Put

$$c_{25} = \frac{(c_3 \log 2 + 1)}{c_2}, \qquad c_{26} = c_{25} \sum_{l=0}^{\infty} 2^{-l/2} = \frac{c_{25}\sqrt{2}}{\sqrt{2}-1}, \tag{46}$$

and introduce the events

$$A_l = \Big\{ \omega : \Delta^{(l)} \geqslant 2^l\, c_{26}\, \tau\, d^{3/2} \log^+ d \Big\}, \qquad l = 1, 2, \dots, \tag{47}$$

where

$$\Delta^{(l)} = \max_{1 \leqslant r \leqslant m_l} \Big| \sum_{j=1}^{r} X_j - \sum_{j=1}^{r} Y_j \Big|. \tag{48}$$

According to (44), (45) and (48), we have

$$\Delta^{(l)} \leqslant \Delta_1 + \dots + \Delta_l. \tag{49}$$

Taking into account the relations (42), (46), (47), (49) and applying the inequality (43) with $x = 2^{(s+l)/2}$, we get

$$\begin{aligned} \mathbf{P}\big\{ A_l \big\} &\leqslant \sum_{s=1}^{l} \mathbf{P}\Big\{ \Delta_s \geqslant 2^{(s+l)/2}\, c_{25}\, \tau\, d^{3/2} \log^+ d \Big\} \\ &\leqslant \sum_{s=1}^{l} \exp\big(-2^{(s+l)/2}\big) \leqslant c \exp\big(-2^{l/2}\big). \end{aligned} \tag{50}$$

The inequality (50) implies that $\sum\limits_{l=1}^{\infty} \mathbf{P}\big\{ A_l \big\} < \infty$, Hence, by the Borel–Cantelli lemma with probability one a finite number of the events A_l occurs only. This implies the equality (1.13) with $c_4 = 2\, c_{26} \big/ \log 2$ (see (41), (47) and (48)). □

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