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ON THE STRUCTURE OF THE CANONICAL FUNDAMENTAL SET FOR THE 2-BRIDGE LINK ORBIFOLDS

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Abstract

Let $\mathcal{O}(p/q, n)$ denote the orbifold with singular set the 2-bridge knot or link p/q and isotopy group cyclic of order n. The paper is devoted to study of the fundamental set of the orbifold $\mathcal{O}(p/q, n)$ in the hyperbolic, Euclidean and spherical three-dimensional spaces. The main goal of the paper is to construct the simplest possible fundamental polyhedron for $\mathcal{O}(p/q, n)$ in the sense that it has just two pairs of mutually congruent curvilinear faces. The fundamental polyhedron is non-convex and consists of $2p$ tetrahedra whose vertices can be defined in a canonical way from the presentation of the orbifold fundamental group.

1. Introduction

The aim of this paper is the study of the structure of the fundamental set for the 2-bridge link orbifolds. A 2-bridge knot or link p/q is determined by a pair of relatively prime integers (p, q) with $0 < q < p$. Two pairs (p, q) and (p', q') determine the same link if and only if $p = p'$ and $qq' \equiv \pm 1$ ($mod\ p$). A hyperbolic (resp. Euclidean, spherical) 2-bridge link orbifold $\mathcal{O}(p/q, n)$ has as its singular set the 2-bridge link p/q and as its underlying space the three-dimensional sphere. Away from the 2-bridge link the orbifold

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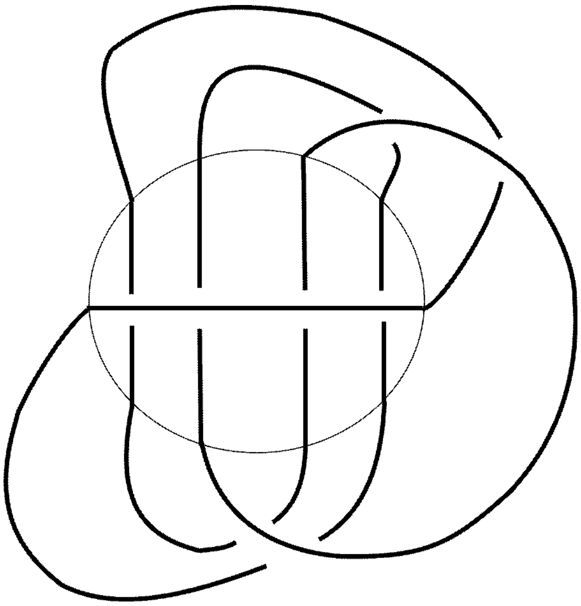

Figure 1. The figure-eight knot.

is locally isometric to hyperbolic (resp. Euclidean, spherical) three dimensional space and on the 2-bridge link it is locally the quotient space of the action of an n-fold rotation.

In Section 2 will be constructed the canonical fundamental set for $\mathcal{O}(p/q,2)$-orbifold in the spherical space. Topologically this construction is well-known [15]. It is very surprising but the same construction can be realized not only in spherical space but in hyperbolic and Euclidean spaces too. It will be do in the next three sections.

In Section 3 the structure of the fundamental set of the figure-eight orbifold $\mathcal{O}(5/2,3)$ will be constructed in Euclidean space in explicit way. More precisely, integer Cartesian coordinates of the vertices of the fundamental polyhedron for $\mathcal{O}(5/2,3)$ will be given.

In Section 4 the geometrical parameters of fundamental set for the figure-eight orbifolds $\mathcal{O}(5/2,n)$, $n>3$ will be computed.

In Section 5 the existence of the conic figure-eight manifold will be considered. Using the above mentioned construction of the fundamental set it will be shown that the conic figure-eight manifold will be realized in hyperbolic, Euclidean and spherical geometries.

In Section 6 an algorithm for producing of the canonical fundamental set for the $\mathcal{O}(p/q,n)$ orbifold will be given. This work was inspired by the papers [6], [9], [12].

2. The canonical fundamental set for orbifold $\mathcal{O}$(p/q,2) in the spherical space

Denote by $\mathbb{S}^3$ the three-dimensional spherical space. Let us consider a lens L bounded