ON TAME REPETITIVE CATEGORIES OF NON-POLYNOMIAL GROWTH

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Abstract. This article introduces a recipe for constructing algebras whose repetitive category is tame of non-polynomial growth. Moreover it contains a list of all the bound poset algebras which are minimal with respect to this property.

Introduction

Throughout let us fix an algebraically closed field k. The term *algebra* always denotes a finite-dimensional k-algebra (associative, with 1) which is moreover basic and connected.

The concept of the *repetitive category* (cf. 1.2) to an algebra was introduced by Hughes/ Waschbüsch at the beginning of the 80ies. It is the main tool in their classification of the representation-finite selfinjective algebras. (For instance, group algebras to finite groups are selfinjective.) Meanwhile it has been proved that tame repetitive categories of polynomial growth provide all the standard selfinjective algebras which are tame of polynomial growth (see [HW], [S]).

The algebras with tame repetitive category of polynomial growth may be divided into two classes: the ones whose repetitive category is isomorphic to the repetitive category of an algebra which is representation-infinite or simply connected, and the remaining ones. The first of these classes consists of the algebras being tilting-cotilting equivalent to a hereditary algebra of Dynkin/Euclidian type resp. to a tubular canonical algebra, and most of these algebras posess particularly nice, explicit descriptions (see [HW], [ANS], [NS], [N], [S], [AS2]). The second class (being the exceptional one) is however not completely understood.

In strict contrast, the present knowledge on the tame repetitive categories of non-polynomial growth is extremely sparse, and, as far as we know, essentially confined to the (rare, but well-understood) special biserial ones, summarized in 1.3.

The aim of these notes will be twofold:

- to present a recipe for constructing algebras with tame repetitive category, the resulting algebras being called *almost gentle* (see 2.1),
- to display a list of the *rpg-critical bound poset algebras* (see 4.2), i.e. the bound poset algebras B (= 'poset algebras with zero relations') s.t. the repetitive category to B is tame of non-polynomial growth, but the repetitive category to any proper full subcategory of B is of polynomial growth.

This article will moreover expose a complete description of all the bound poset algebras with repetitive category of polynomial growth. Any of these algebras whose number of vertices is not among $6, 7, 8, 9, 10$, any rpg-critical bound poset algebra, any pg-critical algebra with more than 6 vertices, and many other well-known classes of algebras with tame repetitive category turn out to be almost gentle.

These notes are organized as follows. Starting with a preliminary section, section 2 is dedicated to the *almost gentle algebras*, and after an introduction to *bound poset algebras* in section 3 (including a complete description of the ones with repetitive category of polynomial growth) we will present in the final section a list of the *rpg-critical bound poset algebras.*

This article is a rather faithful excerpt from the author's thesis [Ro].

1 Preliminaries

1.1 Throughout this article we will adopt the language of [R1] 2.,4.

Let B be a locally bounded k-category, which we will always suppose to be *connected*. Its (countable) object set is denoted by B_0. (Algebras are exactly such categories B s.t. $\#B_0 < \infty$.) B is said to be *triangular*, if the quiver of B does not contain oriented cycles. The definition of *tameness* of B, including its several degrees *domestic, of polynomial growth*, and *of non-polynomial growth*, will be the usual one (see any of [S], [ANS], [NS], ...). Note that, except being of non-polynomial growth, any of these properties of B carries over to the full subcategories of B.

1.2 Let B be an algebra. As introduced by Hughes/ Waschbüsch in [HW], the *repetitive category* $\hat{B}$ to B is given as: $\hat{B}_0 := B_0 \times \mathbf{Z}$ ($\mathbf{Z}$ = the integers), and for any $z \in \mathbf{Z}$, $\alpha, \beta \in B_0$ set

$$\begin{aligned} Hom_{\hat{B}}((\alpha, z), (\beta, z)) &:= Hom_B(\alpha, \beta) \\ Hom_{\hat{B}}((\beta, z), (\alpha, z+1)) &:= \mathcal{D}Hom_B(\alpha, \beta) \end{aligned}$$

and the remaining Hom-spaces in $\hat{B}$ are $= 0$, where $\mathcal{D} := Hom_k(-, k)$. The composition of morphisms in $\hat{B}$ is induced by the one in B, the mappings $Hom_B(\alpha, \beta) \times \mathcal{D}Hom_B(\gamma, \beta) \to \mathcal{D}Hom_B(\gamma, \alpha)$, which sends (φ, ψ) to ($(\xi : \gamma \to \alpha) \mapsto \psi(\xi\varphi)$), and $\mathcal{D}Hom_B(\beta, \alpha) \times Hom_B(\beta, \gamma) \to \mathcal{D}Hom_B(\gamma, \alpha)$, which assigns to (ψ, φ) the map ($(\xi : \gamma \to \alpha) \mapsto \psi(\varphi\xi)$), for α, β, γ in B_0, and the zero map $\mathcal{D}B \times \mathcal{D}B \to 0$. We will usually identify B with the full subcategory in $\hat{B}$ given by $B_0 \times \{0\}$.

It is well-known that $\hat{B}$ is a selfinjective locally bounded k-category. We denote by $\nu_{\hat{B}}$ (or just ν) the canonical shift automorphism of $\hat{B}$ which sends each $(\alpha, z) \in \hat{B}_0$ to $(\alpha, z+1)$. Observe that ν is the unique automorphism ζ of $\hat{B}$ satisfying *top* $\hat{B}(\alpha, -) \cong$ *soc* $\hat{B}(\zeta^{-1}\alpha, -)$ for each $\alpha \in \hat{B}_0$. A convex subcategory in $\hat{B}$ intersecting each $\nu^{\mathbf{Z}}$-orbit in $\hat{B}_0$ exactly in one vertex is called a (ν-) *slice* in $\hat{B}$.