BETTI NUMBERS FOR $p-$ STABLE IDEALS

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Introduction

Let k be an infinite field of characteristic $p > 0$, $R = k[x_1, ..., x_n]$ the polynomial ring with the standard grading $R = \bigoplus_{d \geq 0} R_d$.
Let I $\subset R$ be a monomial ideal and $G(\mathrm{I})$ a minimal set of monomials generating I. Let u be a monomial. Denote by $m(u)$ the maximum integer i such that $x_i | u$. If x_i^s divides u but x_i^{s+1} does not, we write $x_i^s \parallel u$. An ideal I is *stable* if it is monomial and for every monomial $u \in$ I and $i < m(u)$ it holds $x_i \left(u/x_{m(u)} \right) \in$ I. These ideals become well known after Eliahou - Kervaire described their minimal free resolutions (see[EK]). The generic initial ideals are Borel fixed ideals (see[Ei]), which are stable in characteristic zero. When $p = char\ k > 0$, they are not stable and they are called $p-$Borel ideals, their

study being very difficult (see [AH_2]). But the real analog of stable ideals in characteristic $p > 0$ are the $p-$ stable ideals which we introduce below:

We consider on $\mathbb{N}$ the following partial order: we say that $k \leq_p s$ if the binomial coefficient $\begin{pmatrix} s \\ k \end{pmatrix}$ is not divisible by p. Of course, $\leq_0$ is the usual order $\leq$ on $\mathbb{N}$. For a prime number p there is the following explicit description: $k \leq_p s$ if and only if $k_j \leq s_j$ for all $j \geq 0$, where k_j, s_j are defined by the $p-$ adic expansion $k = \sum_{j \geq 0} k_j p^j$, $s = \sum_{j \geq 0} s_j p^j$ of k, respectively s (see [Ei]).

Definition. A monomial ideal I is called $p-$ *stable* if $x_i^k \left(u / x_{m(u)}^k\right) \in$ I for all monomial $u \in$ I, $1 \leq i < m(u)$ and positive integers $k \leq_p s$, s being given by $x_{m(u)}^s \parallel u$.

In the first section we give some properties of $p-$ stable ideals and describe the saturated $p-$stable ideals. In the Section 2 we compute the Koszul homology of a subclass of $p-$ stable ideals, which are called *acceptable* $p-$ *stable* ideals (see Definition 2.1 and Theorem 2.5.). This was done applying the method used by Aramova and Herzog to compute the Koszul homology for Cohen - Macaulay $p-$ Borel principal ideals [AH_2]. As an application of Theorem 2.5 the Betti numbers, the Castelnuovo regularity and the Poincaré series of acceptable $p-$ stable ideals are computed. Our formula for the Castelnuovo regularity is on the line given by Pardue Conjecture (see [Pa]) and it might hold for all products of $p-$powers of stable ideals as we asked in Question 3.8.

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1. $p-$ stable ideals

As already mentioned in Introduction one considers the following partial order on $\mathbb{N}$: