A NOTE ON THE $\mathbb{D}_n$-PATTERN

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Abstract. The $\mathbb{D}_n$-pattern is a class of tame vector space categories where no classification of the isomorphism classes of the indecomposable subspaces seems to be available. We show that the subspace category of a representative of this class is equivalent to the category of representations of a specific bush. For bushes the classification of the isomorphism classes of the indecomposable representations is known. Finally, we extend our results to some multiple vector space categories which are natural generalizations of those in the $\mathbb{D}_n$-pattern.

1. Vector space categories

We fix an algebraically closed field k. As general sources for notation we refer to [Ri2] and [GR]. In particular, by *aggregate* we denote a k-additive category with finite-dimensional morphism spaces such that idempotents split. By $\operatorname{ind} K$ we denote a *spectroid* of K that is a full subcategory which contains exactly one object out of each isomorphism class of indecomposable objects in K.

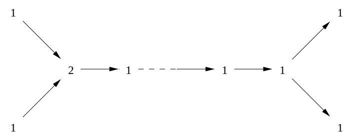
A *vector space category* is a pair (K, M) where K is an aggregate and $M : K \to \operatorname{mod} k$ is a k-linear functor. The *subspace category* $\check{\mathcal{U}}(K, M)$ of (K, M) is again an aggregate whose objects are the triples $X = (X_\omega, \gamma_X, X_0)$ such that X_ω is a finite-dimensional k-space, X_0 is an object of K and $\gamma_X : X_\omega \to M(X_0)$ is a k-linear map. Morphisms $X \to Y$ in $\check{\mathcal{U}}(K, M)$ are pairs $f = (f_\omega, f_0)$ consisting of a k-linear map $f_\omega : X_\omega \to Y_\omega$ and a morphism $f_0 : X_0 \to Y_0$ in K such that $M(f_0)\gamma_X = \gamma_Y f_\omega$. Sometimes one is only interested in the full subcategory $\mathcal{U}(K, M)$ of $\check{\mathcal{U}}(K, M)$ given by all objects X such that γ_X is a monomorphism. We call $\mathcal{U}(K, M)$ the *proper subspace category* of (K, M). The only object in $\operatorname{ind}\check{\mathcal{U}}(K, M)$ not belonging to $\operatorname{ind}\mathcal{U}(K, M)$ is $(k, 0, 0)$.

The vector space category (K, M) is called *faithful* if the functor M acts faithfully on K. We can always produce a faithful version (K_M, M) of a given vector space category (K, M) by defining K_M as $K/\operatorname{Ker} M$ where $\operatorname{Ker} M$ is the ideal of K given by all morphisms of K with $M(f) = 0$. The functor M acts faithfully on K_M in the natural way and there is an obvious full and dense functor $\check{\mathcal{U}}(K, M) \to \check{\mathcal{U}}(K_M, M)$. The indecomposable objects of $\check{\mathcal{U}}(K, M)$ which are mapped to 0 by this functor are precisely the ones which are isomorphic to $(0, 0, X_0)$ where X_0 is an indecomposable object of K satisfying $M(X_0) = 0$.

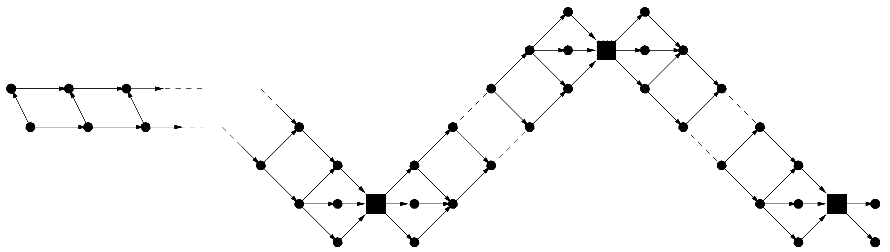
The reason for introducing vector space categories in the representation theory of finite-dimensional algebras is that there are various reduction functors which allow to reduce the study of the finite-dimensional modules over a finite-dimensional algebra to the study of the subspace category of a vector space category (see e.g. [Ri1], [Ri2], [G-V], [Dr1]). A prominent example of such a reduction technique appears for one-point extension algebras where the reduction functor is rather obvious. The one-point extension technique was used in [Ri1] in order to classify tame one-relation algebras. Therefore in this paper the tameness of several vector space categories had to be established. Some of these tameness proofs relied on further reductions such that only tameness could be obtained but the control on the indecomposable objects was lost.

An example of this effect are the vector space categories in the $\mathbb{D}_n$-pattern. *Patterns* are equivalence classes of vector space categories. The underlying equivalence relation was defined in [Ri1] in order to reduce the analysis of the subspace category to a single representative. Namely, two vector space categories (K, M) and (L, N) are said to be equivalent if there are cofinite (i.e. only finitely many indecomposable objects are not contained in the image) embedding functor $\alpha : K \to L$ and $\beta : L \to K$ such that $N\alpha \cong M$ and $M\beta \cong N$.

Canonical representatives of the $\mathbb{D}_n$-pattern, where $n \in \mathbb{N}$, $n \geq 4$, are the vector space categories (K_M, M) given as follows. The category K is the category $\operatorname{mod} H$ of finite-dimensional modules over the tame hereditary algebra $H = k\Delta$ where the underlying graph of the quiver Δ is an extended Dynkin diagram of type $\widetilde{\mathbb{D}}_n$. The functor M equals $\operatorname{Hom}_H(R, -)$ where R is an indecomposable regular module in $\operatorname{mod} H$ which is of regular length 2 and lies in a non-homogeneous tube of rank $n-2$. We choose one representative $\mathcal{D}_n$ which is given by the following quiver where to the vertices we attach the coefficients of the dimension vector which determines R uniquely up to isomorphism. Note, that we consider R as a covariant representation of the quiver.



A faithful vector space category like (K_M, M) is usually presented by the quiver with relations of its spectroid $\operatorname{ind} K_M$ and the dimensions of the spaces $M(X)$ for each object X in $\operatorname{ind} K_M$. If the dimensions of the spaces $M(X)$ are bounded by 2, then one draws a square for X if $\dim_k M(X) = 2$ and a dot for X if $\dim_k M(X) = 1$. With these conventions the data of the vector space category $\mathcal{D}_n$ look as follows:



Since the quiver of $\operatorname{ind} K_M$ is the translation subquiver of the Auslander-Reiten quiver of H supported by the indecomposable modules X satisfying $M(X) \neq 0$, we have to impose the usual mesh relations. It follows from [Dr2] that these are all necessary relations and that $\mathcal{D}_n$ is uniquely determined by the picture. But observe that, since the quiver has two infinite components, one has to consider suitable finite predecessor closed pieces of the first and successor closed pieces of the second component, in order to make use of [Dr2].

2. Multiple vector space categories

A *multiple vector space category* is also a pair (K, M) where K is an aggregate but $M : K \to \operatorname{mod} k^r$ is now a functor with values in the category of modules over some semisimple algebra k^r where $r \in \mathbb{N}$ is called the *number of peaks* (see [Si]). Thus vector space categories are the multiple vector space categories with one peak. The subspace category $\check{\mathcal{U}}(K, M)$ and also the proper subspace category $\mathcal{U}(K, M)$ are defined in the completely analogous way. Note that for an object $X = (X_\omega, \gamma_X, X_0)$ of $\check{\mathcal{U}}(K, M)$ the k^r-module X_ω can be considered as a tuple $(X_\omega^{(1)}, \ldots, X_\omega^{(r)})$ of finite-dimensional k-spaces. Analogously, the functor M becomes a sequence $(M^{(1)}, \dots, M^{(r)})$ of functors $M^{(i)} : K \to \operatorname{mod} k$. Consequently, γ_X can be considered as a tuple $(\gamma_X^{(1)}, \dots, \gamma_X^{(r)})$ of k-linear maps $\gamma_X^{(i)} : X_\omega^{(i)} \to M^{(i)}(X_0)$. In particular, the objects of $\operatorname{ind}\check{\mathcal{U}}(K, M)$ not belonging to $\operatorname{ind}\mathcal{U}(K, M)$ are the objects $(S_i, 0, 0)$ where S_i is the tuple $(0, \dots, 0, k, 0, \dots, 0)$ having k in the i-th position.

Given a multiple vector space category (K, M) with $r \geq 2$ peaks we consider the vector space category $(K, M^{(r)})$. We define a k-linear functor $M' : \check{\mathcal{U}}(K, M^{(r)}) \to \operatorname{mod} k^{r-1}$ by setting $M'(Y_\omega, \gamma_Y, Y_0) = (M^{(1)}(Y_0), \dots, M^{(r-1)}(Y_0))$ and $M'(f_\omega, f_0) = (M^{(1)}(f_0), \dots, M^{(r-1)}(f_0))$. There is an obvious functor Φ from the subspace category of the multiple vector space category (K, M) with r peaks to the subspace category of the multiple vector space category $(\check{\mathcal{U}}(K, M^{(r)}), M')$ with $r - 1$ peaks. Namely, for an object $X = (X_\omega, \gamma_X, X_0)$ of $\check{\mathcal{U}}(K, M)$ we put $\Phi(X) = ((X_\omega^{(1)}, \dots, X_\omega^{(r-1)}), (\gamma_X^{(1)}, \dots, \gamma_X^{(r-1)}), (X_\omega^{(r)}, \gamma_X^{(r)}, X_0))$. Making the obvious definitions of Φ on the morphisms, it is immediate to see that Φ is actually an equivalence of categories. For later use we sum this up as a proposition:

Proposition 2.1. *The functor $\Phi : \check{\mathcal{U}}(K, M) \to \check{\mathcal{U}}(\check{\mathcal{U}}(K, M^{(r)}), M')$ is an equivalence of aggregates which maps $\mathcal{U}(K, M)$ onto $\mathcal{U}(\mathcal{U}(K, M^{(r)}), M')$.*

Of course, the problem for applying this observation is that usually $\check{\mathcal{U}}(K, M^{(r)})$ will be hard to control. Nevertheless, in the case we are interested in this will turn out to be easy. Note, that we may iterate the process. We could actually do this until we finally arrive at an ordinary vector space category with one peak.

3. Tangles, rods and bushes

In [Gf] a classification problem was posed which is now called the Gelfand problem. A first step to the solution was given in [NR] where even a wider class of matrix problems was considered. Complete solutions were presented in [CB], [Bd] and [De]. The problems considered in [Bd] and [De] are very similar. The transfer to the language of clans used in [CB] is explained in [De]. We will use and therefore recall only the notation introduced in [De] because it is based on categorical terms which fit well to our purposes. The point is that in all solutions one has a complete classification of the indecomposable objects in an appropriate category of representations. The parameters of these classifications are combinatorial data which are always certain words in a suitable alphabet together with all the Jordan block matrices over the field k.

Let us now turn to the definitions we need from [De]. A *tangle* over an aggregate K is a pair (N^-, N^+) formed by sequences $N^- = (N_1^-, \dots, N_n^-)$ and $N^+ = (N_1^+, \dots, N_n^+)$ of k-linear functors $N_i^\epsilon : K \to \operatorname{mod} k$ where $n \in \mathbb{N}$. The category $\operatorname{rep}(N^-, N^+)$ of representations of the tangle (N^-, N^+) is an aggregate which has as objects the tuples $(X, f_1, \dots, f_n)$ such that X in K and $f_i : N_i^-(X) \to N_i^+(X)$ are k-linear maps for all $i = 1, \dots, n$. A morphism from $(X, f_1, \dots, f_n)$ to $(Y, g_1, \dots, g_n)$ is a morphism $\mu : X \to Y$ in K such that $N_i^+(\mu) f_i = g_i N_i^-(\mu)$ for all $i = 1, \dots, n$.

In [De] the representations of faithful rodded tangles are classified. We will not give the formal definition of this kind of tangles but use a more combinatorial description which is also presented in [De]. A *rod* is a partially ordered set S such that each element $x \in S$ admits at most one element $y \in S$ which is incomparable to x. Note, that in [De] only finite rods are considered. But, since the objects X appearing in the representations of a tangle can always by considered as objects of a full subaggregate with finite spectroid, this assumption can easily be removed.

If for $n \in \mathbb{N}$ there are given two sequences $S^- = (S_1^-, \dots, S_n^-)$ and $S^+ = (S_1^+, \dots, S_n^+)$ of rods, then one defines S to be the partially ordered set given by the disjoint union of all the S_i^- and S_i^+. The partially ordered set S is considered as a category in the usual way by introducing morphisms $(x \mid y) : x \to y$ for all $x \geq y$ in S. Let kS be the k-linear hull of this category, then the additive hull $\operatorname{add} kS$ of kS is an aggregate with spectroid kS. Each of the partially ordered sets S_i^ϵ has a canonical functor $N_i^\epsilon : S \to \operatorname{mod} k$ which maps the objects $x \in S_i^\epsilon$ to k and the morphisms $(x \mid y)$ to the identity of k. As S is the disjoint union of the S_i^ϵ, one can extend N_i^ϵ by zero to the whole of S. Finally, N_i^ϵ is extended to a k-linear functor $\operatorname{add} kS \to \operatorname{mod} k$ which is again called N_i^ϵ. Hence (N^-, N^+) where $N^- = (N_1^-, \dots, N_n^-)$ and $N^+ = (N_1^+, \dots, N_n^+)$ is a tangle over $K = \operatorname{add} kS$.

To obtain the needed tangles one has to make one more step of construction. For that is required an equivalence relation $\sim$ on S such that each equivalence class consists of at most two elements and, if there are incomparable elements x, y in some S_i^ϵ, then the class of x contains only the single element x. The pair $(S, \sim)$ is called a *bush*. For each equivalence class $\bar{x}$ of an element x of S the object $\tilde{x}$ of $\operatorname{add} kS$ is defined to be the direct sum over all elements of $\bar{x}$. That is $\tilde{x} = x$ if $\bar{x}$ has only one element and $\tilde{x} = x \coprod y$ if $\bar{x}$ consists of two distinct elements x, y. Let us consider the subcategory $k(S, \sim)$ of $\operatorname{add} kS$ given as follows: The objects are the $\tilde{x}$ for $x \in S$. The space of morphisms $\tilde{x} \to \tilde{y}$ is the (direct) sum of all radical morphisms $\tilde{x} \to \tilde{y}$ in kS and a space U which is 0 if $\tilde{x} \neq \tilde{y}$ and otherwise is the 1-dimensional space generated by the identity of $\tilde{x}$. Clearly, $k(S, \sim)$ is not a full subcategory of kS provided there is an equivalence class with 2 elements. Nevertheless, $k(S, \sim)$ is always a spectroid. Its aggregate $\operatorname{add} k(S, \sim)$ is a subcategory of $\operatorname{add} kS$. Hence the functors N_i^ϵ act on $\operatorname{add} k(S, \sim)$ as well. The category of representations of the tangle (N^-, N^+) over the aggregate $\operatorname{add} k(S, \sim)$ will be called category of representations of the bush $(S, \sim)$.

In [De] the representations of any bush are classified. The involved parameters are called asymmetric strings, dimidiate strings, asymmetric bands and dimidiate bands. The corresponding representations are basically easy to calculate. We want to show how we can transfer this knowledge to the subspace category of the vector space category $\mathcal{D}_n$. For this reason we will introduce conditions for a bush to be identified with a multiple vector space category and its category of representations with the corresponding subspace category.

For $\epsilon \in \{-, +\}$ we denote by S^ϵ the disjoint union of all the partially ordered sets S_i^ϵ. The spectroid kS^ϵ and the aggregate $\operatorname{add} kS^\epsilon$ are defined in the obvious way. We call a bush $(S, \sim)$ *separated* if no equivalence class of $\sim$ intersects both S^- and S^+. For a separated bush also $k(S^\epsilon-, \sim)$ and $\operatorname{add} k(S^\epsilon, \sim)$ are well-defined. Remember that a partially ordered set is said to be an antichain if it does not contain any comparable elements.

Proposition 3.1. *Let $(S, \sim)$ be a separated bush such that S^- is an antichain. Then the category of representations of $(S, \sim)$ is equivalent to the subspace category of the vector space category $(\operatorname{add} k(S^+, \sim), M)$ where the sequence $M = (M^{(1)}, \dots, M^{(r)})$ of functors $M^{(i)} : \operatorname{add} k(S^+, \sim) \to \operatorname{mod} k$ is obtained in the following way from the set of equivalence classes $\{C_1, \dots, C_r\}$ of $\sim$ on S^-:*
If $C_i = \{x\}$ consists of a single element $x \in S_p^-$, then $M^{(i)} = N_p^+$.
If $C_i = \{x, y\}$ consists of two elements $x \in S_p^-$ and $y \in S_q^-$, then $M^{(i)} = N_p^+ \coprod N_q^+$.

Proof. For a proof we have to construct an equivalence. We do this only on the objects. Since S^- is an antichain, the category $\operatorname{add} k(S^-, \sim)$ can be identified with the category of modules over the semisimple algebra k^r. This will make the construction on the morphims obvious.

Let $(X, f_1, \dots, f_n)$ be a representation of $(S, \sim)$. To define the image $Y = (Y_\omega, \gamma_Y, Y_0)$ in $\check{\mathcal{U}}(\operatorname{add} k(S^+, \sim), M)$ we start by decomposing X as $X = X^- \coprod X^+$ such that the X^ϵ are in $\operatorname{add} k(S^\epsilon, \sim)$. After putting $Y_0 = X^+$ it remains to define the sequences $(Y_\omega^{(1)}, \dots, Y_\omega^{(r)})$ of k-spaces and $(\gamma_Y^{(1)}, \dots, \gamma_Y^{(r)})$ of k-linear maps $\gamma_Y^{(i)}: Y_\omega^{(i)} \to M^{(i)}(X^+)$.

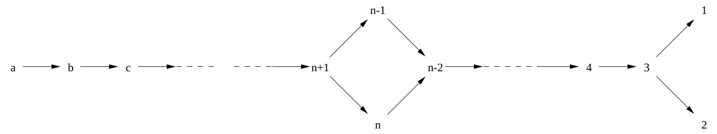
For this purpose choose a representative x_i out of each class C_i. Then $X^- = \coprod_{i=1}^r \tilde{x}_i^{m_i}$ for some non-negative integers m_i. If $C_i = \{x_i\}$ and $x_i \in S_p^-$, we put $Y_\omega^{(i)} = N_p^-(x_i^{m_i})$ and $\gamma_Y^{(i)} = f_p : N_p^-(x_i^{m_i}) \to N_p^+(X^+)$. Note, that $N_p^-(x_i^{m_i})$ may be a proper summand of $N_p^-(X^-)$ because S_p^- may contain an element different from x_i.

If C_i consists of two elements x_i, y_i, then $x_i \in S_p^-$ and $y_i \in S_q^-$ for some $p \neq q$. In this case we put $Y_\omega^{(i)} = N_p^-(x_i^{m_i})$ and observe that $N_p^-(x_i^{m_i}) = N_q^-(y_i^{m_i})$. Thus we can define $\gamma_Y^{(i)} = (f_p, f_q) : N_p^-(x_i^{m_i}) \to N_p^+(X^+) \coprod N_q^+(X^+)$. □

Let us remark that the preimages of the objects $(S_i, 0, 0)$, $i = 1, \dots, r$, under the equivalence of the above proposition are the objects $(\tilde{x}_i, 0, \dots, 0)$.

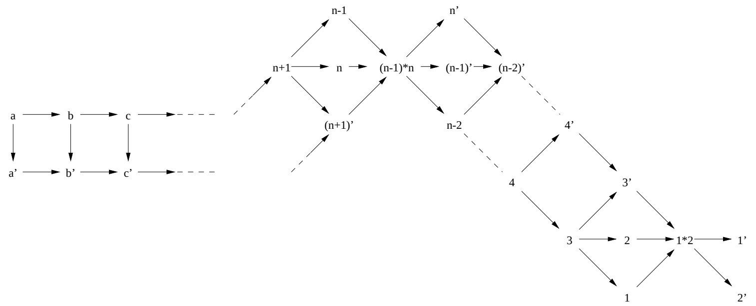
4. The $\mathbb{D}_n$-pattern as bush

We consider the bush $(S, \sim)$ where $n = 1$, the partially ordered set S_1^- consists of two incomparable elements u, v, the equivalence relation is the identical relation and the partially ordered set S_1^+ is given by its Hasse diagram $\Delta(n-2)$ displayed below. Note, that for the right component of S_1^+ the shift be $n-2$ is an order preserving map.



By Proposition 3.1 the category of representations of the bush $(S, \sim)$ is equivalent to the subspace category $\check{\mathcal{U}}(K, M)$ of the multiple vector space category (K, M) with 2 peaks where $K = \operatorname{add} k(S^+, \sim)$ and $M = (M^{(1)}, M^{(2)})$ with $M^{(1)} = M^{(2)} = N_1^+$. Note that the functor N_1^+ is obviously faithful. As observed above we may consider $\mathcal{U}(K, M)$ instead loosing only two trivial indecomposable objects.

Now we make use of Proposition 2.1 and claim that the obtained vector space category with one peak coincides with $\mathcal{D}_n$. To see this, we first have to understand the subspace category $\mathcal{U}(K, M^{(2)})$. Fortunately, this is very easy to calculate since the width of the partially ordered set S_1^+ is 2. Therefore the only indecomposable objects are the $x = (0, 0, x)$ and $x' = (k, 1, x)$ for all $x \in S_1^+$ together with the objects $x * y = (k, (1, 1), x \coprod y)$ for all pairs x, y of incomparable objects in S_1^+. Although S_1^+ is infinite, the category $\mathcal{U}(K, M^{(2)})$ has Auslander-Reiten sequences and the Auslander-Reiten quiver has the following shape where we do not display the (obvious) translation.



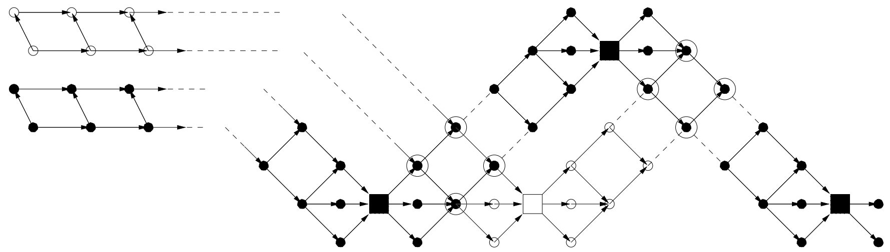
In order to describe a spectroid of $\mathcal{U}(K, M^{(2)})$ by its quiver with relations, one first has to impose the obvious mesh relations on the Auslander-Reiten quiver. But by [Dr2] these are again all relations because M' acts faithfully on $\mathcal{U}(K, M^{(2)})$ as N_1^+ is faithful. Summing up our findings, we obtain the main result of this note.

Theorem 4.1. *The proper subspace category of $\mathcal{D}_n$ is equivalent to the full subcategory of the representations of the bush $(S, \sim)$ defined at the beginning of this section given by all representations which do not admit a direct summand isomorphic to $(u, 0)$ or $(v, 0)$.*

5. Some generalizations

In [DS] multiple one-point extension algebras are investigated concerning tameness. The corresponding module categories can be identified with the subspace categories of multiple vector space categories. We present an example of such an algebra where our reduction technique to a bush can be applied as well to obtain a classification of the indecomposable subspaces of the associated multiple vector space category. Namely, let (K, M) be given by $K = \operatorname{mod} H$ where H is a tame hereditary algebra of type $\widetilde{\mathbb{D}}_n$, $n \geq 6$ and $M = (M^{(1)}, M^{(2)})$ where $M^{(i)} = \operatorname{Hom}_H(R_i, -)$. The R_i are regular indecomposable H-modules of regular length 2 lying both in a tube of rank $n-2$ such that $R_1 = (D\operatorname{Tr})^2 R_2$.

The multiple vector space category (K_M, M) can be described by the quiver of $\operatorname{ind} K_M$ and the dimensions of the spaces $M^{(i)}(X)$ for X in $\operatorname{ind} K_M$ in the analogous way as in the 1 peak situation. In order to distinguish the two functors $M^{(1)}$ and $M^{(2)}$ we use filled points resp. squares for $M^{(1)}$ and empty points resp. squares for $M^{(2)}$.



To construct a bush whose category of representations up to finitely many trivial indecomposables is equivalent to the proper subspace category of this multiple vector space category, we take $n = 2$ and for $i = 1, 2$ let the sets $S_i^- = \{u_i, v_i\}$ consist of 2 incomparable elements. The partially ordered set S_1^+ is given by the Hasse diagram $\Delta(n-3)$. For S_2^+ we take the same Hasse diagram but remove the vertices 1 and 2. But now we introduce a non-trivial equivalence relation by defining that the equivalence classes with 2 elements are precisely those consisting of the elements $(n-3)i$ in S_1^+ and $(n-3)(i-1)+3$ in S_2^+ for all $i \in \mathbb{N}$ After translating this bush into a multiple vector space category with 4 peaks via Proposition 3.1, we apply Proposition 2.1 to the peaks corresponding to the v_i and obtain a multiple vector space category with 2 peaks. It is an easy exercise that this leads to the required (K_M, M).

Let us finally mention that one can deal with analogous examples with more peaks and associated with other patterns in the same fashion.

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