Ray Hölder-continuity for Fractional Sobolev Spaces in Infinite Dimensions and Applications

Jiagang REN
Department of Mathematics
Huazhong University of Science and Technology
Wuhan, Hubei 430074
P.R.China

Michael RÖCKNER
Fakultät für Mathematik
Universität Bielefeld
33615 Bielefeld
Germany

Abstract

We prove Hölder-continuity on rays in the direction of vectors in the (generalized) Cameron-Martin space for functions in Sobolev spaces in L^p of fractional order $\alpha \in (\frac{1}{p}, 1)$ over infinite dimensional linear spaces. The underlying measures are required to satisfy some easy standard structural assumptions only. Apart from Wiener measure they include Gibbs measures on a lattice and Euclidean interacting quantum fields in infinite volume. A number of applications, e.g., to the two-dimensional polymer measure, are presented. In particular, irreducibility of the Dirichlet form associated with the latter measure is proved without restrictions on the coupling constant.

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