COMPOUND POISSON APPROXIMATION FOR DISSOCIATED RANDOM VARIABLES VIA STEIN'S METHOD

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Abstract. In the present paper we consider compound Poisson approximation by Stein's method for dissociated random variables. We present some applications to problems in system reliability such as k-runs, counting isolated vertices in the rectangle lattice on the torus as well as consecutive 2-systems, coloured graphs, birthday problems, connected s-systems and two dimensional consecutive-k-out-of-n systems. In particular our examples have the structure of an incomplete U-statistics. For nonnegative integer valued complete U-statistics improvements of the Poisson approximation results can not be expected in general. We mainly apply techniques from Barbour and Utev, who gave new bounds for the solutions of the Stein equation in compound Poisson approximation in two recent papers.

1. Introduction

Let Γ denote an arbitrary finite collection of indices, usually denoted by α, β and so on. Let X_α be 0-1-valued, possibly dependent, random variables and let $W = \sum_{\alpha \in \Gamma} X_\alpha$. If the X_α are weakly dependent and the value 1 occurs with small probability, Poisson approximation in total variation distance between the law of W, denoted by $\mathcal{L}(W)$, and the Poisson distribution with parameter $\mathbb{E}W$, denoted by $Po(\mathbb{E}W)$, can be successfully established via the Stein-Chen method, which at the same time gives estimates of the approximation error (see the book of Barbour, Holst and Janson [7]). If the random variables X_α can take other positive integer values or if the dependence is stronger (clumps of 1's tend to occur), the compound Poisson approximation should provide better approximation. The importance of developing a compound Poisson approach has been discussed in Aldous [1].

In [3] Barbour, Chen and Loh introduced a Stein equation for compound Poisson approximation. Roos developed the local version of the basic method in [16] and in [15] the coupling approach. The theoretical results were successfully applied to many examples in reliability theory (see [4] and [5]) and to the problem of uniform m-spacings on the unit circle (see [13] and [14]). The solutions of the corresponding Stein equation may in general grow exponentially with the mean of clumps. For a special class of distributions in [3, Theorem 5] a better bound was given which

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is comparable in sharpness to the corresponding bound in the Poisson Stein-Chen method (apart from a logarithmic term in the numerator of the bound). Recently, Barbour and Utev [9] proved bounds for the solutions of the Stein equation with respect to the Kolmogorov distance which enabled them to carry out compound Poisson approximation in this distance with the same efficiency as in the Poisson case. They assumed that the compound Poisson limit distribution has a fourth moment and that it satisfies an aperiodicity condition. In [8] the same authors proved the counterparts in total variation distance under the assumption that the compound Poisson limit distribution is aperiodic of finite exponential moment, which is hardly restrictive.

The contribution of [9] and [8] is fundamental. Barbour and Utev applied the local approach in the way developed by Roos [16] in a more general setting. Consequently, the results derived for indicator random variables in [15] and [16] became a special case of the more general results in [8].

The aim of the present paper is to improve some results stated in [4, 5, 16] by using the new bounds from [9] and [8]. We observe that the random variables in the examples considered up to now are actually dissociated ones. Moreover, we present Stein's method for compound Poisson approximation both in Kolmogorov distance and in total variation distance for this special class of random variables. In view of the theoretical work by Barbour and Utev we provide the following examples and applications: k-runs, isolated vertices in the rectangular lattice on the torus, consecutive 2-systems, coloured graphs, connected-s systems, and the two dimensional consecutive-k-out-of-n system. In [9, Section 5] the k-run example was already discussed briefly. In particular all the examples have the structure of an incomplete U-statistics. After having calculated the bound for compound Poisson approximation for U-statistics, we see that improvements of the Poisson approximation results on U-statistics (see [6]) cannot be expected in general. In the examples we are able to obtain estimates with the right behaviour when the mean of the clump tends to infinity. As a consequence of the good bounds in [9] and [8] no unwanted logarithmic factor and no unwanted e^{λ}, where λ is the mean of the clumps, come into play.

Most of our examples deal with models from reliability theory. In this theory evaluation is an important and integral feature of planing, designing and operating engineering systems. In these models the independent items constitute the system. In the definition of the system there are special critical combinations consisting of underlying independent items. A critical combination is said to fail if all items in it are failed. A failure of a critical combination is dangerous for the system. Let W be the random variable which counts the number of failed critical combinations in the system. The system is said to fail if there are at least m occurrences of failed critical combinations, where $m \geq 1$ is a fixed integer. The reliability of the system is $\mathbb{P}(W \leq m-1)$. Thus estimates of $\mathbb{P}(W \leq m-1)$ are estimates of the reliability