Construction of diffusions on configuration spaces

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Abstract

We show that any square field operator on a measurable state space E can be lifted by a natural procedure to a square field operator on the corresponding (multiple) configuration space $\overline{\Gamma}_E$. We then show the closability of the associated lifted (pre–)Dirichlet forms $\mathcal{E}_\mu^\Gamma$ on $L^2(\overline{\Gamma}_E;\mu)$ for a large class of measures μ on $\overline{\Gamma}_E$ (without assuming an integration by parts formula) generalizing all corresponding results known so far. Subsequently, we prove that under mild conditions the Dirichlet forms $\mathcal{E}_\mu^\Gamma$ are quasi–regular, and that hence there exist associated diffusions on $\overline{\Gamma}_E$, provided E is a complete separable metric space and $\overline{\Gamma}_E$ is equipped with a suitable topology, which is the vague topology if E is locally compact. We discuss applications to the case where E is a finite dimensional manifold yielding an existence result on diffusions on $\overline{\Gamma}_E$ which was already announced in [AKR96a, AKR96b], resp. used in [AKR98, AKR97b]. Furthermore, we present applications in particular detail to the case where E is the free loop space.

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