On 3-regular subgraphs of 4-regular graphs with multiple edges *

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Abstract

For any 4-regular graph G (possibly with multiple edges and loops), we [1] proved recently that, if the number N of distinct Euler orientations of G is such that $N \not\equiv\ 1 \pmod 3$, then G has a 3-regular subgraph. Using this approach here we prove a conjecture of Carsten Thomassen [9] that any 4-regular connected graph with multiple edges and loops has a 3-regular subgraph, whenever the number of vertices of the graph is even.

1 Introduction

The Berge-Sauer conjecture (see [4,5]) says that any simple (no multiple edges and loops) 4-regular graph contains a 3-regular subgraph. This conjecture was proved in [7,9]. In [2,3] the Chevalley-Warning theorem was used to extend this result to the graphs with multiple edges, that are 4-regular plus an edge. Our main result of the previous paper [1] Theorem 2.3, presents

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the sufficient condition for a 4-regular graph with multiple edges to have a 3-regular subgraph. It gives the infinite families of the 4-regular graphs for which we can write out exactly the numbers of distinct Euler orientations. The conjecture of Thomassen [9] says that any 4-regular connected graph with multiple edges and loops has a 3-regular subgraph, whenever the number of vertices is even. In the mentioned above paper [1] we reduce this conjecture to the same conjecture for graphs with only multiple edges. Here we give a proof of this conjecture. So, any 4-regular connected graph G with multiple edges and loops has a 3-regular subgraph, if the number of vertices is even. This result was obtained when the author visited the Department of Mathematics of the University of Bielefeld. I would like to thank Professor Rudolf Ahlswede and Professor C. M. Ringel for the support of this research.

2 Preliminary Results

We consider the finite undirected graphs $G = (V, E)$ with multiple edges and without loops. A graph G is k-regular if every vertex of G is incident with exactly k edges.

Definition 1 *Let G be a 4-regular graph with the n vertices: $v_1, v_2, \cdots, v_n$ and $2n$ edges. An Euler orientation of G is a partition $E = \bigcup_{i=1}^{n} A_i$ of the set E of edges such that:* (i) *any two A_i are disjoint,* (ii) *each A_i has exactly two edges both incident with v_i.*

Two partitions A_i and $A'_i,\ i = 1, ..., n$, of E are different if there is i such that $A_i \neq A'_i$. Denote by $N = N(G)$ the number of different Euler orientations of the graph G. It is well known (see, for example [6]) that any connected graph, where every vertex has an even degree, has an Euler orientation. The following theorem [1] connects the number $N(G)$ of Euler orientations of a 4-regular graph G with the property of G to have a 3-regular subgraph.

Theorem 1 *A 4-regular graph G with n vertices has a 3-regular subgraph if N, the number of distinct Euler orientations of G, is such that $N \not\equiv 1$ (mod 3).*