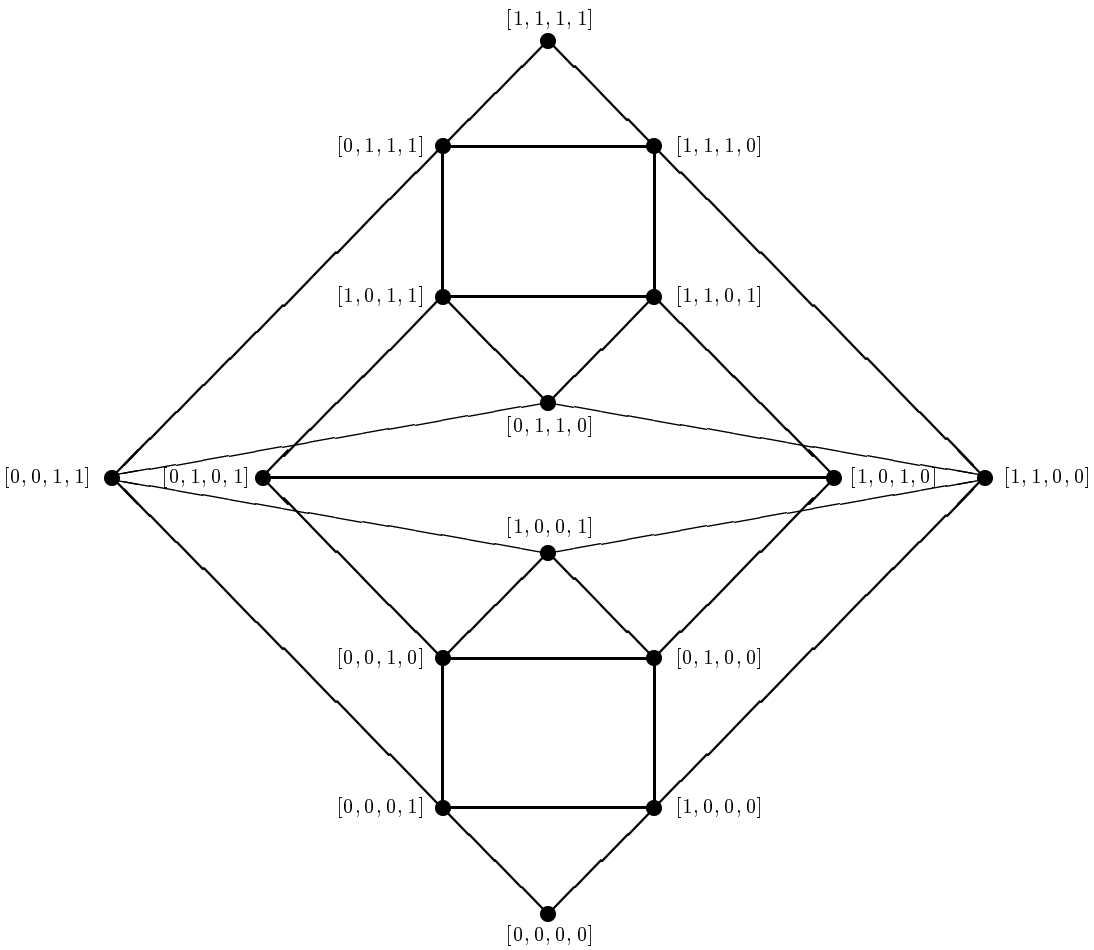

Figure 1: The $\{0,1\}^4_{dB}$

We define the **isoperimetric number $i^n(d)$ of the deBruijn Network $\{0,1,\ldots,d-1\}^n_{dB}$** by

$$i^n(d) := \min_{\substack{S \subset V^n \\ 1 \le |S| \le V^n/2}} \frac{|N(S)|}{|S|}.$$

where $N(S)$ is the set of vertices of $V^n \setminus S$ which are connected to some vertex in S.