Construction of Uniquely Decodable Codes for the Two-User Binary Adder Channel *

Rudolf Ahlswede and Vladimir B. Balakirsky, *Member, IEEE*

Fakultät für Mathematik, Universität Bielefeld,
Postfach 100131, D-33501 Bielefeld 1, Germany

E-mail addresses :
hollmann@mathematik.uni-bielefeld.de
vbal@mathematik.uni-bielefeld.de
vbal@vbal.spb.su

Abstract. A construction of uniquely decodable codes for the two-user binary adder channel is presented. The rates of the codes obtained by this construction are greater than the rates guaranteed by the Coebergh van den Braak and van Tilborg construction and these codes can be used with simple encoding and decoding procedures.

Key words :
Coding, Decoding, Multiple-Access Channel, Adder Channel

*The work was supported in part by the SFB-343, Universität Bielefeld, Germany

1 Introduction

We address the problem of constructing uniquely decodable codes for the two-user binary adder channel. Suppose that two independent users transmit binary codewords of the same length over the channel and the receiver gets a vector obtained by component-wise arithmetic sum of these codewords. The decoder has to decide which codeword was transmitted by each user with the error probability zero.

Systematic investigations of multiple-access channels were initiated by the papers [1], [2] where the achievable rate region for memoryless multiple-access channels under the criterion of arbitrarily small average decoding error probability was found. The boundary of this region for the two-user binary adder channel is defined by the equations

$$R_1 = 1, \quad R_2 = 1, \quad R_1 + R_2 = 1.5,$$

where R_1 and R_2 are the code rates of the users. These equations also give an outer bound on the code rates that can be realized under the criterion of the decoding error probability zero, i.e., the rates of the pair of codes that form a *uniquely decodable code* for the adder channel. The best known lower bound on these rates was proved by Kasami, Lin, Wei, and Yamamura [3] (this bound will be referred to as the KLWY lower bound). The first constructions of specific codes for this channel were obtained by Weldon [4]. The further results in this direction were established by Khachatrian [5], Coebergh van den Braak and van Tilborg [6], and the other authors. Probably, the code construction discovered in [6] gives the best known pairs (R_1, R_2) such that there exist uniquely decodable codes with these rates. This construction will be referred to as the CT-construction.

We will construct two binary codes, $\mathcal{U}$ and $\mathcal{V}$, of length tn, where t and n are fixed integers, in such a way that $(\mathcal{U}, \mathcal{V})$ is a uniquely decodable code for the two-user binary adder channel. Each codeword will be represented as a sequence of binary n-tuples having length t; these n-tuples will be regarded as subblocks. *The main point of our considerations is that we do not only prove the statement of an existence type concerning uniquely decodable codes, but build specific codes for fixed t and n in a regular way. The rates of these codes are located above the KLWY lower bound and these codes can be used in conjunction with simple encoding and decoding procedures.*