Covering Monomial Algebras

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1 Introduction

Although covering has been a standard technique in the representation theory of finite-dimensional algebras for several years now and it has been pointed out by various authors (cf eg [10], [7]) that using covering techniques, the question, whether or not a given algebra has up to isomorphism only finitely many indecomposable representations, is decidable, no attempt at implementing such an algorithm seems to have been made. One problem with coverings when working with a computer is the fact, that in general they tend to be infinite-dimensional, giving rise to infinite combinatorial structures.

Based on an approach of E.Green already presented in [10] that uses a criterion for finite type for tree algebras by Bongartz and Ringel [4], we will in this paper introduce an algorithm, how for a monomial algebra to construct a finite part of its universal covering just large enough to decide, whether the algebra given is of finite type.

The algorithm presented has been implemented and will be part of a future release of the computer algebra system CREP for representation theory of finite-dimensional algebras.

2 Some notation

Let Q be a directed graph, which we will always assume to be connected. We will denote by Q_0 the set of vertices of Q, and for each pair $x, y \in Q_0$ by $Q(x, y)$ the set of arrows in Q starting in x and ending in y. By a *path* of length $m \geq 0$ from a vertex x to a vertex y in Q we will understand a sequence $\varpi = \alpha_1\alpha_2...\alpha_m$ of arrows such that for $i = 1, \ldots, m-1$ the endpoint of α_i coincides with the starting point of α_{i+1}, with the starting point of α_1 being x, the endpoint of α_m being y. Especially, we will for $x \in Q_0$ by e_x denote the unique path of length 0 from x to x.

By a *walk* in Q we will understand a sequence $\omega = (p_m, p_{m+1}, \dots, p_n)$ of paths p_i with $m \in \{0, 1\}$, $m \leq n$, such that for any even index $i \in \{m, \dots, n\}$ the starting point of p_i coincides with that of p_{i+1} and the endpoint of p_i with that of p_{i-1}, provided p_{i+1}, respectively p_{i-1} exist. Introducing for each arrow α in Q say $\alpha \in Q(x, y)$ a formal inverse α^{-1} with starting point y and endpoint x, we may consider such a walk a sequence $\omega = \beta_1\beta_2 \dots \beta_l$ of arrows and formal inverses of arrows in Q, such that for $i = 1, \dots l-1$ the starting point of β_{i+1} coincides with the endpoint of β_i. We will then call the starting point of β_1 the starting point of ω, and the endpoint of β_l the endpoint of ω.

Using the convention $(\alpha^{-1})^{-1} = \alpha$ for any arrow α in Q we call a walk $w = \beta_1\beta_2 \dots \beta_m$ *reduced*, if $\beta_i \neq (\beta_{i+1})^{-1}$ for $i = 1, \dots, m-1$.

3 Monomial algebras and coverings

Let k be a field. For a directed graph Q, denote by kQ the free k-vector space over the set of all paths in Q. We define the product $p \cdot q$ of two paths p and q in Q to be the concatenation pq, if this exists (i.e., if the endpoint of p coincides with the starting point of q,) respectively otherwise to be 0. The k linear closure of this multiplication on its basis makes kQ a k-algebra, the *path algebra* of Q.

By a *monomial algebra* one understands any factor algebra $A = kQ/I$ of the path algebra of a directed graph Q by an ideal I of kQ that admits a generating set $P = \{\rho_i | i \in J\}$ consisting entirely of paths ρ_i in Q. We may assume all of these paths ρ_i to be of length at least 2, since a path of length 0 would 'delete' a vertex, a path of length 1 an arrow of Q, telling us, that we started with the wrong directed graph Q. We will call P a set of relations for Q.

Let now Q be a finite directed graph, P a set of paths in Q, I the ideal of kQ generated by P. Set $A = kQ/I$.

By a *covering* of Q we understand a connected directed graph $\tilde{Q}$ together with a map $F : \tilde{Q} \longrightarrow Q$, mapping vertices to vertices, arrows to arrows, and paths to paths (i.e. if $\alpha\beta$ is a path in $\tilde{Q}$, then $F(\alpha)F(\beta)$ is a path in Q.), having in addition the following properties: