Lectures

V. Bavula, Krull, Gelfand-Kirillov and Filter dimension.
D. Benson, Infinite dimensional modules for finite groups.
P. Eklof, Modules with strange decomposition properties.
A. Facchini, Failure of the Krull-Schmidt property for artinian modules and for serial modules.
R. Göbel, Classifying modules over particular rings. – Can we find a borderline in terms of endomorphism rings?
B. Huisgen-Zimmermann, Algebraic compactness, direct sum decompositions, and representation type.
H. Krause, Finite versus infinite dimensional representations – a new definition of tameness.
N. Kuhn, Generic Representation Theory: a survey of basic structure.
T. Lenagan, Dimension Theory of Noetherian Rings.
A. Martsinkovsky, Cohen - Macaulay Modules and Approximations.
G. Powell, Artinian objects in the category of functors between $\mathbb{F}_2$-vector spaces.
M. Prest, Topological and geometric aspects of the Ziegler spectrum.
J. Rickard, Bousfield Localization for Representation Theorists.
A. Schofield, Birational classification of moduli spaces of representations and the endomorphism rings of bricks.
L. Schwartz, The degree filtration on polynomial functors and unstable modules.
G. Zwara, Tame algebras and degenerations of modules.

Krull, Gelfand-Kirillov and Filter dimension

Vladimir Bavula

In the first lecture a connection will be established between Krull, Gelfand-Kirillov and Filter dimension of simple affine algebras, and an analog of Bernstein's inequality will be proved for the nonzero finitely generated modules over simple affine algebras. These results will be applied to compute the Krull dimension of the ring of differential operators on a smooth affine variety. Detailed proofs will be given.

In the second lecture we give a review of (recent) results on the Krull dimension of skew Laurent extensions and generalized Weyl algebras with left Noetherian coefficients, based on papers of P. Gabriel, K.R. Goodearl, D. Jordan, T.J. Hodges, T.H. Lenagan, J.C. McConnell, J.J. Pettit, R. Rentschler, J.C. Robson, P. Smith, F. Van Oystaeyen, R.B. Warfield and V. Bavula.

These lectures can be considered as an addition to and continuation of the lectures given by Tom Lenagan.

1 Krull, Gelfand-Kirillov and Filter dimensions of simple affine algebras

Let K be a field, a module means a left one, $\otimes = \otimes_K$. Let A_n be the n'th **Weyl algebra**, i. e. the $K-$algebra with $2n$ generators $X_1, \ldots, X_n$, $\partial_1, \ldots, \partial_n$ and relations

$$\partial_i X_j - X_j \partial_i = \delta_{ij}, \text{ the Kronecker delta, } X_i X_j - X_j X_i = \partial_i \partial_j - \partial_j \partial_i = 0.$$

If char $K = 0$, then A_n is a simple Noetherian algebra.

Let K.dim, GK and fil.dim be the *(left) Krull* (in the sense of Rentschler and Gabriel, [RG]), *Gelfand-Kirillov* and *Filter* dimension respectively.

Theorem 1.1 (the Bernstein's inequality, [Be]). *Let A_n be the n'th Weyl algebra over a field of characteristic zero. Then* $\mathrm{GK}(M) \geq n$ *for every nonzero finitely generated A_n-module M.* ∎

For each function f from $\mathbf{N}_0$ to $\mathbf{R}'$, where $\mathbf{R}' = \{r \in \mathbf{R}, r \geq 1\}$ we can associate a number $\gamma(f) \in \mathbf{R}$, a *degree* of f,

$$\gamma(f) = \inf\{r \in \mathbf{R} : \ f(n) \leq n^r \text{ for sufficiently large}\, n >> 0\}.$$