Selfinjective and iterated tilted algebras of type E_6 , E_7 , E_8

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Abstract. This paper is devoted to the construction of a system of representatives of the selfinjective / the iterated tilted algebras of type E_6, E_7, E_8. The two related Pascal programs are integrated into the program package $CREP$ on (combinatorial) representation theory. A few of the output lists are contained in the Appendix.

Introduction

One of the present highlights of the representation theory of finite- dimensional algebras (over an algebraically closed field) is, surely, the theory of the representation- finite (r.f.) selfinjective algebras. According to their stable AR- quivers, these algebras are distributed into classes indexed by the Dynkin graphs, A_n, D_n, and E_6, E_7, E_8 (cf. [Ri1], [Ri2]). In this way, we speak of the *type* of such an algebra. There are general descriptions of the selfinjective algebras of the 'regular' types A_n, D_n, due to Bretscher/ Riedtmann (cf. [BLR] $6, 7$). One aim of this paper is to construct a system of representatives of the selfinjective algebras of the 'exceptional' types E_6, E_7, E_8.

An important subclass of the r.f. selfinjective algebras is given by the r.f. trivial extensions. By grouping the standard r.f. selfinjective algebras, say of Dynkin type Δ, according to the isoclass of their universal covering, these algebras can the parametrized by the (isoclasses of the) trivial extensions of type Δ.

Linking the two items in the title of this paper, we note that, due to Assem/ Happel/ Roldan, the algebras whose trivial extension is of Dynkin type Δ coincide with the iterated tilted algebras of type Δ (see [AHR]). We refer to general descriptions of the iterated tilted algebras of type A_n resp. D_n, given by Assem/ Happel resp. Assem/ Skowronski (see [AH], [AS]). The second aim of this paper is to determine a system of representatives of the iterated tilted algebras of type E_6, E_7, E_8.

In this way, this paper completes the description of

- *all representation-finite selfinjective algebras*
- *all representation-finite trivial extensions*
- *all the algebras whose trivial extension is representation-finite*

0.1 Now let us fix some notation which will be valid for the remainder of this paper. Denote by k a fixed algebraically closed field. All the algebras will be finite- dimensional k- algebras (associative, with 1), and we will furthermore assume that they are basic and connected. Moreover, if an algebra is assumed to be selfinjective, we demand that it is $\neq k$. By a module of an algebra A (resp. a locally bounded category Λ) we will mean a finite-dimensional A- right- module (resp. a finitely generated covariant functor $\Lambda \longrightarrow mod{-}k$). Maps (as well as paths in quivers) will be composed from the left to the right, that is, the composition of two maps $f : X \to Y$, $g : Y \to Z$ will be written as $fg : X \to Y \to Z$ (however, when using an argument $x \in X$, we prefer the more conventional notation fx, $gf\,x$). The *Cartan matrix* C_A of an algbra A is defined as $C_A := (dim_k e_i A e_j)_{i,j} \in \mathcal{M}_n(\mathbf{Z})$, where $1_A = e_1 + \ldots + e_n$ is a fixed decomposition of 1_A into pairwise orthogonal primitive idempotents. Finally, when speaking of a '*system of representatives*' (of certain algebras) we always mean a system of representatives of the isomorphism classes of these algebras.

0.2 We will adopt the language of [R] 2.1, 2.4 resp. 4.1, 4.2 (concerning basic concepts resp. tilting theory) and [BG] 2.1, 2.2 resp. 1, 3 (on locally bounded categories resp. covering theory).

0.3 The present paper is organized as follows. (We remark that, apart from section 1, this paper is essentially a rather complete summary of [Ro] 4.1, 6.1, 6.2, 6.3.)
In *section 1* we will present a few essential facts about simply connected (repetitive) categories and, moreover, an outline of the classifications of the (standard) r.f. selfinjective algebras, due to Bretscher/ Riedtmann resp. Hughes/ Waschbuesch. Combining these two theories, which will be done in *section 2*, leads to a description of the selfinjective algebras of type E_n, $n = 6, 7, 8$, which needs as input data 'just' a system of representatives of the repetitive categories of type E_n and, in case $n = 6$, for each of the categories of this system a mark whether its associated description by quiver and relations allows an involution. In this way, the construction of a system of representatives of the selfinjective algebras of type E_n is reduced to the consideration of certain simply connected algebras which in turn are uniquely determined by their Cartan matrix, and, moreover, that it is possible (and easy) to construct from these various Cartan matrices the associated repetitive categories of type E_n. However, these two fundamental observations were