A Direct Approach to Searching with Lies *

Vladimir B. Balakirsky

The author is currently with the
Electrical Engineering Department,
Eindhoven University of Technology,
P.O.Box 513, 5600 MB Eindhoven, the Netherlands
E-mail addresses :
v.b.balakirsky@ele.tue.nl
vbal@vbal.spb.su

Abstract We introduce a tree structure in such a way that the nodes "contain" integer-valued vectors, and the vector contained in each node is uniquely determined by the arithmetic operation applied to the vectors contained in its sons. Relationships between the distribution of terminal nodes of the tree and the vector contained in the root relates trees to the coverings of the Hamming space by the sets whose cardinalities coincide with the sizes of the Hamming balls. Application of this construction to the searching problem when one of M integers is unknown and has to be found with n questions under the assumption that e lies are allowed in the answers represents many of the known results as corollaries from the properties of the trees and leads to a new upper bound on the number of questions that differs from the known lower bound by $3e$.

Key words :
Search for a pattern, Trees, Coverings

*The work was supported by the SFB-343, Universität Bielefeld, Germany

1 Introduction

Let us consider the following game (see Fig.1). Suppose that there is a board containing two rows and $e+1$ columns numbered by integers $0, \ldots, e$ and a player who is given $c_0 + \ldots + c_e$ coins. The player distributes c_j coins among the rows of the j-th column, $j = 0, \ldots, e$. Then another player, called the devil, puts a pointer either on the first or the second row. After that all coins of the selected row are shifted by 1 position to the left; the coins of the leftmost column are removed from the board. These actions represent the first move of the game. As a result of the move, the player receives new numbers of coins in each column, and the participants make the second move. The game is finished when the board is either empty, or contains only 1 coin, no matter in which column. The goal of the player is to finish the game as fast as possible, while the devil tries to play as long as possible. Some positions that are attained in the game when the player is initially given 4 coins in column 0 and 1 coin in column 1 are shown in Fig.1, where the pointer drawn in the middle of the board means that the devil does not distinguish between the variants. Note that if the player distributes the 4 coins of column 0 into equal halves at the first move, then the devil can make the game consist of 4 moves. However, if these coins are distributed in the proportion $1:3$ and the proportion in column 1 is $1:0$, then the game is finished after at most 3 moves whatever the devil does.

The game described above relates to the problem of the search for an unknown integer m chosen from the set $\{1, \ldots, M\}$, where M is given. Suppose that this integer is chosen by the "devil". We ask him a question, i.e., present some subset $\mathcal{M}$ of the set $\{1, \ldots, M\}$ and receive the answer "yes" if $m \in \mathcal{M}$ and "no" if $m \notin \mathcal{M}$. After that we form the next question based on this and previous answers. Suppose that the devil is allowed to lie by answering "yes" if $m \notin \mathcal{M}$ and "no" if $m \in \mathcal{M}$, but the number of lies is not greater than a given number e. Furthermore, he is allowed to change m during the searching procedure in such a way that the integer he currently has in mind satisfies the rules of the game, i.e., he lied for this integer not more than e times. The positions presented in Fig.1 correspond to the case when $M = 5$, $e = 1$, the question at time instant -1 was : is $m \in \{1\}$, and the answer was "yes". As a result, we count 1 and 0 lies if $m \in \{2, 3, 4, 5\}$ and $m = 1$, respectively, and generate the vector $\mathbf{c}_0 = (4, 1)$. The distribution of coins at time instant 0 in