THE DERIVED EQUIVALENCE CLASSIFICATION OF REPRESENTATION-FINITE SELFINJECTIVE STANDARD ALGEBRAS

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Introduction

Throughout this paper k denotes an algebraically closed field and all algebras, categories and functors are assumed to be k-linear; in addition all algebras are assumed to be connected, basic, finite-dimensional and with identity.

In [15, Theorem 4.2] Rickard classified Brauer tree algebras up to derived equivalence, in connection with Broué's problem [5]. In [1, Theorem 5.8] we extended this classification to representation-finite selfinjective algebras ($\neq k$) of class A_n using a covering technique for derived equivalence developed there (Cf. Membrillo-Hernandez [13]). In this paper we will further extend this classification to the class RFSS of representation-finite selfinjective standard algebras ($\neq k$). Moreover we will show that in the class of representation-finite selfinjective algebras, any standard algebra is not stably equivalent to any non-standard algebra. It immediately follows from this that the class RFSS is closed under stable equivalence. Note that derived equivalent selfinjective algebras are always stably equivalent by Keller-Vossieck [12, 2.3 Example] or Rickard [15, Theorem 2.1]. As a corollary of our results we see that the converse of this statement holds if one of the algebras is in the class RFSS. Namely, for any $\Lambda \in$ RFSS, a selfinjective algebra is derived equivalent to Λ iff it is stably equivalent to Λ.

Section 1 is devoted to a preparation. In particular we gave the precise form of induced display-functors of repetitions for a necessary class of algebras for later use. In section 2 we state our main results, which also give "complete invariants" under derived equivalence for RFSS and a complete set of representatives of the derived equivalence classes in RFSS. In section 3 we introduce computations in a formal additive hull of a spectroid to make the whole computation easy. In section 4 a covering technique developed in [1] is rephrased and a new tool is given to deal with those algebras whose Auslander-Reiten quivers are "twisted". In section 5 we give the proof of the classification theorem, and in section 6 we give the proof of the statement about standard and non-standard algebras.

1. Preliminaries

Dynkin graphs considered in this paper are only A_n ($n \geq 1$), D_n ($n \geq 4$) or E_n ($n = 6, 7, 8$); and all quivers are assumed to be *locally finite*, i.e., for each vertex x, there are only finitely many arrows which start or stop at x.

We excluded the 1-dimensional algebra k from RFSS because (i) the stable Auslander-Reiten quiver of k is empty, which is different from any algebra in RFSS; and (ii) as easily seen the isoclass of k in the class of selfinjective algebras is closed under derived equivalence; namely if Λ is a selfinjective algebra then $\Lambda \cong k$ iff Λ is derived equivalent to k.

For an additive category $\mathcal{A}$, we denote by $\mathcal{C}(\mathcal{A})$ and by $\mathcal{C}^b(\mathcal{A})$ the category of differential complexes and the category of bounded differential complexes in $\mathcal{A}$, respectively; and by $\mathcal{H}(\mathcal{A})$ and $\mathcal{H}^b(\mathcal{A})$ the corresponding homotopy categories. When $\mathcal{A}$ is an exact category, we denote by $\mathcal{D}(\mathcal{A})$ and by $\mathcal{D}^{\mathrm{b}}(\mathcal{A})$ the corresponding derived categories. For a triangulated category $\mathcal{T}$, and a class of objects E in $\mathcal{T}$, $\operatorname{tri}(E)$ denotes the smallest full triangulated subcategory of $\mathcal{T}$ closed under direct summands and isomorphisms that contains E.

1.1. **Spectroids.** As in [1] we regard every algebra as a finite spectroid. Recall from Gabriel-Roiter [7] that a small category A is called a *spectroid* (= a *locally finite-dimensional category* in [6]) if the following three conditions are satisfied:

(i) Distinct objects of A are not isomorphic;
(ii) Every object of A has a local endomorphism algebra; and
(iii) The space $A(x, y)$ is finite-dimensional for every $x, y \in A$.

For a spectroid A, A is called *finite* if A has only a finite number of objects; and A is called *locally bounded* if for every $x \in A$, there are only finitely many $y \in A$ such that $A(x, y) \neq 0$ or $A(y, x) \neq 0$.

For a spectroid A, we denote by $\operatorname{Mod} A$ the category of all (right) A-modules; by $\operatorname{mod} A$ the full subcategory of $\operatorname{Mod} A$ consisting of finitely presented objects; and by $\operatorname{pro} A$ the full subcategory of $\operatorname{Mod} A$ consisting of finitely generated projective objects. In addition, $\underline{\operatorname{mod}}\, A$ denotes the (projectively) stable category of $\operatorname{mod} A$, and we set $D := \operatorname{Hom}_k(-, k)$.

1.2. **Repetitions.** We recall the definition of the repetition of a spectroid. Let A be a spectroid.

(1) We define a spectroid, the *repetition* $A^{\mathbb{Z}}$ of A as follows (Cf. [10], [8], [7]). Objects are the pairs $x^{[n]} := (x, n)$ with $x \in A$ and $n \in \mathbb{Z}$.

$$A^{\mathbb{Z}}(x^{[n]}, y^{[m]}) := \begin{cases} \{f^{[n]} := (f, n) | f \in A(x, y)\} & \text{if } m = n; \\ \{\varphi^{[n]} := (\varphi, n) | \varphi \in DA(y, x)\} & \text{if } m = n + 1; \text{ and} \\ 0 & \text{otherwise.} \end{cases}$$

The composition

$$A^{\mathbb{Z}}(y^{[m]}, z^{[l]}) \times A^{\mathbb{Z}}(x^{[n]}, y^{[m]}) \to A^{\mathbb{Z}}(x^{[n]}, z^{[l]})$$

is given as follows:

(i) If $m = n, l = m$, then this is the composition of A:

$$A(y, z) \times A(x, y) \to A(x, z).$$

(ii) If $m = n, l = m + 1$, then this is given by the right A-module structure of $DA(-, ?)$:

$$DA(z, y) \times A(x, y) \to DA(z, x).$$