INVERSE TRIDIAGONAL Z–MATRICES

J. J. McDonald *
Department of Mathematics and Statistics
University of Regina
Regina, Saskatchewan S4S 0A2

R. Nabben †
Fakultät für Mathematik
Universität Bielefeld
Postfach 10 01 31
33501 Bielefeld, Germany

M. Neumann ‡
Department of Mathematics
University of Connecticut
Storrs, Connecticut 06269–3009

H. Schneider §
Department of Mathematics
University of Wisconsin
Madison, Wisconsin 53706

M. J. Tsatsomeros *
Department of Mathematics and Statistics
University of Regina
Regina, Saskatchewan S4S 0A2

November 11, 1996

*Work supported by an NSERC Research Grant.
†Research supported partially by the Deutsche Forschungsgemeinschaft and NSF Grant DMS-9424346.
‡Work supported by NSF Grant DMS-9306357.
§Work supported by NSF Grant DMS-9424346.

Abstract

In this paper, we consider matrices whose inverses are tridiagonal Z–matrices. Based on a characterization of symmetric tridiagonal matrices by Gantmacher and Krein, we show that a matrix is the inverse of a tridiagonal Z–matrix if and only if, up to a positive scaling of the rows, it is the Hadamard product of a so called weak type D matrix and a flipped weak type D matrix whose parameters satisfy certain quadratic conditions. We predict from these parameters to which class of Z–matrices the inverse belongs to. In particular, we give a characterization of inverse tridiagonal M–matrices. Moreover, we characterize inverses of tridiagonal M–matrices that satisfy certain row sum criteria. This leads to the cyclopses that are matrices constructed from type D and flipped type D matrices. We establish some properties of the cyclopses and provide explicit formulae for the entries of the inverse of a nonsingular cyclops. We also show that the cyclopses are the only generalized ultrametric matrices whose inverses are tridiagonal.