Vertex Set Partitions Preserving Conservativeness

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Abstract

Let G be an undirected graph and $\mathcal{P} = \{X_1, \ldots, X_n\}$ be a partition of $V(G)$. Denote by $G/\mathcal{P}$ the graph which has vertex set $\{X_1, \ldots, X_n\}$, edge set E, and is obtained from G by identifying vertices in each class X_i of the partition $\mathcal{P}$. Given a conservative graph $(G, \mathbf{w})$, we study vertex set partitions preserving conservativeness, i. e. those for which $(G/\mathcal{P}, \mathbf{w})$ is also a conservative graph. We characterize the conservative graphs $(G/\mathcal{P}, \mathbf{w})$ where $\mathcal{P}$ is a terminal partition of $V(G)$ (a partition preserving conservativeness which is not a refinement of any other partition of this kind). We prove that many conservative graphs admit terminal partitions with some additional properties. The obtained results are then used in new unified short proofs for a co-NP characterization of Seymour graphs [1], a theorem of Korach and Penn [5], a theorem of Korach [4], and a theorem of Kostochka [6].