On sphere–packing bound, capacity and related results for Poisson channel *

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Abstract—A new approach for getting upperbounds for the capacity and the error exponent of the Poisson channel is proposed. It is based on a new sphere–packing lowerbound for the error probability. Strong converse for the coding theorem is also established.

Index Terms—Poisson channel, sphere–packing bound, linear bound, capacity, strong converse of coding theorem.

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I. Introduction

We consider the Poisson (direct–detection photon) channel [5], [15] which is described as follows. The channel input is a waveform $S(t)$, $0 \le t \le T$, which satisfies

$$0 \le S(t) \le A < \infty \,, \tag{1.1}$$

where the parameter A is the peak power. The waveform $S(\cdot)$ defines a Poisson counting process $x(t)$ with "intensity" equal to $S(t)$ (we assume here that the "dark current" $\lambda_0 = 0$). Thus $x(t)$, $0 \le t \le T$, is a random process with independent increments such that $x(0) = 0$ and for any $0 \le t_1 \le t_2 \le T$

$$\Pr\left\{x(t_2) - x(t_1) = j\right\} = \frac{e^{-\Lambda}\Lambda^j}{j!} \,, \quad j = 0, 1, 2, \dots \,,$$

where

$$\Lambda = \int_{t_1}^{t_2} S(\tau)\, d\tau \;.$$

Let $\mathcal{X}_T$ denote the space of right continious (i.e. $x(t+) - x(t) = 0$ for any $0 \le t \le T$) nondecreasing step functions $x(t)$, $0 \le t \le T$, with $x(0) = 0$ and $x(t) - x(t-) = 0$ or $x(t) - x(t-) = 1$. Then with probability 1 any path (trajectory) $x(t)$, $0 \le t \le T$, of a Poisson process with finite intensity belongs to $\mathcal{X}_T$.

A code with parameters (M, T, σ, ϵ) is defined by the following:

a) a set of M waveforms $S_m(t)$, $0 \le t \le T$, which satisfy the "peak power constraint" (1.1) and "the average power constraint"

$$\frac{1}{T}\int_0^T S_m(t)\, dt \le \sigma A \,, \quad m = 1, \dots, M \tag{1.2}$$

(of course, $0 \le \sigma \le 1$);

b) a "decoder" mapping $D : \mathcal{X}_T \to \{1, 2, \dots, M\}$;

c) the average error probability P_e satisfies condition

$$P_e = \frac{1}{M}\sum_{i=1}^{M} \Pr\left\{D(x_0^T) \ne m \,|\, S_m(.)\right\} \le \varepsilon \,.$$

The rate R of the code (in nats per second) is $(1/T)\ln M$.