Revised and Enlarged Version

Irreducible Modules of Quantized Enveloping Algebras at Roots of 1, II

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0. Introduction

A central topic in representation theory is to understand the irreducible modules. In this paper we are concerned with finite dimensional irreducible modules of quantized hyperalgebras and of hyperalgebras. Recently, big progress has been made on the characters of the irreducible modules, namely the Lusztig's conjectural formula for the characters is true for quantized hyperalgebras and is true for hyperalgebras when the characteristic of the groud field is sufficiently large, see [AJS, KL2, KT, L6].

In [X2] we see that certain monomials in a quantized hyperalgebra U_ξ (resp. a hyperalgebra $\mathfrak{U}_\mathfrak{k}$) at a root of 1 are interesting in realizing the finite dimensional irreducible U_ξ-modules (resp. $\mathfrak{U}_\mathfrak{k}$-modules). The realization seems useful in understanding the irreducible modules, for example, based on the realization and assuming char$\mathfrak{k}$=2, Xu and Ye constructed explicitly certain bases for finite dimensional irreducible $\mathfrak{U}_\mathfrak{k}$-modules for type A_4, B_3, C_3, A_5, see [XY, Y1, Y2] and then get the irreducible characters. Until now it seems difficult to get the information by other methods.

One purpose of this paper is to give a unified proof and to romove a restriction for the results in [X2, §5]. Due to technical reasons, the proof in [X2] is lengthy and involves case by case analysis, what is worse, it doesnot work for type G_2. In this paper we will give a simple and unified proof for the main results in [X2, §5], the proof works for all types. To do this we work in the category $\mathcal{O}$ of Bernstein-Gelfand-Gelfand, see §2.

Another purpose is to give some applications of the main results in [X2], such as (a) the description of the socle of a quantized Frobenius kernel (see 3.3), (b) a realization of the tensor product of two irreducible modules of a quantized Frobenius kernel (see 3.5), (c) calculation of the dimensions of some irreducible modules for type A (see 4.5-6). In [X4-5] we show that the main results in [X2] can be used to find out some (quasi) primitive elements in a Verma modules. For type $A_2,\ B_2$ we can find all (quasi) primitive elements in a Verma module of a quantized Frobenius kernel and in a Weyl module, see [X4-5].

The contents of the paper are as follows. In section 1 we introduce the basic notation and recall some basic facts. In section 2 we give a unified proof for the main results of [X2,§5], this section is based on the preprint RIMS-1067. In section 3 we describe the socle of the regular representation of $\mathbf{u}$ (the quantized Frobenius kernel) and realize the tensor product of two irreducible $\mathbf{u}$-modules as a submodule of the regular module, this section is based on part of preprint RIMS-1087. In section 4 we consider some submodules and quotient modules of the Verma modules of $\mathbf{u}$. These modules seem interesting, for example, by means of them we can compute the dimensions of irreducible modules for type A.

All results in §§3-4 have their counterparts for hyperalgebras, we will not state them in the paper.

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1. Preliminaries

1.1. Let U be the quantized enveloping algebra over $\mathbf{Q}(v)$ (v an inderterminate) corresponding to a Cartan matrix (a_{ij}) of rank n. Then U is an associative $\mathbf{Q}(v)$-algebra with