Lower Bounds on the Code Rate for a Model of Data Transmission with Side Information *

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Abstract Asymptotic lower bounds on the code rate when the encoder and decoder have some partial knowledge about the positions, where the errors may occur during transmission of a codeword are derived.

Key words :
Coding, Decoding, Localized Errors, Side Information

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1 Introduction

Suppose that a message m of some source has to be delivered to the receiver using a q-ary block code of length n and at most t bits of the transmitted codeword can be changed. In particular, this situation may occur when the participants, Encoder and Decoder, are included into a network and another user transmits his data at t time instants $i_1, \ldots, i_t \in \{1, \ldots, n\}$ in accordance with some protocol. Obviously, Encoder and Decoder solve the communication problem by using a code with the minimal distance at least $2t + 1$. We consider possible extensions of this problem and assume that Encoder and Decoder know some sets $\mathcal{E}_{t_e}, \mathcal{E}_{t_d} \supseteq \{i_1, \ldots, i_t\}$ of fixed cardinalities $t_e, t_d \geq t$. The case $t_e = t$ and $t_d = n$ (Encoder knows all positions where errors may occur, while Decoder is ignorant) corresponds to data transmission over a channel with localized errors. The coding problem for this channel was formulated in [1] as a generalization of the problem for memories with defective cells [2] and it was intensively studied last years [3], [4], [5], [6] for $q = 2$. The case $t_e > t$ and $t_d = n$ was also considered [4] and called data transmission over a channel with partially localized errors.

A basic idea developed in the analysis of the situation when Encoder has side information is a reservation of lists of codewords for every message instead of one codeword. Having received $\mathcal{E}_{t_e}$, Encoder selects one of the entries of the list assigned to m and sends it. If Decoder is ignorant about $i_1, \ldots, i_t$, then he has to construct a union of the Hamming balls of radius t around all points of the Hamming space corresponding to codewords reserved for a particular message and considers this union as a decoding region for that message.

The case $t_d < n$ means that Decoder has some side information and can change decoding regions of every message depending on $\mathcal{E}_{t_d}$. In particular, if $t_d = t$, then he knows all positions where errors may occur, like for data transmission over erasure channel. Therefore he punctures positions belonging to $\mathcal{E}_t$ and looks for a codeword coinciding with the received word at the remaining positions. If, in addition, $t_e = t$, then Encoder also knows $\{i_1, \ldots, i_t\}$ and any of q^{n-t} messages can be noiselessly delivered. If $t_e = n$, then Encoder has to use a code with the minimal distance at least $t + 1$ (instead of $2t + 1$ as for the case $t_d = n$), since Decoder's side information makes it possible to find out all positions where the transmitted code symbols are not changed.