Strong uniqueness for a class of infinite dimensional Dirichlet operators and applications to stochastic quantization

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Abstract

Strong and Markov uniqueness problems in L^2 for Dirichlet operators on rigged Hilbert spaces are studied. An analytic approach based on a–priori estimates is used. The extension of the problem to the L^p-setting is discussed. As a direct application essential self–adjointness and strong uniqueness in L^p is proved for the generator (with initial domain the bounded smooth cylinder functions) of the stochastic quantization process for Euclidean quantum field theory in finite volume $\Lambda \subset \mathbb{R}^2$.

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1 Introduction

The theory of Dirichlet forms is a rapidly developing field of modern analysis which has intimate relationships with potential theory, probability theory, differential equations and quantum physics. We refer to the monographs [16], [17], [20] and [32] where the theory of Dirichlet forms with applications to different branches of analysis and probability theory is presented. Though the abstract general theory is well developed, specific analytic questions remain open when one studies concrete situations. In this paper we mainly discuss Dirichlet forms and corresponding Dirichlet operators on infinite-dimensional state spaces. In particular, we are concerned with the classical Dirichlet form of gradient type on a separable Hilbert space with a probability measure. Initially, the form is defined on some "minimal" domain $\mathcal{D}$.

The first analytic problem which arises when one studies such forms is closability. This problem is well understood and necessary and sufficient conditions have been found (cf. [7]). The operators associated to the Dirichlet forms generate Markov semigroups (see e.g., [17], [20], [32]) on the corresponding L^2-space. If one assumes that the same domain $\mathcal{D}$ is contained in the domain of the generator, the next natural question which arises is whether the extension in L^2 of the generator restricted to $\mathcal{D}$ is unique. There are at least two different statements of the uniqueness problem in this context. The first is the so–called Markov uniqueness problem when one asks whether the extension generating a Markov semigroup is unique. This problem is completely solved for the finite dimensional case in [41] where Markov uniqueness was obtained under the most general conditions. The situation is quite different in the infinite dimensional case. We refer to [11], [12] for the best results in this direction known so far.

One speaks of strong uniqueness for the Dirichlet form resp. operator when there is only one lower bounded self-adjoint extension of the Dirichlet operator originally defined on $\mathcal{D}$. As is well–known this is equivalent to essential selfadjointness. This problem was addressed in many papers such as e.g. [1], [2], [3], [14], [31], [43], [46] (see also the references therein). The results obtained in [1], [3] and in [31] turn out to be incomparable, although they are all expressed in terms of conditions on the logarithmic derivative of the measure.

The aim of this paper is to investigate several "uniqueness problems" connected with Dirichlet forms resp. Dirichlet operators on rigged Hilbert spaces. The main result (Theorem 1) concerns strong uniqueness in L^2. The method we use is inherited from [31] (see also [1], [2], [3]) and based upon an a–priori estimate of the cylindric smooth solutions to the corresponding parabolic equations. This estimate is a generalization of the estimate obtained in [31] to the case when the logarithmic derivative of the measure contains two terms: one of them satisfies the conditions of [31] and the other is modeled according to that in [3] (see also [1], [2]). The method of this paper is purely analytic, in contrast to that in [3]. It consists of the reduc-