The Display of a Formal p-Divisible Group

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Introduction

We fix throughout a prime number p . Let R be a commutative unitary ring. Let $W(R)$ be the ring of Witt vectors. The ring structure on $W(R)$ is functorial in R and has the property that the Witt polynomials are ring homomorphisms:

$$\begin{array}{rccl} \mathbf{w}_n : & W(R) & \longrightarrow & R \\ & (x_0, \ldots x_i, \ldots) & \mapsto & x_0^{p^n} + p x_1^{p^{n-1}} + \ldots + p^n x_n \end{array}$$

Let us denote the kernel of the homomorphism $\mathbf{w}_0$ by I_R . The Verschiebung is a homomorphism of additive groups:

$$\begin{array}{rccl} {}^V : & W(R) & \longrightarrow & W(R) \\ & (x_0, \ldots x_i, \ldots) & \mapsto & (0, x_0, \ldots x_i, \ldots) \end{array}$$

The Frobenius endomorphism ${}^F : W(R) \to W(R)$ is a ring homomorphism. The Verschiebung and the Frobenius are functorial and satisfy the defining relations:

$$\begin{array}{rcll} \mathbf{w}_n({}^F x) & = & \mathbf{w}_{n+1}(x), & \text{for } \; n \geq 0 \\ \mathbf{w}_n({}^V x) & = & p\mathbf{w}_{n-1}(x), & \text{for } \; n > 0, \qquad \mathbf{w}_0({}^V x) = 0. \end{array}$$

Moreover the following relations are satisfied:

$$ {}^F\,{}^V = p, \qquad {}^V({}^F x y) = x\,{}^V y, \quad x, y \in W(R)$$

We note that $I_R = {}^V W(R)$.

Let P_1 and P_2 be $W(R)-$ modules. An ${}^F-$ linear homomorphism $\phi : P_1 \to P_2$ is a homomorphism of abelian groups, which satisfies the relation $\phi(wm) = {}^F w \phi(m)$, where $m \in P$, $w \in W(R)$. Let

$$\phi^\sharp : W(R) \otimes_{F, W(R)} P_1 \to P_2$$

be the linearization of ϕ. We will call ϕ an ^{F} -linear epimorphism respectively an ^{F} -linear isomorphism if $\phi^\sharp$ is an epimorphism respectively an isomorphism.

The central notion of these notes is that of a display. The name was suggested by the displayed structural equations for a reduced Cartier module introduced by Norman [N]. In this introduction we will assume that p is nilpotent in R .