On incidence algebras associated with regular cell decomposition of $\mathbb{S}^n$

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Abstract

We associate with each regular cell decomposition of $\mathbb{S}^n$ certain finite-dimensional incidence algebra. We obtain the global dimension of such algebras and prove that they are Koszul algebras. We also obtain a global dimension of some quotients of such algebras.

1 Introduction

The problem to construct a projective resolution for simple modules over finite-dimensional algebras is one of the central problem in ring theory. This question was investigated by several authors, see for example [9, 7, 11, 8, 16] and references therein.

In particular one of the central problems is to obtain minimal projective resolution which gives an information about a global dimension of an algebra.

Although it is known different methods for constructing a projective resolutions for simple modules over path algebras and their quotients ([1, 2]) there is no common methods for constructing a minimal resolutions (see also [14] for an injective resolution). There exists only some hard conditions under which it is known that classical resolution is minimal ([9]).

Moreover, usually answer is given in terms of some geometric realization of a path algebra ([10, 11, 14, 3]) which leads to rather complicated calculations in non-trivial cases.

In the subsequent paper we start our investigations in opposite direction. We define by some geometrical reason a class of incidence algebras. All those algebras are defined by the poset of cells of a regular cell decomposition of $\mathbb{S}^n$. It is easy to see that classical results lead to enormous number of difficult calculations to obtain resolutions of simple modules over such algebras. Nevertheless, we obtain minimal projective resolution for simple modules over such algebras in easy way using some special properties of regular cell decomposition.

Moreover, it happened that all our algebras belong to the class of Koszul algebras ([9, 4]) which play an important role in the study of quantuum groups ([12]).

Let us briefly describe a structure of the paper. In section 2 we collect all necessary preliminaries on incidence algebras. In section 3 we define our main object — an incidence algebra associated with regular cell decomposition of $\mathbb{S}^n$. In section 4 we construct projective resolution for simple modules over such algebras and prove that all our algebras are Koszul algebras. In section 5 we investigate a global dimension of some zero relation algebras those are quotient of our algebras. Finally in section 6 we illustrate our results by several examples.

Finally we have to note that all the results of this paper are inspired by [5], namely by famous BGG-resolution of simple finite-dimensional module over simple finite-dimensional complex Lie algebra.

2 Preliminaries

Let $\mathbb{F}$ denotes a fixed field of arbitrary characteristics.

Let P be a finite partially ordered set (poset) with a partial order $<$. In a natural way we assosiate with P its quiver (i.e. a finite oriented graph) Q as follows: the set Q_0 of vertices (points) of Q coincides with P and there is an arrow from x to y, $x, y \in Q_0$ if and only if $y < x$ and there is no $u \in Q_0$ such that $y < u < x$. Such a quiver is usually called oriented quiver ([6]). We will denote by Q_1 the set of arrows of the quiver Q.

A path of length m on Q is a sequence of arrows $\alpha_1, \dots, \alpha_m$ written $p = \alpha_m \alpha_{m-1} \dots \alpha_1$, such that the initial point of α_{i+1} coincedes with the terminal point of α_i for all $1 \leq i \leq m-1$. We also assume that to every $x \in Q_0$ corresponds a "trivial path" e_x with x as the initial and terminal point. We will denote by B the set of all pathes in Q.

Consider a path algebra $\mathbb{F}Q$ of a quiver Q. This is an $\mathbb{F}$-algebra with basis set B and a multiplication defined as follows:

1. $e_x^2 = e_x$ for any $x \in Q_0$;
2. $e_x \cdot p = p \cdot e_y = p$ if y is the initial point of p and x is the terminal point of p;
3. $p \cdot g = 0$ if the initial point of p does not coincide with the terminal point of g;
4. $p \cdot g = pg$ if the initial point of p coincides with the terminal point of g.

Let I be a two-sided ideal in $\mathbb{F}Q$ generated by all differences $p - g$ where $p, g \in B$; p and g have common initial and common terminal points.

The quotient algebra $\mathcal{A} = \mathbb{F}Q/I$ is called the incidence algebra of the poset P ([6]).

For $x, y \in P$ we will write $[x, y]$ for the following set: $\{z \in P \,|\, x < z < y\} \cup \{x, y\}$ in the case $x < y$ or $x = y$ and we put $[x, y] = \emptyset$ otherwise. We will also denote by $[\cdot, x]$ the set $\{z \in P \,|\, z < x\} \cup \{x\}$

In this paper by $\mathcal{A}$-module we mean finite-dimensional $\mathcal{A}$-module.

It is well-known (se for example [13]) that any module V over $\mathcal{A}$ can be considered as a set of $\mathbb{F}$-spaces V_x, $x \in Q_a$ and a set of linear operators $C(V, f)$, $f \in Q_1$ that satisfies all necessary relation caused by the factorization by I.

For an $\mathcal{A}$-module V we will denote by $\operatorname{Supp} V$ the set $\{x \in Q_0 \,|\, V_x \neq 0\}$.

Simple modules $S(x)$ over $\mathcal{A}$ are in $1-1$ correspondense with $x \in Q_0$ and have rather simple structure: $\operatorname{Supp} S(x) = \{x\}$ and $\dim(S(x))_x = 1$. Thus $C(S(x), f) = 0$ for any $e(x) \neq f \in B$ and $C(S(x), e_x)$ is the identity map.

Ther is also $1-1$ correspondense between indecomposable projectives $P(x)$ and $x \in Q_0$. The structure of $P(x)$ is also rather simple: $\operatorname{Supp} P(x) = [\cdot, x]$, $\dim(P(x))_y = 1$ for any $y \in [\cdot, x]$ and for a fixed base in each $(P(x))_y$ an operator $C(P(x), f)$ is a unit matrix if and only if the initial and terminal points of f are in $\operatorname{Supp} P(x)$. Othervise $C(P(x), f)$ is zero.

Clearely any path a with the initial point x and terminal point y defines a linear operator $a(V) : V_x \to V_y$ for any $\mathcal{A}$-module V. Fof two $\mathcal{A}$-modules V and W a homomorhpism $\varphi : V \to W$ is a collection of linear operators $\varphi_x : V_x \to W_x$, $x \in P$ such that for any path a with the initial point x and terminal point y holds $\varphi_y a(V) = a(W)\varphi_x$.

3 Definition of the algebras

Let K be a regular cell decomposition of the sphere $\mathbb{S}^n$ ([15]), $n > 0$. We will denote by K_i, $0 \le i \le n$ the set of i-cells in K. Consider a poset $P = P(K)$ with elements $\emptyset$, L, $e \in K_i$, $0 \le i \le n$ and the relation $<$ defined as follows:

1. $\emptyset < x$ for any $\emptyset \ne x \in P$;
2. $x < L$ for any $S \ne x \in P$;
3. $x < y$, $x, y \in K$ if and only if $x \in y$.

For the rest of the paper $\mathcal{A} = \mathcal{A}(K)$ will denotes an incidence algebra of the poset P. We will call $\mathcal{A}$ regular cell algebra (RC-algebra).

Since by definition $\emptyset$ is a minimal and S is a maximal element we have $[\cdot, x] = [e, x]$ for any $x \in P$ and $[e, S] = P$.

For $0 \le i \le n$ we will denote $P_i = K_i$ and set $P_{-1} = \{\emptyset\}$, $P_{n+1} = \{S\}$.

4 Projective resolution and global dimension

The aim of this section is to construct the minimal projective resoluition of a simple $\mathcal{A}$-modules and to obtain a global dimension of $\mathcal{A}$. As a corollary we obtain that $\mathcal{A}$ is a Koszul algebra.

Since for any $x \in P$ an algebra $\mathcal{A} \sum_{y \le x} e_y$ is again RC-algebra (trivial for $x = \emptyset$, coincides with $\mathcal{A}$ for $x = L$ and a RC-algebra with respect to a regular cell decomposition of the bound of x for $x \in K$) it is sufficient to construct the minimal projective resolution for a simple module $S(L)$.

Lemma 1. *Let $x, y \in P$, $x < y$ then*

$$\dim \operatorname{Hom}_{\mathcal{A}}(P(x), P(y)) = 1$$

and each non-zero homomorphism is monomorphism.

Proof. Let v_x (v_y) denotes some generating element of $P(x)$ ($P(y)$) i.e. $0 \neq v_x \in P(x)_x$ ($0 \neq v_y \in P(y)$). Let a be some path with the initial point y and terminal point x which is exists since $x < y$.

Let $\varphi \in \mathrm{Hom}_{\mathcal{A}}(P(x), P(y))$. Then $\varphi(v_x) \in P(y)_x$ and thus $\varphi(v_x) = cav_y$ for some $c \in \mathbb{C}$. Since v_x generates $P(x)$ each $c \in \mathbb{C}$ defines homomorhish $\varphi_c \in \mathrm{Hom}(P(x), P(y))$ by $\varphi_c(v_x) = cav_y$.

We obtain $\dim \mathrm{Hom}(P(x), P(y)) = 1$. One see that any non-zero homomorhism is monomorphism. $\square$

For any $x \in P$ fix v_x — some generator of the module $P(x)$.

Set $C_i = \bigoplus_{x \in P_i} P(x)$, $-1 \leq i \leq n+1$. By lemma 1 any homomorphism $\varphi : C_i \to C_{i+1}$, $-1 \leq i \leq n$ is uniquely defined by a complex matrix

$$d = (d_{yx})_{y \in P_{i+1}}^{x \in P_i}.$$

Extend our cell-complex K to cell-decomposed ball K' by adding one $n+1$-cell L in a natural way. Fix an orientation for each $x \in K'$ ([15]) and set $[x : y]$ be the incidence numbers with respect to this orientation. Denote by

$$d_i = (d_{yx})_{y \in P_{i+1}}^{x \in P_i}, \quad 0 \leq i \leq n$$

the complex matrix such that $d_{yx} = [y : x]$. We also set

$$d_{-1} = (d_{x\emptyset})_{x \in P_0}, \quad d_{x\emptyset} = 1 \text{ for all } x \in P_0.$$

As it was mentioned d_i, $-1 \leq i \leq n$ defines some homomorphism $d_i : C_i \to C_{i+1}$.

Denote by $p : C_{n+1} \to S(L)$ a canonical projection.

Theorem 1. *The sequence*

$$0 \to C_{-1} \xrightarrow{d_{-1}} C_0 \xrightarrow{d_0} C_1 \xrightarrow{d_1} \dots \xrightarrow{d_{n-1}} C_n \xrightarrow{d_n} C_{n+1} \xrightarrow{p} S(L) \to 0 \tag{1}$$

is the minimal projective resolution of $S(L)$.

We divide the proof of this theorem into several steps.

Lemma 2. *The sequence* (1) *is a complex (i.e. $d_{i+1} \circ d_i = 0$ for $-1 \leq i \leq n-1$ and $p \circ d_n = 0$).*

Proof. $p \circ d_n = 0$ is obvious.

By [15, Theorem 6.2] for any $x \in P_i$, $y \in P_{i+2}$ the number of $z \in P_{i+1}$ such that $x < z < y$ is either 2 or 0 for each $0 \leq i \leq n-1$. By [15, Theorem 6.6] for any $0 \leq i \leq n-1$ and any $x \in P_i$, $y \in P_{i+2}$ such that there exists two distinct z_1, $z_2 \in P_{i+1}$ with $x < z_j < y$, $j = 1, 2$ holds

$$[y : z_1][z_1 : x] + [y : z_2][z_2 : x] = 0.$$

At the same way by [15, Theorem 6.6] for any $x \in P_1$ with faces $z_1, z_2 \in P_0$ holds

$$[x : z_1] + [x : z_2] = 0.$$

We obtain that the product of matrieces $d_{i+1}d_i = 0$ for all $-1 \le i \le n$. This completes the proof. $\square$

Proposition 1. *The sequence* (1) *is exact.*

Proof. Clearely it is sufficient to show that for any fixed $x \in P$ the induced sequence

$$0 \to (C_{-1})_x \to (C_0)_x \to \cdots \to (C_n)_x \to (C_{n+1})_x \to S(L)_x \to 0 \tag{2}$$

is exact. And this is obviously true for $x = L$.

To prove this we will construct a new complex. Suppose that $x \in P_k$, $x \ne L$. For $i \ge k$ set A_i to be a free abelian group with the base a_y, $y \in P_i$, $x < y$. Let $\delta_i : A_i \to A_{i+1}$, $k \le i \le n$ be a homomorphism defined by the matrix

$$\delta_i = ([y : z])^{z \in P_i, x<z}_{y \in P_{i+1}, x<y}.$$

One can see that complex (2) is exact if and only if complex

$$0 \to A_k \xrightarrow{\delta_k} A_{k+1} \xrightarrow{\delta_{k+1}} \dots \xrightarrow{\delta_{n-1}} A_n \xrightarrow{\delta_n} A_{n+1} \to 0 \tag{3}$$

is exact.

Let X denotes a cell subcomplex of K formed by the union of all y, $y > x$ and set $Y = X/x$. Then homology of the sequence (3) coincides with homology of some regular cell complex which is homotopic to Y. Thus they coincides with a reduced homology of the topological space Y. Since Y is homotopic to the one-point space we conclude that the sequence (3) and thus the sequence (1) is exact. This completes the proof. $\square$

Proof of Theorem 1. Clearely each C_i is projective for $-1 \le i \le n+1$. By lemma 2 and proposition 1 the sequence (1) is a projective resolution of $S(L)$. So we need only to prove that it is minimal. This follows from the fact that $\operatorname{Ker} d_i \subset \operatorname{Rad} C_{i+1}$, $-1 \le i \le n$. $\square$

Corollary 1. $\operatorname{Gl.dim} \mathcal{A} = n + 2$.

Proof. Follows from the minimality of the resolution (1). $\square$

let A be a path algebra of some finite quiver Q and I be an ideal of A generated by all arrows. Set $A' = A/I$. A' is semi-simple algebra isomorphic to finite product of copies of $\mathbb{F}$, one for each vertex. Define an Ext-algebra $E(A)$ by

$$E(A) = \sum_{i \ge 0} \operatorname{Ext}^i_A(A', A').$$

An algebra A is called Koszul algebra if the Ext-algebra $E(A)$ is generated by elements of degree 0 and 1 (here we mean the standard grading). For more ditails on Koszul algebras see [12, 4].

Corollary 2. *$\mathcal{A}$ is a Koszul algebra.*

Proof. Clearly $\mathcal{A}$ is quadratic. By [12] $\mathcal{A}$ should be a Koszul algebra if the i-th component in the minimal projective resolution of $\mathcal{A}'$ is generated by elements of degree i. This follows immediately from theorem 1. $\square$

5 Global dimension of some quotient algebras

In this section we obtain a global dimension for some quotient algebras of $\mathcal{A}$. Namely for the quotients that are defined by one zero relation. We note that a quotient of algebras by zero relationes were studied by different authors, see for example [8] where it was investigated a global dimension of some quotients of a path algebras of tree-like quivers by zero relations. In [8] were obtained an upper bound for a global dimension. Her we give complete answer for quotients of RC-algebras.

Consider an algebra $\mathcal{A}$ defined in section 3. Let a be some path with the initial point x and terminal point y, $x, y \in P$. Let $I = \mathcal{A}a\mathcal{A}$. Denote by $\mathcal{A}_a$ an algebra $\mathcal{A}/I$.

We note some difference with [8]. We will also consider the case when a is single arrow from Q. This is not trivial case (as it was it [8]) since $\mathcal{A}$ is not a path algebra of Q but is a quotient by some commutative relations.

Theorem 2. $\operatorname{Gl.dim} \mathcal{A}_a = n + 2$.

To prove this we need some auxilary lemmas. We recall that all the modules considered in this section are $\mathcal{A}_a$-modules.

Lemma 3. *For $z > x$ or $z = x$, $z \in P$ holds*

$$\operatorname{Supp} P(z) = \{u \in P \mid u < z, u \not< y\}.$$

Proof. Follows from the fact that the path a is zero in $\mathcal{A}_a$. $\square$

Lemma 4. *For $z \not> x$, $z \in P$ holds* $\operatorname{Supp} P(z) = [\emptyset, z]$.

Proof. Since I is generaded by a it follows that $u \notin \operatorname{Supp} P(z)$ if and only if $u \not< z$ or there exists a path with initial point z and with terminal point u wich contains a as a subpath. Thus $z > x$ that contradicts with the assumptions of our lemma. $\square$

Proof of the theorem 2. We will consider four different cases.

Case 1. Assume that $x = L$ and a length of a (i.e. number of arrows in a) is greater than 1. Then for any $z \in P$, $z \neq x$ a projective resolution of $S(z)$ can be constructed by theorem 1 and thus $\operatorname{Gl.dim} \mathcal{A}_a \geq n+1$. We will try again to construct projective resolution for $S(L)$.

Let C_i, $-1 \leq i \leq n+1$, $i \neq n$ and d_i, $1 \leq i \leq n-2$ be defined as in section 4. Set

$$C_n = \bigoplus_{z \in P_n} P(z), \quad C'_n = P(y) \oplus \bigoplus_{z \in P_n} P(z).$$

Denote by d_{n-1} the map $C_{n-1} \to C_n$ defined by matrix

$$([u:z])_{u\in P_n}^{z\in P_{n-1}}$$

and by d'_{n-1} the map $C_{n-1} \to C'_n$ defined at the following way: the matrices elements defines a map from $P(z)$ to $P(u)$, $z \in P_{n-1}$, $u \in P_n$ and a map $P(z) \to P(y)$ is trivial for all $z \in P_{n-1}$. As usually we denote by $p : P(L) \to S(L)$ the canonical projection.

Consider the following sequence:

$$0 \to C_{-1} \xrightarrow{d_{-1}} C_0 \xrightarrow{d_0} \dots \xrightarrow{d_n} C_{n+1} \xrightarrow{p} S(L) \to 0. \tag{4}$$

One can see that the sequence (4) is a complex. Lets try to find a homology of this complex. As it done in proposition 1 for any $z \in P$ consider the corresponding comlex

$$0 \to (C_{-1})_z \to (C_0)_z \to \dots \to (C_n)_z \to (C_{n+1})_z \to (S(L))_z \to 0. \tag{5}$$

For $z \not< y$ it follows from the proof of proposition 1 that complex (5) is exact. For $z < y$ one obtain that the only $\operatorname{Ker} d_n / \operatorname{Im} d_{n-1}$ is one-dimensional.

Thus there exists such a homomorphism $d'_n : C'_n \to C_{n+1}$ that the complex

$$0 \to C_{-1} \xrightarrow{d_{-1}} \dots \xrightarrow{d_{n-2}} C_{n-1} \xrightarrow{d'_{n-1}} C'_n \xrightarrow{d'_n} C_{n+1} \xrightarrow{p} S(L) \to 0$$

is exact. It gives us resolution for $S(L)$ and case 1 follows.

Case 2. Let $x = L$ and $y \in P_n$. The proof is analogues to the proof of case 1 with substitution

$$C_n = \bigoplus_{z\in P_n, z\neq y} P(z).$$

Case 3. Assume that $x \neq L$ and a is a path of length at least 2. We will prove that sequense (1) constructed in section 4 is a projective resolution for $S(L)$. Since the length of a is greater than 1 all the maps d_i, $-1 \leq i \leq n$ are well-defined. For the exactness it is sufficient to prove that sequence (2) is exact. For the point $z \not< y$ this follows from proposition 1.

Let X denotes a cell subcomplex of K formed by union of all u, $u > z$ and $u \not> y$. Set $Y = X/z$. One can see that the exactness of sequence (2) is equivalent to the vanishing of reduced simplicial homology of Y. But this is true since Y is homotopic to point.

Case 4. Finally asume that $x \neq L$ say $x \in P_k$ and a is an arrow. Consider sequence 1 with the only difference that we put $d_{xy} = 0$. At the same way as in the case 3 one obtain that this sequence is exact.

The minimality of each resolution is obvious. $\square$

6 Examples

The aim of this section to give some visual examples of RC-algebras and algebras those are similar to RC-algebras but have another properties.

6.1 $n = -1$

It is not too clear what does S^{-1} mean. We set that in this case K is empty an thus P has the form that is shown on figure 1. In this case all resolutions are obviousely:

$$0 \to P(\emptyset) \to P(L) \to S(L) \to 0$$

and

$$0 \to P(\emptyset) \to S(\emptyset) \to 0.$$

Thus $\mathrm{Gl.\,dim}\,\mathcal{A} = 1$.

Figure 1

6.2 $n = 0$

The only possible decomposition of the S^0 leads to the poset P shown on the figure 2. One have the following resolution for $S(L)$:

$$0 \to P(\emptyset) \to P(x) \oplus P(y) \to P(L) \to S(L) \to 0.$$

Fixing the orientation we can obtain for example

$$d_{-1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad d_0 = \begin{pmatrix} 1 & -1 \end{pmatrix}.$$

Thus $\mathrm{Gl.\,dim}\,\mathcal{A} = 2$.

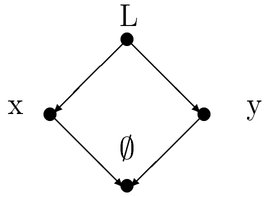

Figure 2

6.3 $n = 1$

There is infinitely many different possibilities to construct regular cell decomposition of S^1. All they give us minimal incidence algebras of global dimension 3. They coinsides with the incidense algebras of posets Q_n from [11] and thus we will omitt them.

6.4 $n = 2$

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Let K be a cell decomposition of S^2 shown on figure 3. Then P has a form shown on figure 4. Fixing an orientation we can choose for example following d_i:

$$d_{-1} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}, \quad d_0 = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix}, \quad d_1 = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}, \quad d_2 = \begin{pmatrix} 1 & -1 \end{pmatrix}.$$

Thus we obtain the following resolution for $S(L)$:

$$0 \to P(\emptyset) \to P(x) \oplus P(y) \to P(z) \oplus P(u) \to P(I) \oplus P(II) \to P(L) \to S(L) \to 0$$

and conclude that $\mathrm{Gl.\,dim}\,\mathcal{A} = 4$.

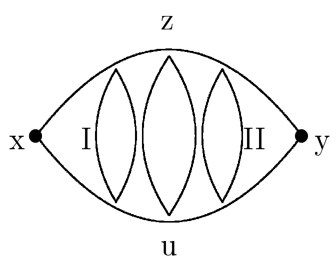

Figure 3

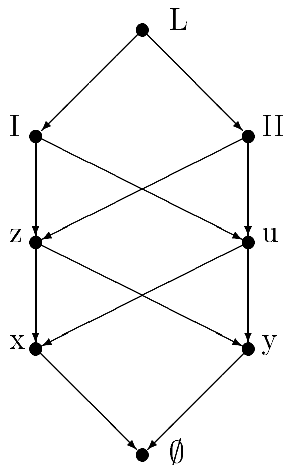

Figure 4

6.5 First example of non RC-algebra

Consider a cell complex shown on figure 5. This is the non-regular cell decomposition of S^1. Then corresponding poset P is shown on figure 6. One have

$$0 \to P(y) \to P(L) \to S(L) \to 0$$

is a resolution and thus $\mathrm{Gl.\,dim}\,\mathcal{A} = 1 \neq 3$.

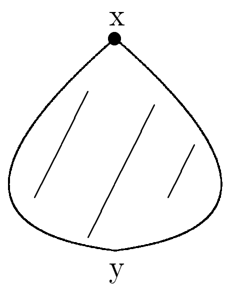

Figure 5

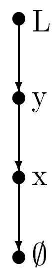

Figure 6

6.6 Second example of non RC-algebra

Consider the cell decomposition of the Möbius strip shown on figure 7. The corresponding P is shown on the figure 8. Unfortunately one obtain the following resolution:

$$0 \to P(A) \oplus P(A) \oplus P(B) \oplus P(B) \to P(l) \oplus P(m) \oplus P(n) \to P(L) \to S(L) \to O.$$

Thus $\mathrm{Gl.\,dim}\,\mathcal{A} = 2 \neq 3$.

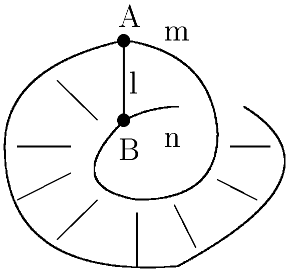

Figure 7

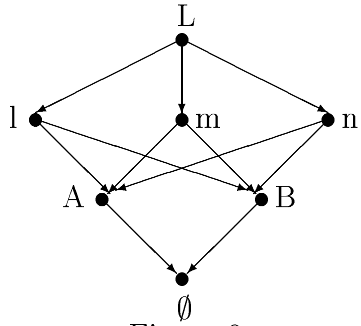

Figure 8

7 Acknowledgments

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