CONTRIBUTIONS TO THE DIAGNOSIS OF COMPUTING NETWORKS

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One of the important and interesting problems of large multi–processor computing networks is the problem of automatic self–diagnosis. The fundamental model for such systems capable of automatic fault diagnosis, developed by Preparata, Metze and Chien, is the following. A system S consists of n units $v_1, v_2, \dots, v_n$ and a prescribed connection assigment that assigns each unit to test a subset of other units. The outcome of a test, in which v_i tests v_j, is denoted by a_{ij}, where $a_{ij} = 1$, if unit v_i finds v_j to be faulty, and $a_{ij} = 0$, if unit v_i finds unit v_j to be nonfaulty. If the testing unit is faulty then the outcome is unreliable. The set of test outcomes $\{a_{ij}\}$ represents the syndrome of the system.

A diagnostic process allowing identification of at least one faulty unit is called sequential diagnosis or diagnosis with repair.

Definition 1. A system of n units is sequentially t–fault diagnosable, if at least one faulty unit can be identified without replacement provided the number of faulty units present does not exceed t.

If one application of the set of tests is sufficient to identify all faulty units, one–step diagnosis is said to occur.

Definition 2. A system of n units is one–step t–fault diagnosable, if all faulty units within the system can be identified without replacement, provided the number of faulty units does not exceed t.

The diagnostic model in a graph theoretical representation is a directed graph $G(U, A)$, called the diagnostic graph of system S, where U is the set of nodes and A the set of arcs.

For a given system S of units $v_1, v_2, \dots, v_n$ each node in U is identified with a unit of S. The graph G has a directed arc (v_i, v_j) from v_i to v_j if and only if unit v_i tests unit v_j. Set A defines the connection of the system. Following one application of the set of tests for units of S, every arc in A is labeled with the test outcome. The set of arc labels represents the syndrome of S.

In works [1–5] so called symmetrical systems (graphs) were investigated, in which the presence of testing link (arc) (v_i, v_j) implies the presence of testing link (arc) (v_j, v_i) . Computing networks in most cases are symmetric systems.

In given works also were investigated problems of so–called partial diagnosis of the systems, which is as intermediate between sequential diagnosis and 1–step diagnosis.

Consider the situation, where the number of faults exceeds 1–step diagnostic capability of the system, i.e. the situation, where reveal of all faulty units of the system is impossible. Then naturally there arises a problem of finding the maximum number of units with known state within the system for given syndrome, i.e. to produce a partial diagnosis.

Let n be the number of units of the system and let the number of faults does not exceed t . Denote by $n_t(G_n)$ the maximum number of units of the graph G_n , for which their state at any configuration with not more than t–faults can be determined for sure. Thus, in the case of sequential diagnosis $n_t(G_n) \geq 1$, and in the case of 1–step diagnosis $n_t(G_n) = n$.

Let's state the main results of works [1–5].

1. Sequential diagnosis

Let $t^{seq.}(G_n)$ be the sequential diagnostic capability of the graph G_n and $t^{seq.}(n) = \min t^{seq.} \ (G_n)$, where the minimum is taken over all n–nodes connected graphs. The exact lower bound of sequential diagnostic capability in the class of all n–nodes connected graphs is characterized by the following theorem ($]x[$ – the minimum integer not less than x).

Theorem 1. $t^{seq.}(n) =] \ 2 \cdot \sqrt{n} \ [-3$.

The proof of Theorem 1 is simultaneously algorithm for sequential diagnosis of n–nodes graph G_n , when the number of faults doesn't exceed $] \ 2 \cdot \sqrt{n} \ [-3$. Furthermore, an example of G_n , which is not sequentially diagnosable at presence of $] \ 2 \cdot \sqrt{n} \ [-2$ faults, is given.

Mentioned theorem is obtained in the class of all n–nodes connected graphs. Obviously, that value of sequential diagnostic capability can be increased, if it is limited by consideration of a smaller class of the graphs. In particular, if the graph is n–node chain–L_n and n–node cycle C_n , then there is the following

Theorem 2.

a) $t^{seq.}(L_n) =] \ 2 \cdot \sqrt{2} \cdot \sqrt{n+4} \ [-7$

b) $t^{seq.}(C_n) =] \ 2 \cdot \sqrt{2} \cdot \sqrt{n} \ [-5, n > 3$.

This theorem shows that sequential diagnostic capability of the systems of linear and cyclic structure approximately $\sqrt{2}$ times exceeds sequential diagnostic capability of the "worst" n–node connected graph.

In the following theorem the dependence of diagnostic capability of the graph on minimum local degree of its nodes is given.

Theorem 3.

Sequential diagnostic capability of the graph G_n (may be not connected) satisfies the inequality

$$t^{seq.}(G_n) \geq \min\left\{p(G_n), \left[\frac{n-1}{2}\right]\right\},$$

where $p(G_n)$ is the minimum local degree of the nodes of graph G_n .

Note, that the statement of the theorem 3 was known before as necessary condition of 1–step t diagnosis.

2. 1–STEP DIAGNOSIS

Let $t^{(1)}(G)$ be 1–step diagnostic capability of the graph G . The following statement is true.

Theorem 4.

$$t^{(1)}(G) = \min_{T, G_i^T}\{|T| + t^{seq.}(G_i^T)\},$$

where the minimum is taken over all node sections T of the graph G and corresponding components G_i^T , and $t^{seq.}(G_i^T)$ is the sequential diagnostic capability of the component G_i^T .

Two sufficient conditions of 1–step diagnosis are obtained. Let $p(G)$ be the minimum local degree of the graph G .

Theorem 5.

1–step diagnostic capability $t^{(1)}(G)$ of the graph G satisfies the inequality

$$t^{(1)}(G) \geq \min\left\{\frac{p(G)+\Delta}{2}, \left[\frac{n-1}{2}\right]\right\},$$

where Δ is the minimum node section of the graph G .

Theorem 6.

$$t^{(1)}(G) \geq \min\left\{\frac{3\cdot p(G)-n}{2}+1, \left[\frac{n-1}{2}\right]\right\}.$$

From Theorem 6 it follows, that at $p(G) \geq \left]\, \frac{2}{3}\cdot n - 1 \,\right[$ 1–step diagnostic capability attains maximum possible value, equal to $\left[\frac{n-1}{2}\right]$.

Note also, that the result of Theorem 5 can not be improved for $t < \frac{n}{4}$ in the sense that there can be given an example of a graph G' , with $p(G') = 2\cdot n - |\Delta| - 1$, which is not 1–step t–diagnosable . When $\frac{n}{4} \leq t \leq \left[\frac{n-1}{2}\right]$, the result of Theorem 6 should be used in the above–mentioned interval is also not improvable.

In the presented works the problems of construction of optimal 1–step t–diagnosable graphs are also considered. Optimal graphs in the meaning of 1–step diagnosis are called the graphs with minimum number of arcs at given number of nodes and diagnostic capability t . It is known, that for 1–step t–diagnosing it is necessary to carry out the two following conditions: $2t < n$ and $t \leq p$, where p is the minimal local degree of the nodes of the graph.

Therefore the construction of 1–step t–diagnosable n–nodes t–regular graphs, solves propounded problem (t–regular is called the graph, whose local degrees of all nodes are equal to t).

Let $v_0, v_1, \ldots, v_{n-1}$ be the nodes of the graph G_n .

a.) $t = 2k$. The nodes v_i and v_j of the graph G_n are joined by the arc e_{ij} iff $(i-j) \equiv m (\mathrm{mod}\ n)$, $m = \pm\, 1, \pm\, 2, \ldots, \pm\, k$.

b.) $t = 2k+1, 2/n$. The nodes v_i and v_j of the graph are connected by the arc e_{ij} iff $(i-j) \equiv m (\mathrm{mod}\ n)$, $m = \pm\, 1, \pm\, 2, \ldots, \pm\, k, \frac{n}{2}$.

It is obvious that in both cases constructed graphs are t–regular and symmetric. The proof, that these graphs are t–diagnosable , is based on Menger's theorem, utilization of which allows to show, that minimum node section of constructed graph is equal to t , and then, taking in account Theorem 4, one gets a statement.

3. Partial diagnosis

In the presented works there are considered two classes of the graphs, for which the values of $n_t(G_n)$ were obtained. The first class includes connected graphs with $p(G_n) \geq t$, and $n > 2t$.

Theorem 7.

Let $p(G_n) \geq t$, $n > 2t$ and Δ be minimum node section of the graph G_n . Then $n_t(G_n) \geq \max\{n - 2 \cdot t + 2 \cdot \Delta, p(G_n) + 1\}$.

The second class are the graphs of linear structure, i.e. $G_n = L_n$ (sequential diagnostic capability of n–node chain was found in Theorem 2).

Theorem 8.

$$n_t(L_n) \geq \begin{cases} 6 - t + 2 \cdot \sqrt{2n - 6t + 8}, & \frac{3 \cdot \sqrt{8n+17} - 11}{4} \leq t \leq 2 \cdot \sqrt{2} \cdot \sqrt{n+4} - 6 \\ t - \frac{\sqrt{t^2 - 4n + 4t}}{2}, & 2 \cdot \sqrt{n+1} - 2 \leq t < \frac{3 \cdot \sqrt{8n+17} - 11}{4} \\ \frac{n - t^2}{4}, & 7 \leq t < 2 \cdot \sqrt{n+1} - 2 \\ n - 2 \cdot (t-1), & 1 \leq t < 7. \end{cases}$$

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