LARGE DEVIATIONS FOR PARTIAL SUMS U-PROCESSES IN DEPENDENT CASES

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Abstract. The large deviation principle (LDP) is known to hold for partial sums U–processes of real-valued kernel functions of i.i.d. random variables X_i. We prove an LDP when the X_i are independent but not identically distributed or fulfill some Markov dependence or mixing conditions. Moreover we give a general condition which suffices for the LDP to carry over from the partial sums empirical processes LDP to the partial sums U-processes LDP for kernel functions satisfying an appropriate exponential tail condition.

1. Introduction

Let $\{X_n, n \in \mathbb{N}\}$ be a sequence of random variables on a probability space $(\Omega, \mathcal{A}, P)$ with Polish state space S. Dembo and Zajic [3] proved an LDP for

$$L_n(t) = \frac{1}{n} \sum_{i=1}^{[nt]} \delta_{X_i}, \quad t \in [0,1], \tag{1.1}$$

where the X_i fulfill Markov dependence or mixing conditions. They observed that the special conditions, which guarantee the LDP to hold for the empirical measures of Polish space valued random variables suffice for the LDP to carry over to the corresponding partial sums processes $L_n(\cdot)$. One application of these results considers the calculation of how large delays are built up in a single server queue in the presence of dependent customer interarrival times. For other applications see [3] and [19].

In [13] the LDP is proved for partial sums U-processes, i.e.

$$U_n(t) = \frac{1}{\binom{n}{m}} \sum_{C_m^{[nt]}} h(X_{i_1}, \dots, X_{i_m}), \quad t \in [0,1], \tag{1.2}$$

where the X_i are i. i. d. random variables and h is a measurable, $\mathbb{R}^d$-valued function, called kernel function, which is symmetric, i.e. invariant under permutation of its arguments. For $k, m \in \mathbb{N}$ C_m^k denotes the set $\{(i_1, \dots, i_m) : 1 \le i_1 < \dots < i_m \le k\}$. The LDP holds in $L_\infty([0,1], (\mathbb{R}^d, \|\cdot\|))$, the space of almost surely bounded, measurable $\mathbb{R}^d$-valued functions on $[0,1]$, where $[0,1]$ is equipped with

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Lebesgue measure and the underlying topology is the supremum norm topology. As a payoff, applications in the linear case were extended to higher order statistics. Thus LDP results for multi-server queues or cost functions with several variables were obtainable.
The LDP for U-statistics of degree $m \in \mathbb{N}$, i.e.

$$U_n := U_n(1) = \frac{1}{\binom{n}{m}} \sum_{C_m^n} h(X_{i_1}, \dots, X_{i_m}),$$

with h and C_m^n as above, is proved in the i. i. d. case for $\mathbb{R}^d$-valued h and Polish space valued X_i in [12] and under weaker moment conditions in [10]. In [13] the LDP for $\{U_n(\cdot)\}_{n \in \mathbb{N}}$ is proved under the same exponential tail conditions for the kernel function as for U-statistics.

In this paper we discuss the LDP for the partial sums U-processes for dependent X_i as well as the LDP for another class of processes, the so called V-processes or von-Mises processes, given by

$$V_n(t) = \frac{1}{n^m} \sum_{D_m^{[n\,t]}} h(X_{i_1}, \dots, X_{i_m}), \quad t \in [0,1], \tag{1.3}$$

where $D_m^n := \{(i_1, \dots, i_m) : 1 \leq i_1, \dots, i_m \leq n\}$ and h is as above. The corresponding statistics are given by the V-statistics or von Mises-statistics given by

$$V_n = \frac{1}{n^m} \sum_{D_m^n} h(X_{i_1}, \dots, X_{i_m}).$$

Due to the symmetry of h, U_n and V_n mainly differ by the diagonal terms, where at least two of the indices $i_1, \dots, i_m$ are equal.
For weakly dependent sequences $\{X_n, n \in \mathbb{N}\}$ invariance principles and the weak convergence of appropriately normalized sample path U-processes and von Mises-processes to Brownian motion or limit processes expressible as multiple Wiener integrals were studied in [6]. Notice that U-processes indexed by a class of measurable functions are studied for example in [1].

We will proceed as follows: as in the i.i.d. situation first we will prove that the m-fold product $L_n^{\otimes m}(\cdot)$ of $L_n(\cdot)$ satisfies a large deviation principle in a suitable topology, if $L_n(\cdot)$ satisfies an LDP for the underlying sequence X_i. Next we will apply a generalized version of the well-known contraction principle using an approximation of the kernel function worked out in [13]. On the other hand a special moment condition (2.4, stated in Section 2) has to be verified to guarantee that the partial sums U-process with the approximating kernel has the same large deviation behaviour as the partial sums U-process we are interested in.

In our examples we will check Condition 2.4 for a suitable modification of $U_n(\cdot)$. The remaining part of the proof is then to verify that these modifications have a