ON TAME REPETITIVE CATEGORIES OF NON-POLYNOMIAL GROWTH

by Bernd Roggon

Abstract. This article introduces a recipe for constructing algebras whose repetitive category is tame of non-polynomial growth. Moreover it contains a list of all the bound poset algebras which are minimal with respect to this property.

Introduction

Throughout let us fix an algebraically closed field k. The term *algebra* always denotes a finite-dimensional k-algebra (associative, with 1) which is moreover basic and connected.

The concept of the *repetitive category* (cf. 1.2) to an algebra was introduced by Hughes/ Waschbüsch at the beginning of the 80ies. It is the main tool in their classification of the representation-finite selfinjective algebras. (For instance, group algebras to finite groups are selfinjective.) Meanwhile it has been proved that tame repetitive categories of polynomial growth provide all the standard selfinjective algebras which are tame of polynomial growth (see [HW], [S]).

The algebras with tame repetitive category of polynomial growth may be divided into two classes: the ones whose repetitive category is isomorphic to the repetitive category of an algebra which is representation-infinite or simply connected, and the remaining ones. The first of these classes consists of the algebras being tilting-cotilting equivalent to a hereditary algebra of Dynkin/Euclidian type resp. to a tubular canonical algebra, and most of these algebras posess particularly nice, explicit descriptions (see [HW], [ANS], [NS], [N], [S], [AS2]). The second class (being the exceptional one) is however not completely understood.

In strict contrast, the present knowledge on the tame repetitive categories of non-polynomial growth is extremely sparse, and, as far as we know, essentially confined to the (rare, but well-understood) special biserial ones, summarized in 1.3.

The aim of these notes will be twofold:

- to present a recipe for constructing algebras with tame repetitive category, the resulting algebras being called *almost gentle* (see 2.1),
- to display a list of the *rpg-critical bound poset algebras* (see 4.2), i.e. the bound poset algebras B (= 'poset algebras with zero relations') s.t. the repetitive category to B is tame of non-polynomial growth, but the repetitive category to any proper full subcategory of B is of polynomial growth.

This article will moreover expose a complete description of all the bound poset algebras with repetitive category of polynomial growth. Any of these algebras whose number of vertices is not among $6, 7, 8, 9, 10$, any rpg-critical bound poset algebra , any pg-critical algebra with more than 6 vertices , and many other well-known classes of algebras with tame repetitive category turn out to be almost gentle.

These notes are organized as follows. Starting with a preliminary section, section 2 is dedicated to the *almost gentle algebras* , and after an introduction to *bound poset algebras* in section 3 (including a complete description of the ones with repetitive category of polynomial growth) we will present in the final section a list of the *rpg-critical bound poset algebras.*

This article is a rather faithful excerpt from the author's thesis [Ro] .

1 Preliminaries

1.1 Throughout this article we will adopt the language of [R1] 2. ,4.

Let B be a locally bounded k-category, which we will always suppose to be *connected* . Its (countable) object set is denoted by B_0 . (Algebras are exactly such categories B s.t. $\# B_0 < \infty$.) B is said to be *triangular* , if the quiver of B does not contain oriented cycles. The definition of *tameness* of B , including its several degrees *domestic, of polynomial growth*, and *of non-polynomial growth* , will be the usual one (see any of [S], [ANS], [NS], ...) . Note that, except being of non-polynomial growth, any of these properties of B carries over to the full subcategories of B .

1.2 Let B be an algebra. As introduced by Hughes/ Waschbüsch in [HW] , the *repetitive category* $\hat{B}$ to B is given as: $\hat{B}_0 := B_0 \times \mathbf{Z}$ ($\mathbf{Z}$ = the integers) , and for any $z \in \mathbf{Z}$, $\alpha, \beta \in B_0$ set

$$\begin{aligned} Hom_{\hat{B}}((\alpha, z), (\beta, z)) &:= Hom_B(\alpha, \beta) \\ Hom_{\hat{B}}((\beta, z), (\alpha, z+1)) &:= \mathcal{D}Hom_B(\alpha, \beta) \end{aligned}$$

and the remaining Hom-spaces in $\hat{B}$ are $= 0$, where $\mathcal{D} := Hom_k(-, k)$. The composition of morphisms in $\hat{B}$ is induced by the one in B , the mappings $Hom_B(\alpha, \beta) \times \mathcal{D}Hom_B(\gamma, \beta) \to \mathcal{D}Hom_B(\gamma, \alpha)$, which sends (φ, ψ) to ($(\xi : \gamma \to \alpha) \mapsto \psi(\xi\varphi)$) , and $\mathcal{D}Hom_B(\beta, \alpha) \times Hom_B(\beta, \gamma) \to \mathcal{D}Hom_B(\gamma, \alpha)$, which assigns to (ψ, φ) the map ($(\xi : \gamma \to \alpha) \mapsto \psi(\varphi\xi)$) , for α, β, γ in B_0 , and the zero map $\mathcal{D}B \times \mathcal{D}B \to 0$. We will usually identify B with the full subcategory in $\hat{B}$ given by $B_0 \times \{0\}$.

It is well-known that $\hat{B}$ is a selfinjective locally bounded k-category. We denote by $\nu_{\hat{B}}$ (or just ν) the canonical shift automorphism of $\hat{B}$ which sends each $(\alpha, z) \in \hat{B}_0$ to $(\alpha, z+1)$. Observe that ν is the unique automorphism ζ of $\hat{B}$ satisfying *top* $\hat{B}(\alpha, -) \cong$ *soc* $\hat{B}(\zeta^{-1}\alpha, -)$ for each $\alpha \in \hat{B}_0$. A convex subcategory in $\hat{B}$ intersecting each $\nu^{\mathbf{Z}}$-orbit in $\hat{B}_0$ exactly in one vertex is called a (ν-) *slice* in $\hat{B}$.

Let ω be a source in B. Then $S_\omega^+ B$ is defined to be the full subcategory in $\hat{B}$ given by $(B_0 \setminus \{\omega\}) \cup \{\nu\omega\}$. $S_\omega^+ B$ is called a *Hughes-Waschbüsch-reflection* of B. It is of course a slice in $\hat{B}$. Canonically $\widehat{(S_\omega^+ B)} \cong \hat{B}$. We have the following classification of the triangular repetitive categories:

Proposition. ([HW]) *Let A, B be triangular algebras. Then the following assertions are equivalent: (i) $\hat{B} \cong \hat{A}$ (ii) B is a slice in $\hat{A}$. (iii) There is an admissible sequence of Hughes-Waschbüsch-reflections carrying A into B.*

By [TW] triangular algebras with isomorphic repetitive categories are tilting-cotilting equivalent.

We will quite frequently consider the Galois covering ([DS])

$$\hat{B} \longrightarrow T(B) := \hat{B}/{\nu_{\hat{B}}}^{\mathbf{Z}} \quad .$$

$T(B)$ is called the *trivial extension* of B, which is a symmetric algebra whose k-dimension is twice of $dim_k B$ (see [HW] 2.). Note that for another algebra A s.t. $\hat{A} \cong \hat{B}$ we have $T(A) \cong T(B)$. By [DS], $T(B)$ being tame (resp. of polynomial growth/ domestic) forces $\hat{B}$ to be tame (resp. of polynomial growth/ domestic).

1.3 A bound quiver (Q, I) is called *special biserial*, if at most two arrows in Q start (resp. end) at any vertex, and if for each arrow α in Q there is at most one $\beta \in Q_1$ and at most one $\gamma \in Q_1$ s.t. $\beta\alpha \notin I$ and $\alpha\gamma \notin I$. A special biserial bound quiver (Q, I) is said to be *gentle*, if in addition I is generated by paths of length $= 2$, and for any $(\alpha : \sigma \to \sigma') \in Q_1$ there is at most one $\beta \in Q_1$ ending in σ and at most one $\gamma \in Q_1$ starting in σ' with $\beta\alpha \in I$ and $\alpha\gamma \in I$. Finally a locally bounded category B is called *special biserial* (resp. *gentle*), if B is triangular and $B \cong kQ/I$, where (Q, I) is special biserial (resp. gentle).

By [WW] special biserial categories are tame. The special biserial repetitive categories are easily described.

Proposition. ([AS1] 1.3, [PS] 2.9, [N], [R2]) *Let B be a triangular algebra. Then $\hat{B}$ is special biserial iff B is gentle. In this case $\hat{B}$ is of polynomial growth (resp. domestic), iff $T(B)$ is of polynomial growth (resp. domestic), iff B contains at most one cycle.*

2 Almost gentle algebras

2.1 We start up with the definition of an almost gentle algebra.

This needs an auxiliary construction. Let $\mathcal{F}$ be the quiver of type D_4 in subspace orientation according to its midpoint. Take an algebra A and an A-module E. We glue together the one-point-extension $A[E]$ and $\mathcal{F}$ by identifying the extension vertex to E with a fixed source of $\mathcal{F}$. The resulting algebra is denoted by $A[E, \mathcal{F}]$. Dually we define $[E, \mathcal{F}^{op}]A$ as the glueing of $[E]A$ and $\mathcal{F}^{op}$ by identifying the coextension vertex to E with a fixed sink of $\mathcal{F}^{op}$.

Let C be an algebra. Choose two integers $s,t \geq 0$, and C-modules $E_1,\ldots,E_s$, $F_1,\ldots,F_t$. We inductively define a sequence of algebras $C_1 := C$, $C_2,\ldots,C_{s+t+1}$:

$$\begin{aligned} C_{i+1} &:= C_i[E_i\,,\mathcal{F}] \qquad \text{for } i=1,\ldots,s \\ C_{j+s+1} &:= [F_j\,,\mathcal{F}^{op}]C_{j+s} \quad \text{for } j=1,\ldots,t \end{aligned}$$

Set $B := C_{s+t+1}$. Denote by $\alpha_1,\ldots,\alpha_{2(s+t)}$ the different vertices in $B_0 \setminus C_0$ having exactly one neighbour in B. Restricting the choices for C we require that the full subcategory given by $B_0 \setminus \{\alpha_1,\ldots,\alpha_{2(s+t)}\}$ is gentle. (Clearly, the case $s=t=0$ provides exactly the gentle algebras.)

Now an algebra A is called *almost gentle*, provided there exists an algebra B as above and a sequence of algebras $A_1 := B$, $A_2,\ldots, A_r = A$ (for some $r \geq 1$) s.t. for $j=2,\ldots,r$ the algebra A_j is tilting-cotilting equivalent to A_{j-1}, or A_j is a full subcategory of A_{j-1}, or $A_j = {A_{j-1}}^{op}$.

Immediate examples of almost gentle algebras are the gentle algebras and the hereditary algebras of type D or $\tilde{D}$.

2.2 For our purposes the main property of the almost gentle algebras is this one.

Theorem. ([Ro] 6.2) *Let k be an algebraically closed field. Then the repetitive category of an almost gentle k-algebra is tame.*

The proof of this Theorem will essentially rest on the following two results:

- ([AP]) Let A, B be tilting-cotilting equivalent algebras. Then $T(A)$ is tame iff $T(B)$ is tame.
- ([C-B]) Clannish algebras are tame.

Slightly strengthening [C-B] a *clannish algebra* is a (maybe non-basic) algebra $A = kQ/I$ where Q is a finite quiver with a distinguished set Sp of loops (the 'special loops') in Q_1 s.t. the following holds:
(i) At most two arrows start (resp. end) at any vertex. (ii) For any $\alpha \in Q_1 \setminus Sp$ there is at most one $\beta \in Q_1$ and at most one $\gamma \in Q_1$ s.t. $\beta\alpha \notin I$ and $\alpha\gamma \notin I$. (iii) There exists a set $\mathcal{Z}$ of paths in Q (of length ≥ 2) which neither start or end in some $\epsilon \in Sp$ nor involve the square of some $\epsilon \in Sp$, and moreover pairs λ_ϵ, μ_ϵ of distinct non-zero elements in k being indexed over the loops $\epsilon \in Sp$, s.t. I is generated by $\mathcal{Z} \amalg \{\,(\epsilon-\lambda_\epsilon)(\epsilon-\mu_\epsilon) \mid \epsilon \in Sp\,\}$.

Furthermore we will use another construction. For an algebra B define

$$\bar{T}(B) := T(B)\,/\,soc\,T(B) \quad .$$

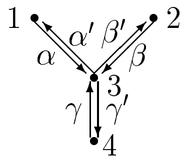
Then the indecomposable $\bar{T}(B)$-modules coincide with the indecomposable non-projective $T(B)$-modules. In particular $T(B)$ is tame iff $\bar{T}(B)$ is tame. If B is gentle, then $T(B)$ is special biserial, and $\bar{T}(B)$ is clannish with $Sp = \emptyset$.

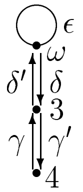
Having enumerated its main ingredients let us now carry out the

Proof. (of the Theorem) Observe that it will be sufficient to consider the algebra B from 2.1 and to show that $\bar{T}(B)$ is tame.

The following calculation concerning the quiver $\mathcal{F}$ (as in 2.1) is of great importance. Fix two non-zero elements $\lambda \neq \mu$ in our base field k. Besides $\bar{T}(\mathcal{F})$ consider the subsequent clannish algebra $C(\mathcal{F})$ depending on λ, μ and with $Sp = \{\epsilon\}$:

$\bar{T}(\mathcal{F})$





$C(\mathcal{F})$

Here $\bar{T}(\mathcal{F})$ is bound by:

$$0 = \alpha\beta' = \alpha\gamma' = \beta\alpha' = \beta\gamma' = \gamma\alpha' = \gamma\beta' = \gamma'\gamma = \gamma\gamma' = \alpha\alpha' = \beta\beta' = \alpha'\alpha = \beta'\beta \ ,$$

and $C(\mathcal{F})$ is bound by:

$$0 = (\epsilon - \lambda)(\epsilon - \mu) = \delta\gamma' = \gamma\delta' = \gamma'\gamma = \gamma\gamma' = \delta\delta' = \delta'\delta = \delta'\epsilon\,\delta \ .$$

As an easy computation shows, we get a k-linear functor

$$\mathcal{H} \ : \quad \bar{T}(\mathcal{F}) - mod \longrightarrow C(\mathcal{F}) - mod$$

resulting from the following rule $M \mapsto \mathcal{H}M$ (canonically extending to natural transformations):

$$\begin{aligned} \mathcal{H}M\,(\omega) &:= M(1) \oplus M(2) \\ \mathcal{H}M\,(\delta) &:= M(\alpha) \oplus M(\beta) \\ \mathcal{H}M\,(\delta') &:= M(\alpha') \oplus M(\beta') \\ \mathcal{H}M\,(\epsilon) &:= \lambda \cdot id_{M(1)} \oplus \mu \cdot id_{M(2)} \end{aligned}$$

and leaving unchanged $\mathcal{H}M\,(3) := M(3)$, $\mathcal{H}M\,(4) := M(4)$, $\mathcal{H}M\,(\gamma) := M(\gamma)$, $\mathcal{H}M\,(\gamma') := M(\gamma')$.

Conversely, take a $C(\mathcal{F})$-module N. Then the semisimplicity of the map $N(\epsilon) \in End_k\, N(\omega)$ yields the decomposition

$$N(\omega) \ = \ ker(N(\epsilon) - \lambda) \oplus ker(N(\epsilon) - \mu) \ .$$

Setting $N_1 := ker(N(\epsilon) - \lambda)$ and $N_2 := ker(N(\epsilon) - \mu)$ we consider the canonical injections/ projections associated to $N_1 \oplus N_2$ (where $i = 1, 2$):

$$\iota_i : \ N_i \to N_1 \oplus N_2 \ , \ \ \pi_i : \ N_1 \oplus N_2 \to N_i \quad .$$

As a direct calculation reveals, we obtain a k-linear functor

$$\mathcal{H}^\star \quad : \quad C(\mathcal{F}) - mod \longrightarrow \bar{T}(\mathcal{F}) - mod$$

resulting from the subsequent assignment $N \mapsto \mathcal{H}^\star N$:

$$\begin{aligned}
\mathcal{H}^\star N\,(1) &:= N_1 \\
\mathcal{H}^\star N\,(2) &:= N_2 \\
\mathcal{H}^\star N\,(\alpha) &:= \iota_1 N(\delta) \\
\mathcal{H}^\star N\,(\beta) &:= \iota_2 N(\delta) \\
\mathcal{H}^\star N\,(\alpha') &:= N(\delta')\pi_1 \\
\mathcal{H}^\star N\,(\beta') &:= N(\delta')\pi_2
\end{aligned}$$

and leaving unchanged $\mathcal{H}^\star N\,(3) := N(3)$, $\mathcal{H}^\star N\,(4) := N(4)$, $\mathcal{H}^\star N\,(\gamma) := N(\gamma)$, $\mathcal{H}^\star N\,(\gamma') := N(\gamma')$.

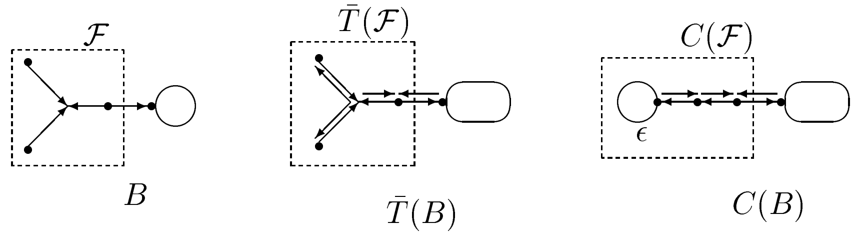
(Let us just demonstrate the equalities $\mathcal{H}^\star N\,(\beta'\beta) = 0$ and $\mathcal{H}^\star N\,(\alpha'\alpha) = 0$. From $\delta'\epsilon\,\delta = 0 = \delta'\delta$ we derive $\delta'(\epsilon-\lambda)\delta = 0 = \delta'(\epsilon-\mu)\delta$. Therefore $0 = N(\delta'(\epsilon-\lambda)\delta) = N(\delta')(0_{N_1} \oplus (\mu-\lambda)\cdot id_{N_2})N(\delta) = (\mu-\lambda)\cdot \mathcal{H}^\star N\,(\beta'\beta)$, and since $\lambda \neq \mu$ we get $\mathcal{H}^\star N\,(\beta'\beta) = 0$, and similarly the other equality.)

By definition of $\mathcal{H}$, $\mathcal{H}^\star$ we have natural isomorphisms

$$\mathcal{H}\,\mathcal{H}^\star \cong id \ , \ \ \mathcal{H}^\star\,\mathcal{H} \cong id \ .$$

As an immediate consequence of this computation we will obtain a clannish algebra $C(B)$ s.t. $\bar{T}(B)$ and $C(B)$ are Morita-equivalent.
Namely, to obtain $C(B)$ we replace in $\bar{T}(B)$ any full subcategory $\bar{T}(D)$, with $D = \mathcal{F}$ or $\mathcal{F}^{op}$ being one of the $s+t$ connected components of the full subcategory in B given by $B_0 \setminus C_0$, by the clannish algebra $C(\mathcal{F})$ using that $T(\mathcal{F}) = T(\mathcal{F}^{op})$. Indeed $C(B)$ is a clannish algebra, since the full subcategory B' in B given by $B_0 \setminus \{\alpha_1, \ldots, \alpha_{2(s+t)}\}$ as in 2.1 is gentle, and hence $\bar{T}(B')$ is a clannish algebra with $Sp = \emptyset$. To get the required Morita-equivalence we canonically extend the functors $\mathcal{H}$ and $\mathcal{H}^\star$ (from above) defined on any $\bar{T}(D)$ and the corresponding $C(\mathcal{F})$ by setting elsewhere the identity.
Therefore $\bar{T}(B)$ is tame. This finishes the proof. □

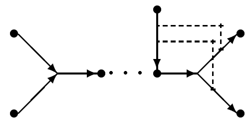
As an illustration, we sketch the previous replacement leading from $\bar{T}(B)$ to $C(B)$ in the particular (but characteristic) case $s = 1$ and $t = 0$.



2.3 Let us insert an application concerning the pg-critical algebras introduced in [NoS]. (Many more examples of almost gentle algebras can be found in [Ro] 6.)

Lemma. *Any pg-critical algebra B s.t. $\#B_0 \neq 6$ is almost gentle. In particular $\hat{B}$ is tame of non-polynomial growth.*

Proof. For any integer m the list in [NoS] immediately provides an almost gentle pg-critical algebra with at least m vertices. Note that any pg-critical algebra with more than 6 vertices is up to duality tilting-cotilting equivalent to an algebra of the shape



and that these algebras form an ascending chain of full subcategories. □

3 Bound poset algebras and the polynomial growth case

3.1 By a *bound poset category* we will mean a locally bounded category of the form $A = k[S]/I$, where S is a countable poset (throughout identified with its Hasse diagram), $k[S]$ denotes the associated incidence category, and I is an ideal in $k[S]$ generated by paths of length ≥ 2. In case $\#S < \infty$ we prefer to call $k[S]/I$ a *bound poset algebra*.

Observe that A is uniquely determined, and also conveniently reconstructable, by its Cartan invariants $(dim_k\, A(\alpha,\beta))_{\alpha,\beta\in A_0} \in \{0,1\}^{A_0\times A_0}$.

Well-known and important examples of bound poset algebras are the completely separating algebras introduced in [D].

3.2 Let $A = k[S]/I$ be a bound poset algebra, and let T be a convex subcategory in A containing no zero relations. Following [D], we denote by k_T the subsequent A-module: on vertices set $k_T(\alpha) := k$ for $\alpha \in T_0$, $k_T(\alpha) := 0$ otherwise, and for some arrow $\xi : \beta \to \gamma$, with $\beta, \gamma \in T_0$, let $k_T(\xi) := id_k$.

We have explicit control over the possible one-point (co)extensions of A inside the class of bound poset algebras, only needing the data (S, I).

Lemma. *Let A be an algebra and M be an A-module. Then the following are equivalent: (i) $A[M]$ is a bound poset algebra. (ii) A is a bound poset algebra and $M = k_T$ for some convex subcategory T in A without zero relations.*

Corollary. *Let A be a bound poset algebra. Then $\hat{A}$ is a bound poset category $= k[\hat{S}]/\hat{I}$ given as follows: (i) For $\alpha,\beta \in A_0$ there is an arrow $\beta \to \nu\alpha$ in $\hat{S}$ iff there is a non-extensible non-zero path $\alpha \rightsquigarrow \beta$ in S. (ii) For $\alpha, \beta \in \hat{A}_0$ a path $(w : \alpha \rightsquigarrow \beta)$ in $\hat{S}$ is $\neq 0$, iff it allows an extension $(w\bar{w} : \alpha \rightsquigarrow \beta \rightsquigarrow \nu_{\hat{A}}\alpha)$ in $\hat{S}$.*

Note that if $A = k[S]/I$ is gentle, then the bound poset description (S, I) is gentle, and in addition for $\hat{A} = k[\hat{S}]/\hat{I}$ as above $(\hat{S}, \hat{I})$ is special biserial.

3.3 Inside the class of bound poset algebras the description of the algebras with repetitive category of polynomial growth has been completed. By [Ro] 9.1 any such bound poset algebra is simply connected or gentle. While the gentle case is indeed obvious by 1.3, the non-gentle case now looks as follows:

Theorem. *Let k be an algebraically closed field. Suppose that B is a non-gentle bound poset k-algebra. Then $\hat{B}$ is of polynomial growth, iff $T(B)$ is of polynomial growth, iff B is tilting-cotilting equivalent to a hereditary k-algebra of type D, E, $\tilde{D}$, $\tilde{E}$ resp. to a tubular canonical k-algebra of type* $(3,3,3), (4,4,2), (6,3,2)$.

Corollary. *Suppose that B is a bound poset algebra s.t. $\hat{B}$ is of polynomial growth and $\# B_0 \notin \{6,7,8,9,10\}$. Then B is almost gentle.*

In particular only *finitely many* bound poset algebras with repetitive category of polynomial growth are not almost gentle. Notice that the tameness of these repetitive categories can already be obtained from the tameness of the trivial extensions to the three tubular canonical algebras of type $(3,3,3), (4,4,2), (6,3,2)$. We expect that these three algebras are not almost gentle.

4 The rpg-critical algebras

4.1 An algebra B is called *rpg-critical*, if $\hat{B}$ itself is tame of non-polynomial growth, but there is no proper full subcategory K of B s.t. $\hat{K}$ is also of non-polynomial growth.

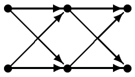
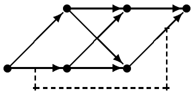
If A is an algebra s.t. $\hat{A}$ is tame, then clearly $\hat{A}$ is of non-polynomial growth, iff some full subcategory of A is rpg-critical. In this sense, the rpg-critical (bound poset) algebras form the *lower boundary* of the (bound poset) algebras with tame repetitive category of non-polynomial growth.

We call two algebras *repetitive equivalent*, iff their repetitive categories are isomorphic or antiismorphic. (Recall from 1.2 the equivalent meanings of repetitive equivalence for triangular algebras.)
By 1.2, 1.3, 3.2 any of the following properties of a triangular algebra carries over to its repetitive equivalence class: being a bound poset algebra, being rpg-critical, being gentle, or being almost gentle.

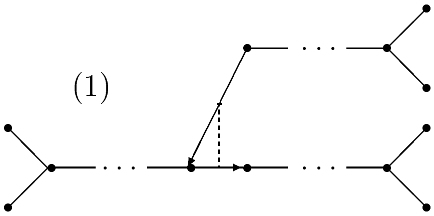
4.2 Our description of the rpg-critical bound poset algebras splits as this:

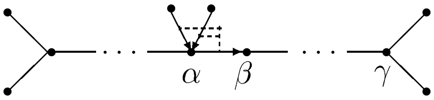
Let us start with the untypical case. From 1.3 the description of the gentle rpg-critical bound poset algebras is indeed obvious.

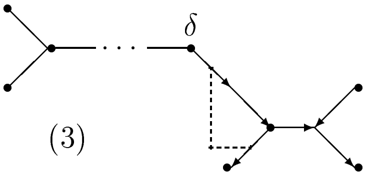
The main result of [Ro] now asserts that the non-gentle rpg-critcal algebras can be completely described by nine infinite families of bound poset algebras and the following two bound poset algebras $\mathcal{G}$ and $\mathcal{G}'$.

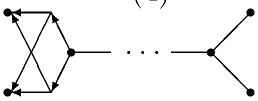
$\mathcal{G}$  $\mathcal{G}'$ 

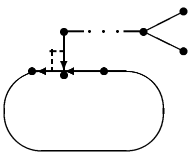
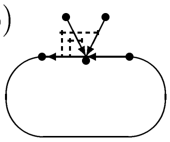
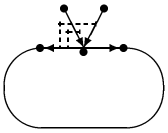
Theorem. *Let k be an algebraically closed field. The bound poset k-algebras from the families $(1)-(9)$, and the k-algebras $\mathcal{G}$ and $\mathcal{G}'$ are non-gentle rpg-critical. Conversely, for any non-gentle rpg-critical bound poset k-algebra $B \neq \mathcal{G}$ or $\mathcal{G}'$ there exists an algebra A from these 9 families s.t. A and B are repetitive equivalent.*

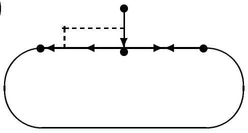
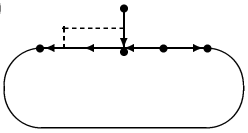
(1) 

(2) 

(3) 

(4) 

(5)  (6)  (7) 

(8)  (9) 

(Pictures (1) – (4) display pg-critical bound poset algebras. Here the unoriented edges have to be oriented s.t. the resulting algebra does not contain a pg-critical proper full subcategory or $\mathcal{G}$, which is actually easy to check. Picture (3) may contain just 8 vertices (i.e. we double the $\tilde{D}_5$-extension vertex δ), and in picture (2) we may have $\alpha = \gamma$ meaning that we double the $\tilde{D}_m$-coextension vertex β.

In pictures (5) – (9) the drawn oval means a non-oriented cycle, bound by zero relations of length $= 2$. Moreover all the non-zero paths there of length ≥ 2 are drawn. Finally in picture (5) the convex subcategory K outside the cycle is hereditary $= A_3$ or D_m s.t. if for $\sigma \in K_0$ the full subcategory given by all the vertices except σ is connected, then this subcategory is gentle.)

4.3 *About the proof.* First take an algebra B from our list. It is not difficult to see that $\hat{B}$ is not of polynomial growth ([Ro] 5.5). Moreover, a direct calculation based on [NoS] reveals that the repetitive category to any proper full subcategory of B is domestic ([Ro] 5.6). Finally we observe that B is almost gentle using 2.3 and [Ro] 8.3, 6.4 concerning family (8), (9), and therefore B is rpg-critical.

Verifying that any non-gentle rpg-critical bound poset algebra is repetitive equivalent to an algebra from our list is a lengthy combinatorial development which covers big parts of [Ro]. So we will restrict ourselves to comment on the key idea concerning our restriction to bound poset algebras: Suppose that B is a triangular rpg-critical algebra. Then up to duality $B = A[M]$ for a (connected) algebra A (cf. [Ro] 4.1), and $\hat{A}$ is of polynomial growth. So we will usually have an explicit description of A (for instance, in case A is representation-infinite), but we will mostly not have enough knowledge about the module category of A in order to list all the possibilities for M. If now B is a bound poset algebra, then A is a bound poset algebra as well, and by 3.2 we can read all the possibilities for M directly from the bound poset description of A.

4.4 Let us append a few rather immediate applications of our Theorem.

From 4.3 we obtain that the repetitive category to a proper full subcategory of an rpg-critical bound poset algebra is even domestic, and moreover this

Proposition. *Any rpg-critical bound poset algebra is almost gentle.*

Proposition. ([Ro] 9.3, cf. 3.3) *Suppose that B is an rpg-critical bound poset algebra. Then any non-gentle proper full subcategory of B is tilting-cotilting equivalent to a hereditary algebra of type D or $\tilde{D}$.*

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