Coding for a Model of Random-Access Memories of a Computer and Write-Unidirectional Memories *

Vladimir B. Balakirsky
Member, IEEE

The author is currently with the
Fakultät für Mathematik, Universität Bielefeld,
Postfach 100131, D-33501 Bielefeld 1, Germany
on leave from the Data Security Association "Confident",
193060 St.-Petersburg, Russia
E-mail addresses :
vbal@mathematik.uni-bielefeld.de
vbal@vbal.spb.su

Abstract. The OR-NOT operation $\mathbf{x} \vee \overline{\mathbf{y}}$ between two binary vectors $\mathbf{x}$ and $\mathbf{y}$ of the same length is viewed as an operation between the new and old content of a random-access memory used to increase the speed of the access to the memory. Coding procedures that allow us to recover $\mathbf{x}$ based on $\mathbf{x} \vee \overline{\mathbf{y}}$ are developed and connections of this model of the memory with the model of write-unidirectional memory are established.

Key words :
Random-Access Memory, Write-Unidirectional Memory, Coding, Decoding

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1 Introduction and Statement of the Problem

The study of different models of the memory from the point of view of information theory was initiated by Kuznetsov and Tsybakov [1], who introduced coding problems for the memory with defective cells (some cells always output fixed bits regardless of information that was stored). Rivest and Shamir [2], Wolf, Wyner, Ziv, and Körner [3], and the other authors analyzed a model that was called "Write-Once Memory" - WOM : if $\mathbf{x}_t \in \{0,1\}^n$ is a binary vector to be stored in the WOM containing a binary vector $\mathbf{y}_{t-1} \in \{0,1\}^n$, then

$$\begin{aligned} \mathbf{y}_0 &= 0^n, \\ \mathbf{y}_t &= \mathbf{x}_t \vee \mathbf{y}_{t-1}, \quad t = 1, 2, \ldots, \end{aligned} \tag{1}$$

where 0^n denotes the all-zero vector of length n. Another model, the so called "Write-Unidirectional Memory" - WUM, was introduced by Borden [4] and Willems and Vinck [5] :

$$\begin{aligned} \mathbf{y}_0 &= 0^n, \\ \mathbf{y}_t &= \begin{cases} \mathbf{x}_t \vee \mathbf{y}_{t-1}, & \text{if } t \text{ is odd,} \\ \mathbf{x}_t \wedge \mathbf{y}_{t-1}, & \text{if } t \text{ is even,} \end{cases} \quad t = 1, 2, \ldots \end{aligned} \tag{2}$$

Transformations (2) are unidirectional in the sense that one can replace either some 0's in the memory with 1's when t is odd, or some 1's with 0's when t is even. A more general model, where updating the memory should be realized under cost constraints ("Write-Efficient Memory" - WEM), was introduced by Ahlswede and Zhang [6]. There also exists a model of "Write-Isolated Memory" - WIM, where no change of two consecutive positions of $\mathbf{y}_{t-1}$ is allowed, introduced by Cohen and Zemor [7].

The list of models mentioned above is not complete ("the reason for this blossoming are evoluting technology, fashion... and existing letters !" [8]), but all of them are oriented to an effective storage of the data that can be viewed as some operation with external memory of a computer. In the considerations below we are mainly concerned with internal or random-access memory where fast data exchange is important. Nevertheless, there are close