Superimposed Codes and ALOHA system

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Abstract–**We study an application of superimposed codes [1, 2, 3] to the ALOHA-system [4] with a central station. These codes increase the transmission rate [4].**

1 Statement of the problem

Let a system contain M terminal *stations* and let a *multiple-access channel* (MAC) [5] connect these M stations to the *central station* (CS). Each terminal station has a source. The sources can generate binary sequences called *information packets*. Each information packet has the same length K, and these packets should be transmitted. The CS is interested in the *contents* of the packet. The CS is not interested in a terminal station transmitted this packet. For instance, such situation can occur in a *reference communication system* (RCS) (see **Fig. 1**) which contains a *feedback broadcast channel* (FBC) [5]. If the CS receives a *request*, then it uses the FBC to transmit the *answer* to this request to all M stations simultaneously.

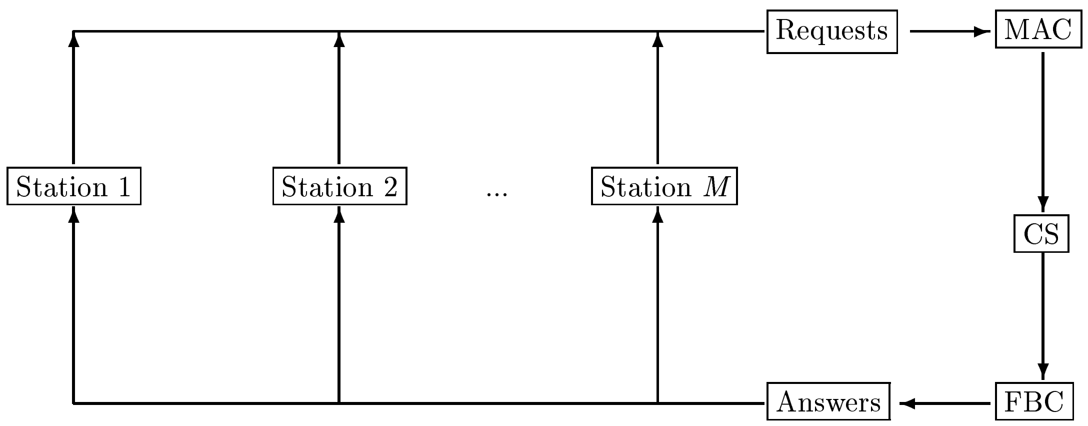

Fig.1. A block-scheme of the RCS.

2 Notations and definitions

Introduce $t = 2^K$ and enumerate all 2^K possible information packets by integers from 1 to t. Let an integer $N \geq K$ and let $X = \|x_i(u)\|$, $i = 1, 2, \ldots, N$, $u = 1, 2, \ldots, t$, be a binary $(N \times t)$-matrix (*code*). The column $\mathbf{x}(u) = (x_1(u), x_2(u), \ldots, x_N(u)) \in \{0,1\}^N$, $u = 1, 2, \ldots, t$ is called a *code packet*, corresponding to the information packet (request) with the number u. Suppose that the time is divided into the *windows* of equal lengths. For all M stations (*synchro*), each window is divided into N units. Within every window each station has one of two possibilities:

- the station can *be silent*,

- the station can send *one* of t code packets, transmitting *one binary symbol for the time unit*.

Let $\mu > 0$ be a fixed parameter. Put $p = \mu/M$, $0 < p < 1$. Suppose that the sources are independent. During the time of a given window, the source of the m-th station $m = 1, 2, \ldots, M$ randomly chooses between two possibilities:

- with probability $p = \mu/M$, the source generates one information packet which will be sent by the m-th station during the next window using the corresponding code packet,
- with probability $1 - p$, the source does not generate any packet, and the m-th station is silent during the next window.

Let the random variable ξ_μ be the number of code packets, which were transmitted during a given window. If M is sufficiently large, then ξ_μ has the Poisson distribution with parameter μ:

$$\Pr\{\xi_\mu = n\} = \frac{\mu^n e^{-\mu}}{n!}, \quad n = 0, 1, 2, \ldots. \tag{1}$$

3 Mathematical models of MAC

For the RCS, the deterministic MAC [5] (with n inputs and one output) is described by the function

$$f_n = f_n(a_1, a_2, \ldots, a_n),\ a_i \in \{0, 1\}.$$

If the MAC's inputs (signals of n transmitting stations) are binary symbols $a_1, a_2, \ldots, a_n$, then the MAC's output (signal received by CS) is denoted by $f_n(a_1, a_2, \ldots, a_n)$. We consider two models of MAC corresponding to two types of *modulation* for transmission of binary symbols. Let $\sum_{i=1}^n a_i$ denote the arithmetic sum, i.e., the number of 1's in the sequence $a_1, a_2, \ldots, a_n$. The *disjunct model* (D–model)

$$f_n(a_1, a_2, \ldots, a_n) = \begin{cases} 1, & \sum_{i=1}^n a_i \neq 0, \\ 0, & \sum_{i=1}^n a_i = 0, \end{cases} \tag{2}$$

corresponds to the *impulse modulation* (IM), and the *symmetrical disjunct model* (SD–model)

$$f_n(a_1, a_2, \ldots, a_n) = \begin{cases} 1, & \sum_{i=1}^n a_i = n, \\ 0, & \sum_{i=1}^n a_i = 0, \\ *, & 1 \leq \sum_{i=1}^n a_i \leq n-1, \end{cases} \tag{3}$$

where the symbol " $*$ " denotes an *erasure*, describes the *frequency modulation* (FM). The adequacy of these models is evident. Function (2) is called the *Boolean sum* of binary symbols $a_1, a_2, \ldots, a_n$. By analogy, function (3) will be called *symmetrical Boolean sum* of $a_1, a_2, \ldots, a_n$.

4 Superimposed (s,L,N)-codes

We say that column $\mathbf{z}$ *covers* column $\mathbf{x}$ (or $\mathbf{x}$ is covered by $\mathbf{z}$) if the component-wise Boolean sum of $\mathbf{z}$ and $\mathbf{x}$ is equal $\mathbf{z}$. Let $1 \leq s < t$, $1 \leq L \leq t - s$ be integers.

Definition [2]. An $(N \times 2^K)$-matrix X is called a *list-decoding superimposed code* (LDSC) for the D–model of *length N, size $t = 2^K$, strength s* and *list-size L* if the Boolean sum of any s-subset of columns (codewords) X can cover not more than $L - 1$ columns that are not