ON STABLE EQUIVALENCES OF MORITA TYPE BETWEEN REPRESENTATION-FINITE SELFINJECTIVE ALGEBRAS

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Introduction

Throughout this paper k denotes an algebraically closed field, and all algebras considered here are assumed to be basic, connected, finite-dimensional k-algebras with identity. For an algebra Λ, we denote by $\operatorname{mod}\Lambda$ and by $\underline{\operatorname{mod}}\,\Lambda$ the category of finite-dimensional (right) Λ-modules and its stable category, respectively. Recall that for two algebras Λ and Π, an equivalence $\underline{\operatorname{mod}}\,\Lambda \to \underline{\operatorname{mod}}\,\Pi$ is called a *stable equivalence* from Λ to Π. The algebras Λ and Π are said to be *stably equivalent* if there exists a stable equivalence between them. Following Broué [4] (see also Linckelmann [5]) a stable equivalence $\phi : \underline{\operatorname{mod}}\,\Lambda \to \underline{\operatorname{mod}}\,\Pi$ is called *of Morita type* if there exist bimodules ${}_\Lambda M_\Pi, {}_\Pi N_\Lambda$ such that the following three conditions are satisfied:

(1) ${}_\Lambda M, M_\Pi, {}_\Pi N$ and N_Λ are projective modules;
(2) (a) $M \otimes_\Pi N \cong \Lambda \oplus P$ as Λ-bimodules for some projective Λ-bimodule P, and
(b) $N \otimes_\Lambda M \cong \Pi \oplus Q$ as Π-bimodules for some projective Π-bimodule Q; and
(3) ϕ is lifted to the functor $\text{-} \otimes_\Lambda M$, i.e., the diagram

$$\begin{array}{ccc} \operatorname{mod}\Lambda & \xrightarrow{\text{-}\otimes_\Lambda M} & \operatorname{mod}\Pi \\ {\scriptstyle \mathrm{can}}\downarrow & & \downarrow{\scriptstyle \mathrm{can}} \\ \underline{\operatorname{mod}}\,\Lambda & \xrightarrow[\phi]{} & \underline{\operatorname{mod}}\,\Pi \end{array}$$

commutes up to natural isomorphisms.

Note that the ϕ above is lifted to a Morita equivalence if and only if it is of Morita type with $P = 0 = Q$ in the definition. An example of a stable equivalence of Morita type is given by the loop functor over a selfinjective algebra, as we shall see later. In particular, the loop functor over a (non-semisimple) selfinjective Nakayama algebra gives an example of a stable quivalence of Morita type that is not lifted to a Morita equivalence. Since the condition to be of Morita type is rather strong, most stable equivalences seem to be not of Morita type. But in constructing stable equivalences one usually uses those of Morita type. Therefore it seems that only few examples of stable equivalences not of Morita type are known. In looking for such examples in the class of representation-finite selfinjective algebras, we realized that by contraries it is impossible to find them in most cases in this class. Namely, we obtained the following.

Main Theorem. *Let Λ and Π be selfinjective algebras. Assume that Λ is representation-finite and not of type $(D_{3m}, s/3, 1)$ with $m \geq 2$, $s \nmid 3$. Then all stable equivalences between Λ and Π are of Morita type.*

See the next section for the definition of types of representaion-finite selfinjective algebras. At present we do not know whether there exist stable equivalences not of Morita type in the remaining cases.

1. Preliminaries

Note that Main Theorem holds trivially if Λ is semisimple. Therefore throughout the rest of the paper we assume that all selfinjective algebras considered here are *not semisimple*, or equivalently that they are not isomorphic to the one-dimensional algebra k.

We denote by D the usual k-dual functor $\mathrm{Hom}_k(\text{-}, k)$.

Recall from [1] the definition of the *type* $\mathrm{typ}(\Lambda)$ of a representation-finite selfinjective algebra Λ. By Riedtmann [8] it is known that the stable Auslander-Reiten quiver of Λ has the form $\mathbb{Z}\Delta/\langle g\tau^{-r}\rangle$, where Δ is a Dynkin graph, r is a natural number and g is an automorphism of the quiver $\mathbb{Z}\Delta$ with a fixed vertex. In this case, we set $\mathrm{typ}(\Lambda) := (\Delta, f, t)$, where $f := r/m_\Delta$ and t is the order of g. Here m_Δ is equal to n, $2n-3$, 11, 17 or 29 according as $\Delta = A_n, D_n, E_6, E_7$ or E_8, respectively. Note that if n is the number of vertices of Δ, then nf is equal to the number of isoclasses of simple Λ-modules (Bretscher et al. [3]). We recall the following.

Proposition 1.1 ([1]). *The set of all the types of representation-finite selfinjective algebras ($\not\cong k$) is equal to the disjoint union of the following sets:*
$\{(A_n, s/n, 1) | n, s \in \mathbb{N}\}$;
$\{(A_{2p+1}, s, 2) | p, s \in \mathbb{N}\}$;
$\{(D_n, s, 1) | n, s \in \mathbb{N}, n \geq 4\}$;
$\{(D_{3m}, s/3, 1) | m, s \in \mathbb{N}, m \geq 2, 3 \nmid s\}$;
$\{(D_n, s, 2) | n, s \in \mathbb{N}, n \geq 4\}$;
$\{(D_4, s, 3) | s \in \mathbb{N}\}$;
$\{(E_n, s, 1) | n = 6, 7, 8, s \in \mathbb{N}\}$; *and*
$\{(E_6, s, 2) | s \in \mathbb{N}\}$.

Notice that non-standard algebras have type $(D_{3m}, 1/3, 1)$ for some $m \geq 2$. Hence all the algebras in Main Theorem are standard algebras.

Recall that algebras Λ and Π are said to be *derived equivalent* (Rickard [6]) if the bounded derived categories of $\mathrm{mod}\,\Lambda$ and $\mathrm{mod}\,\Pi$ are equivalent as triangulated categories. By Rickard [7], there exists a stable equivalence of Morita type between two selfinjective algebras if they are derived equivalent. With this in mind we have the following:

Theorem 1.2 ([1]). *Let Λ and Π be representation-finite standard selfinjective algebras. Then the following are equivalent:*

1. *Λ and Π are derived equivalent;*
2. *There exists a stable equivalence of Morita type between Λ and Π;*
3. *Λ and Π are stably equivalent; and*
4. *$\mathrm{typ}(\Lambda) = \mathrm{typ}(\Pi)$.*

For each type (Δ, f, t) we gave a representative algebra $\Lambda(\Delta, f, t)$ that is a representation-finite standard selfinjective algebra of type (Δ, f, t) (see [1, 2.4 and 2.5]). We set $\mathrm{can}(\Lambda) := \Lambda(\mathrm{typ}(\Lambda))$.

2. Reduction to self-equivalences of representative algebras

Lemma 2.1. *Let Λ, Λ' and Λ'' be algebras.*

1. *If $\phi : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Lambda'$ is a stable equivalences of Morita type, then so is any quasi-inverse of ϕ; and*
2. *If both $\phi : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Lambda'$ and $\phi' : \underline{\mathrm{mod}}\,\Lambda' \to \underline{\mathrm{mod}}\,\Lambda''$ are stable equivalences of Morita type, then so is the composite $\phi'\phi : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Lambda''$.*

Proof. This is straitforward. □

The following theorem, the proof of which will be given in the next section, is an essential part of the proof of Main Theorem.

Theorem 2.2. *Let Λ be a representation-finite selfinjective algebra with $\Lambda = \mathrm{can}(\Lambda)$. If Λ is not of type $(D_{3m}, s/3, 1)$ with $m \geq 2$, $s \nmid 3$, then every stable self-equivalence of Λ is of Morita type.*

Proof of Main Theorem using Theorem 2.2. Let $\phi : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Pi$ be a stable equivalence from Λ to Π. Then $\mathrm{can}(\Lambda) = \mathrm{can}(\Pi) =: \Gamma$. By Theorem 1.2 there exist stable equivalences $\zeta : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Gamma$ and $\eta : \underline{\mathrm{mod}}\,\Pi \to \underline{\mathrm{mod}}\,\Gamma$ of Morita type. Then by Theorem 2.2, $\xi := \eta\phi\zeta^{-1} : \underline{\mathrm{mod}}\,\Gamma \to \underline{\mathrm{mod}}\,\Gamma$ is a stable equivalence of Morita type because $\Gamma = \mathrm{can}(\Gamma)$. Hence $\phi = \eta^{-1}\xi\zeta$ is also of Morita type by Lemma 2.1. □

3. Proof of Theorem 2.2

In this section we give a proof of Theorem 2.2. We recall the following from Auslander and Reiten [2, Corollary 3.3].

Lemma 3.1. *Let A and B be finite-dimensional algebras over a perfect field K. Then for a finite-dimensional A-B-bimodule X the following are equivalent.*

1. *X is a projective bimodule.*
2. *X is a projective right B-module and for any finite-dimensional right A-module C, $C \otimes_A X$ is a projective B-module.*

In particular, if X is projective as an A-B-bimodule, then so is $C \otimes_A X$ for any finite-dimensional right A-module C.

The following is well-known.

Lemma 3.2. *Let K be a commutative ring and R and S be K-algebras. We regard R-S-bimodules as left $R \otimes_K S^{op}$-modules in a natural way. If M is an R-S-bimodule, X is a left R-module and Y is a left S-module, then we have a natural isomorphism*

$$\mathrm{Hom}_R(M \otimes_S Y, X) \xrightarrow{\sim} \mathrm{Hom}_{R\otimes_K S^{op}}(M, \mathrm{Hom}_K(Y, X)).$$

Proof. The isomorphism from the right to the left is defined by

$$\alpha \mapsto (m \mapsto (y \mapsto \alpha(m \otimes y))),$$

and the converse is defined by

$$\beta \mapsto (m \otimes y \mapsto (\beta(m))(y))$$

for each α in the left term, each $m \in M$, each $y \in Y$ and each β in the right term. $\square$

Lemma 3.3. *Let A and B be finite-dimensional algebras over a field K. If X is a finite-dimensional injective left A-module and Y is a finite-dimensional injective right B-module, then $X \otimes_k Y$ is an injective A-B-bimodule. In particular, if both A and B are selfinjective algebras, then so is $A \otimes_k B^{op}$.*

Proof. The assertion follows from the following isomorphisms:

$$\begin{aligned}\operatorname{Hom}_{A\otimes_k B^{\mathrm{op}}}(\text{-}, X \otimes_k Y) &\cong \operatorname{Hom}_{A\otimes_k B^{\mathrm{op}}}(\text{-}, \operatorname{Hom}_k(D(Y), X))\\ &\cong \operatorname{Hom}_A(\text{-} \otimes_B D(Y), X).\end{aligned}$$

Here the last isomorphism is given by the lemma above. $\square$

Lemma 3.4. *Let Λ be a selfinjective algebra. Then both the loop functor $\Omega_\Lambda : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Lambda$ and the Auslander-Reiten translate $\tau_\Lambda : \underline{\mathrm{mod}}\,\Lambda \to \underline{\mathrm{mod}}\,\Lambda$ are stable equivalences of Morita type.*

Proof. Let $\mu : \Lambda \otimes_k \Lambda^{\mathrm{op}} \to \Lambda$ be the multiplication of Λ. Put $I := \operatorname{Ker}\mu$ and consider the exact sequence

$$(*) \qquad 0 \longrightarrow I \xrightarrow{\text{incl}} \Lambda \otimes_k \Lambda^{\mathrm{op}} \xrightarrow{\mu} \Lambda \longrightarrow 0$$

of Λ-bimodules. Then since Λ is projective as a right (resp. a left) Λ-module, $(*)$ splits in $\mathrm{mod}\,\Lambda$ (resp.in $\mathrm{mod}\,\Lambda^{\mathrm{op}}$). Thus I is projective as a right (resp. a left) Λ-module. By the lemma above the algebra $\Lambda \otimes_k \Lambda^{\mathrm{op}}$ is selfinjective. Let E be an injective hull of Λ as a left $\Lambda \otimes_k \Lambda^{\mathrm{op}}$-module. Put $J := E/\Lambda$ and consider the exact sequence

$$(**) \qquad 0 \longrightarrow \Lambda \xrightarrow{\text{incl}} E \xrightarrow{\text{can}} J \longrightarrow 0$$

of Λ-bimodules. Then since Λ is injective as a right (resp. a left) Λ-module, $(**)$ splits in $\mathrm{mod}\,\Lambda$ (resp. in $\mathrm{mod}\,\Lambda^{\mathrm{op}}$). Thus J is projective as a right (resp. a left) Λ-module because so is E. Now since I is a projective right Λ-module, the sequence $I \otimes_\Lambda (**)$:

$$0 \longrightarrow I \cong I \otimes_\Lambda \Lambda \longrightarrow I \otimes_\Lambda E \longrightarrow I \otimes_\Lambda J \longrightarrow 0$$

is an exact sequence of Λ-bimodules. Note that $I \otimes_\Lambda E$ is still a projective Λ-bimodule by Lemma 3.1. Hence we have $I \otimes_\Lambda J \cong \Omega^{-1}_{\Lambda\otimes_\Lambda \Lambda^{\mathrm{op}}}(I) \cong \Lambda$ in $\underline{\mathrm{mod}}(\Lambda \otimes_k \Lambda^{\mathrm{op}})^{\mathrm{op}}$. Therefore $I \otimes_\Lambda J \oplus P' \cong \Lambda \oplus P''$ for some projective Λ-bimodules P', P''. If A is a direct summand of P' as a Λ-bimodule, then Λ turns out to be a projective Λ-bimodule, and $X \cong X \otimes_\Lambda \Lambda$ is projective for each $X \in \mathrm{mod}\,\Lambda$. Thus Λ is semisimple, a contradiction. Hence Λ is a direct summand of $I \otimes_\Lambda J$ because Λ is connected. Accordingly, we have

$$I \otimes_\Lambda J \cong A \oplus P$$

for some projective Λ-bimodule P. Similarly, we have

$$J \otimes_\Lambda I \cong A \oplus Q$$

for some projective Λ-bimodule Q. Therefore I and J induce a stable equivalence of Morita type. It is well-known [2] and easy to see that Ω_Λ is lifted to the functor $\text{-}\otimes_\Lambda I$. As a consequence Ω_Λ is a stable equivalence of Morita type. Finally noting that the Nakayama functor $\text{-}\otimes_\Lambda D\Lambda$ is a Morita equivalence in this case, it follows from $\tau_\Lambda \cong \Omega_\Lambda^2 \circ (\text{-}\otimes_\Lambda D\Lambda)$ that τ_Λ is also a stable equivalence of Morita type by Lemma 2.1. □

For a representation-finite standard selfinjective algebra Λ, let $\operatorname{ind}\Lambda$ be the full subcategory of $\operatorname{mod}\Lambda$ formed by a fixed complete set of representatives of isoclasses of indecomposable Λ-modules. Identify $\operatorname{ind}\Lambda$ with the mesh category $k(\Gamma_\Lambda)$ by a fixed isomorphism. Also identify the mesh category $k({}_s\Gamma_\Lambda)$ with the stable category $\underline{k(\Gamma_\Lambda)}$ in a natural way. Then the stable category $\underline{\operatorname{ind}}\,\Lambda$ is identified with $k({}_s\Gamma_\Lambda)$.

Lemma 3.5. *Let Λ and Π be representation-finite standard selfinjective algebras, and let $\phi : \underline{\operatorname{mod}}\,\Lambda \to \underline{\operatorname{mod}}\,\Pi$ a stable equivalence. If the isomorphism $\phi' : k({}_s\Gamma_\Lambda) \to k({}_s\Gamma_\Pi)$ of the mesh categories induced from ϕ is lifted to an isomorphism of the mesh categories $k(\Gamma_\Lambda) \to k(\Gamma_\Pi)$, then ϕ is lifted to a Morita equivalence.*

Proof. This is straitforward. □

The following proposition makes it possible to prove our theorem by a combinatorial argument.

Proposition 3.6. *Let Λ and Π be representation-finite standard selfinjective algebras, and let $\phi : \underline{\operatorname{mod}}\,\Lambda \to \underline{\operatorname{mod}}\,\Pi$ a stable equivalence. Then ϕ is lifted to a Morita equivalence if and only if ϕ maps the configuration of Λ into that of Π, i.e., for any projective indecomposable Λ-module P, there exists a projective indecomposable Π-module Q such that $\phi(\operatorname{rad} P) \cong \operatorname{rad} Q$.*

Proof. ($\Rightarrow$). Assume that ϕ is lifted to a Morita equivalence Φ. Let P be an indecomposable projective Λ-module. Then $Q := \Phi(P)$ is an indecomposable projective Π-module. Since the inclusion map $\alpha : \operatorname{rad} P \to P$ is a source map, $\Phi(\alpha) : \Phi(\operatorname{rad} P) \to Q$ is a source map. Hence $\Phi(\operatorname{rad} P) \cong \operatorname{rad} Q$ in $\operatorname{mod}\Pi$. Thus $\phi(\operatorname{rad} P) \cong \operatorname{rad} Q$ in $\underline{\operatorname{mod}}\,\Pi$.

($\Leftarrow$). Fix isomorphisms $k({}_s\Gamma_\Lambda) \to \underline{k(\Gamma_\Lambda)}$ and $k({}_s\Gamma_\Pi) \to \underline{k(\Gamma_\Pi)}$. Then ϕ induces an isomorphism $\phi' : k({}_s\Gamma_\Lambda) \to k({}_s\Gamma_\Pi)$, and ϕ' induces a quiver isomorphism $\psi : {}_s\Gamma_\Lambda \to {}_s\Gamma_\Pi$. Assume that ϕ maps the configuration of Λ into that of Π. Let P be an indecomposable projective Λ-module, and consider an almost split sequence

$$0 \longrightarrow \operatorname{rad} P \xrightarrow{\binom{f_1}{f_2}} P \oplus \operatorname{rad} P/\operatorname{soc} P \xrightarrow{(g_1, g_2)} P/\operatorname{soc} P \longrightarrow 0.$$

By assumption $\phi(\operatorname{rad} P) \cong \operatorname{rad} Q$ for some indecomposable projective Π-module $Q := Q_P$. Since $g_2 f_2 = -g_1 g_1 : \operatorname{rad} P \to P/\operatorname{soc} P$ factors through the projective module P, any representative h in $\phi(\underline{g_2 f_2})$ factors through a projective Π-module. But since Q is a projective cover of $Q/\operatorname{soc} Q$, h factors through Q. Now denote the mesh corresponding to the almost split sequence above by

$$\begin{tikzcd}
 & P \arrow[dr, "\beta_P"] & \\
x \arrow[ur, "\alpha_P"] \arrow[r, "\alpha_1"] \arrow[ddr, "\alpha_n"'] & y_1 \arrow[r, "\beta_1"] & z \\
 & \vdots & \\
 & y_n \arrow[uur, "\beta_n"'] &
\end{tikzcd} .$$

Then for each $i = 1, \dots, n$ we have

$$\begin{aligned} \phi'(\alpha_i) &= s_i \overline{\psi(\alpha_i)} + \mu_i \\ \phi'(\beta_i) &= t_i \overline{\psi(\beta_i)} + \nu_i \end{aligned} \tag{1}$$

for some $s_i, t_i \in k^\times := k\backslash\{0\}$ and some $\mu_i, \nu_i \in \operatorname{rad}^2 k(\Gamma_\Pi)$, where $\overline{\text{(-)}}$ denotes the coset modulo the mesh ideal. Further by the above $h := \sum_{i=1}^n \phi'(\beta_i)\phi'(\alpha_i)$ factors through the projective cover Q of $\psi(z)$, say $h = -\eta_P \zeta_P$. Then we have

$$\eta_P \zeta_P + \sum_{i=1}^n \phi'(\beta_i)\phi'(\alpha_i) = 0. \tag{2}$$

Therefore we can define a functor $\Phi : k(\Gamma_\Lambda) \to k(\Gamma_\Pi)$ by $\Phi := \phi'$ on ${}_s\Gamma_\Lambda$; and $\Phi(P) := Q_P$ and $\Phi(\alpha_P) := \zeta_P$ and $\Phi(\beta_P) := \eta_P$ for each projective vertex P in Γ_Λ. Then by construction Φ is a lift of ϕ'. By Lemma 3.5 it is enough to show that Φ is an isomorphism. Since Φ induces a quiver isomorphism $\Psi : \Gamma_\Lambda \to \Gamma_\Pi$ that is an extension of ψ, we have $\dim_k k(\Gamma_\Lambda)(x, y) = \dim_k k(\Gamma_\Pi)(\Phi(x), \Phi(y))$ for all vertices x, y of Γ_Λ. Thus we have only to show that Φ is full. For each projective vertex P in Γ_Λ we can write

$$\begin{aligned} \zeta_P &= s_P \overline{\Psi(\alpha_P)} + \mu_P \\ \eta_P &= t_P \overline{\Psi(\beta_P)} + \nu_P \end{aligned} \tag{3}$$

for some $s_P, t_P \in k$ and some $\mu_P, \nu_P \in \operatorname{rad}^2 k(\Gamma_\Pi)$. Suppose that $s_P t_P = 0$ for some P. Then (1), (2) and (3) show that $\sum_{i=1}^n s_i t_i \overline{\psi(\beta_i)\psi(\alpha_i)} \in \operatorname{rad}^3 k(\Gamma_\Pi)$. But since each mesh relation is a sum of paths of length 2, this forces to have $\Psi(\beta_P)\Psi(\alpha_P) = 0$ in $k\Gamma_\Pi$, a contradiction. Therefore $s_P t_P \neq 0$, i.e., $s_P \neq 0$ and $t_P \neq 0$ for all projective vertices P in Γ_Λ. This shows that $\overline{\Psi(\alpha_P)}$ and $\overline{\Psi(\beta_P)}$ are in the image of Φ for all projective vertices P in Γ_Λ because ϕ' is an isomorphism. Thus for all arrows α in Γ_Π, $\overline{\alpha}$ are in the image of Φ, i.e., Φ is full. Hence Φ is an isomorphism. $\square$

Proof of Theorem 2.2. Assume that $\Lambda = \operatorname{can}(\Lambda)$, and that $\operatorname{typ}(\Lambda) \neq (D_{3m}, s/3, 1)$ for any $m \geq 2$ and s with $3 \nmid s$. Let $\phi : \underline{\operatorname{mod}}\, \Lambda \to \underline{\operatorname{mod}}\, \Lambda$ be a stable equivalence. We show that ϕ is of Morita type. As above ϕ induces an automorphism ϕ' of $k({}_s\Gamma_\Lambda)$. Put $\operatorname{typ}(\Lambda) = (\Delta, f, t)$. Then ${}_s\Gamma_\Lambda$ is identified with $\mathbb{Z}\Delta/\langle g \rangle$ for some admissible automorphism g of $\mathbb{Z}\Delta$; and ϕ' induces a quiver automorphism ψ of $\mathbb{Z}\Delta/\langle g \rangle$. Lift ψ to an automorphism $\tilde{\psi}$ of $\mathbb{Z}\Delta$, thus we have a commutative diagram

$$\begin{tikzcd} \mathbb{Z}\Delta \arrow[r, "\tilde{\psi}"] \arrow[d] & \mathbb{Z}\Delta \arrow[d] \\ \mathbb{Z}\Delta/\langle g\rangle \arrow[r, "\psi"'] & \mathbb{Z}\Delta/\langle g\rangle, \end{tikzcd}$$

where the vertical morphisms are the canonical one. By the classification of automorphisms of $\mathbb{Z}\Delta$ due to Riedtmann [8], $\tilde{\psi} = \tau^a\eta$ up to conjugates, where $a \in \mathbb{Z}$ and η is an automorphism either satisfying $\eta^2 = \tau$ or having an order 1, 2 or 3. If either η is the identity or $\operatorname{typ}(\Lambda) \neq (A_n, s/n, 1)$ for any n, s, then η can be chosen in such a way that it stabilizes the configuration of Λ because of the symmetry of $\operatorname{can}(\Lambda)$ (see [1, 2.4 and 2.5]). This means that $\xi := \tau_\Lambda^{-a}\phi$ maps the configuration of Λ into itself. Hence by Proposition 3.6, ξ is lifted to a Morita equivalence. Thus $\phi \cong \tau_\Lambda^a\xi$ is of Morita type by Lemmas 3.4 and 2.1.

Now assume that η is not the identity and $\operatorname{typ}(\Lambda) = (A_n, s/n, 1)$ for some n, s. Then Λ is the selfinjective Nakayama algebra of Loewy length $n+1$ with exactly s simple modules; and η has order 2 or satisfies $\eta^2 = \tau$. In this case the loop functor Ω_Λ maps the indecomposable module of length t to the indecomposable module of length $n+1-t$ for all $1 \leq t \leq n$. This means that $\xi := \Omega_\Lambda\tau_\Lambda^b\phi$ maps the configuration of Λ into itself for some $b \in \mathbb{Z}$. Hence by Proposition 3.6, ξ is lifted to a Morita equivalence. Thus $\phi \cong \tau_\Lambda^{-b}\Omega_\Lambda^{-1}\xi$ is of Morita type.

Acknowledgment

I would like to thank H. Krause for asking me to give an example of a stable quivalence not of Morita type, from which the present work has started, and S. Takashima for showing me his idea of a proof of Lemma 3.4 from which the present proof of it essentially came because the original proof was not complete. This work was done while I visited the University of Bielefeld in 1999. I would like to thank Professor C. M. Ringel for the hospitality, and Sonderforschungsbereich 343 for the support.

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