THE THEORY OF GENERALIZED DIRICHLET FORMS AND ITS APPLICATIONS IN ANALYSIS AND STOCHASTICS

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Abstract. We present an introduction (also for non–experts) to a new framework for the analysis of (up to) second order differential operators (with merely measurable coefficients and in possibly infinitely many variables) on L^2–spaces via associated bilinear forms. This new framework, in particular, covers both the elliptic and the parabolic case within one approach. To this end we introduce a new class of bilinear forms, so–called *generalized Dirichlet forms*, which are in general neither symmetric nor coercive, but still generate associated C_0–semigroups. Particular examples of generalized Dirichlet forms are symmetric and coercive Dirichlet forms (cf. [FOT], [MR1]) as well as time dependent Dirichlet forms (cf. [O1]). We discuss many applications to differential operators that can be treated within the new framework only, e.g. parabolic differential operators with unbounded drifts satisfying no L^p–conditions, singular and fractional diffusion operators. Subsequently, we analyze the probabilistic counterpart. More precisely, we identify necessary and sufficient analytic properties of a generalized Dirichlet form on an arbitrary topological state space to be associated with a nice strong Markov process, e.g. a diffusion. These results extend previous results on the existence of associated processes in the elliptic as well as the parabolic case.

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