1 Introduction

A three dimensional manifold M is called hyperbolic if its universal cover is three dimensional hyperbolic space $\mathbf{H}^3$, so M may be viewed as

$$M \cong \Gamma \backslash \mathbf{H}^3,$$

the orbit space of some lattice $\Gamma \cong \pi_1(M)$ in the group $Iso^+(\mathbf{H}^3)$ of orientation preserving isometries of hyperbolic space. For large classes of manifolds hyperbolicity is proven: In [Otal] the case of M fibering over the circle is settled, a case which conjecturally is fairly general. The proofs do not produce lattice representations of the funtamental group of the manifold in question. To do so for some classes of manifolds is the aim of the present paper. For further study of a hyperbolic manifold detailed information in this direction is indispensible. In addition although in general being non arithmetic hyperbolic lattices carry interesting arithmetic features, due to the fact that their trace fields are algebraic number fields.

The manifolds M treated here are assumed to fiber over the circle with fibre T_0 a once punctured torus. So

$$M = M(f) = (T_0 \times [0,1]) / \sim$$

where f is some homeomorphism of T_0 onto itself which is the identity close to the puncture, and "$\sim$" means the identification

$$(p,0) = (f(p),1)$$

for all $p \in T_0$. The fundamental group $\pi_1(T_0)$ is free with rank 2; we normalize a pair $\{\xi, \eta\}$ of generators such that their commutator

$$\lambda = \eta^{-1}\xi^{-1}\eta\xi$$

is represented by a loop around the puncture. The homeomorphism f will induce an automorphism f_* of $\pi_1(T_0)$. It is easy to see that it is only f_* which matters for the construction of the threemanifold $M(f)$. So we understand f to be an automorphism of $\langle \xi, \eta \rangle$. It inherits, from its topological origin, that

$$f(\lambda) = \lambda.$$

The fundamental group $\pi_1(M(f))$ has the structure of a semidirect product:

$$\begin{array}{rcll}\pi_1(M(f)) & \cong & \langle \xi, \eta \rangle \rtimes \langle \mu \rangle & \\ & = & \langle \xi, \eta, \mu | & f(\xi) = \mu \xi \mu^{-1} \\ & & & f(\eta) = \mu \eta \mu \rangle\end{array}$$

and it is this group for which a lattice representation $\Gamma(f) < Iso^+(\mathbf{H}^3)$ has to be constructed and studied (in case $M(f)$ is hyperbolic). We proceed by in general first constructing the so called deformation variety $C(f)$ of $\pi_1(M(f))$ which is, whenever it exists, an algebraic curve, and then singling out that point on $C(f)$ which corresponds to the lattice representation. The last step requires, in general, computations in high degree algebraic number fields.

Our model for hyperbolic geometry in dimension three is Poincaré's upper half space model:

$$\mathbf{H}^3 = \{(z, r) | z \in \mathbf{C}, r > 0\}$$

with

$$Iso^+(\mathbf{H}^3) \cong PSL_2(\mathbf{C}).$$

When it comes down to constructing fundamental domains explicitly we use the action of $PSL_2(\mathbf{C})$ on $\mathbf{H}^3$ described by the formula

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix} (z, r) = \left(\frac{(az + b)(\overline{cz + d}) + a\bar{c}r^2}{|cz + d|^2 + |c|^2 r^2}, \frac{r}{|cz + d|^2 + |c|^2 r^2} \right).$$

We shall deal with representations in $SL_2(\mathbf{C})$ instead of $PSL_2(\mathbf{C})$, exclusively, this being justified as the groups to be represented will not contain torsion.

Finally, for $f \in Aut\langle \xi, \eta \rangle$ we denote its Anosov monodromy by $A(f)$. It is defined to be the automorphism $\bar{f}$ on $\langle \xi, \eta \rangle$ abelianized, induced by f:

$$\bar{f} : (\bar{\xi}, \bar{\eta}) \longrightarrow (\bar{\xi}, \bar{\eta}) A(f)$$

where $\bar{\xi}, \bar{\eta}$ are ξ, respectively η, mod $\langle \xi, \eta \rangle'$. So $A(f) \in SL_2(\mathbf{Z})$ is a modular matrix.

2 Character varieties and deformation curves

We collect some standard material which is basic for the following.
A character, more precisely an ordinary, unimodular degree two character on