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Main Lectures

The main lectures will present important recent advances in the subject and decisive relations to other parts of mathematics. The aim of these lectures is to outline developments which are of interest to a general audience; they should be comprehensible not only for experts, but for mathematicians say with some working knowledge in algebra, including graduate students.

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Computer algebra session

Peter Dräxler (Bielefeld, Germany): "Computer algebra packages for problems in representation theory"

Short Communications

The organizers have used all their efforts to arrange the program in such a way that it allows as many of the partipants as possible to report on their new results. Due to lack of time, a decisive selection had to be made. For the benefit of the participants, all the abstracts obtained in time (including those not scheduled in the program) are included in this collection.

István Agoston (Budapest, Hungary): "A generalization of quasi-hereditary algebras"

Toma Albu (Milwaukee, USA): "Double infinite chain condition, and generalized deviations of posets and algebras"

Lidia Angeleri Hügel (München, Germany): "Tilting and cotilting classes in Mod(R)"

Pham Ngoc Anh (Budapest, Hungary): "Hall polynomials for $\tilde{A}_n$"

Regina Maria de Aquino (Sao Paulo, Brasil) "Over Koszul-Tiltings Algebras"

Vyacheslav Artamonov (Moscow, Russia): "Quantum polynomials and their representations"

Tsunenobu Asai (Kinki University, Osaka, Japan): On the number of crossed homomorphisms"

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Michael Barot (UNAM, Mexico): "Non-negative unit forms and algebras"

Manfred Bauch (Bayreuth, Germany): "Non-regular sincere objects in factorspace categories"

Vladimir Bavula (Edinburgh, Scotland): "The simple holonomic modules over rings of differential operators on surface"

Apostolos Beligiannis (Chalkis, Greece): "Gorenstein categories"

Grzegorz Bobinski (Torun, Poland): "Geometry of directing modules over tame algebras"

Vitalij Bondarenko (Kiev, Ukraine): "On an extension of the problem about representations of bundles of semichains"

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Flávio Ulhoa Coelho (Sao Paulo, Brasil): "Algebras with small homological dimensions"

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Yuriy Drozd (Kiev, Ukraine, and Strasbourg, France): "Classification of quadratic modules"

Xianneng Du (Hefei, China): "Tilting modules of dual extension algebras and induced torsion theries"

Ole Enge (Trondheim, Norway): "On quasitilted algebras"

Joerg Feldvoss (Hamburg, Germany): "Stable Categories of Tame Local Symmetric Algebras"

Edward Green (Blacksburg, USA): "T-Koszul algebras"

Sergei Grudsky (Rostov-on-Don, Russia): "Toeplitz operators with symbol having generalized almost-periodic discontinuities"

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Dieter Happel (Chemnitz, Germany): "Hochschild cohomology of piecewise hereditary algebras"

Lutz Hille (Hamburg, Germany): "Moduli Spaces of Quiver Representations"

Thorsten Holm (Magdeburg, Germany): "Twisted bimodules and Hochschild cohomology"

Youichi Iida (Tokyo, Japan): "Normal extensions and induced characters of 2-groups"

Osamu Iyama (Kyoto, Japan): " A characterization of Auslander-Reiten quivers of orders of finite representation type"

Otto Kerner (Düsseldorf, Germany): "Almost serial regular Auslander-Reiten components for wild hereditary algebras"

Mark Kleiner (Syracuse, USA): "Orthogonal subcategories and computation of almost split sequences for relatively projective and prinjective modules"

Klara Kögerler (Bielefeld, Germany): "On algebras with complete preprojective components"

Justyna Kosakowska (Torun, Poland): "Sincere posets of finite prinjective type"

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Héctor Merklen (Sao Paulo, Brasil): "Generators of the Grothendieck group"

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Luise Unger (Paderborn, Germany): "Standardly stratified algebras and tilting"

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Hailou Yao (Beijing, China): "On the Auslander-Reiten quiver with oriented cycles of representation-finite algebras"

Tomoyuki Yoshida (Hokkaido, Japan): "Categorical aspects of generating functions"

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Pu Zhang (Hefei, China): "Representations as elements in affine composition algebras and quantum groups"

Shunhua Zhang (Jinan, China): "Some results about tame Ringel-Hall algebras"

Yingbo Zhang (Beijing, China): "Number of parameters of module varieties in a wild category"

Bin Zhu (Bielefeld, Germany): "A Combinatorial Description of Coils"

Alexander Zimmermann (Amiens, France): "Derived equivalences, orders and group rings"

Rita Zuazua (UNAM, Mexico): "Tree of reduction algorithms"

Alexandr N. Zubkov (Omsk, Russia): "Classical groups adjoint actions invariants"

Matrix problems arising in the classification of homotopy types

Hans Joachim Baues

The basic numerical invariants of a polyhedron or a manifold are the dimension and the degree of connectedness. For example an n-sphere has dimension n and is (n-1)-connected. Moreover it is a classical result that any n-dimensional plyhedron X which is (n-1)-connected is homotopy equivalent to a one point union of n--spheres.

Let A(n,k) be the homotopy category of (n-1)-connected (n+k)--dimensional compact polyhedra and let F(n,k) be the full subcategory of A(n,k) consisting of polyhedra with torsion free homology. In the stable range n>k the categories A(k)=A(n,k) and F(k)=F(n,k) do not depend on n and are additive categories in which the sum is the one point union of spaces. The classification of indecomposable objects in A(k) and F(k) leads to problems in representation theory which are tame for small k. For example A(0)=F(0) has only one indecomposable object given by the sphere.

In joint work with Y. Drozd we show that for k<6 the number I(k) of all indecomposable objects in F(n) is finite; the number is:

I(o)=1 I(1)=2 I(2)=4 I(3)=7 I(4)=67 I(5)=451 I(6)=? I(7)=? I(8)=? I(9)=? I(k)=infinite if k>9

For k<6 we give a complete list of indecomposable polyhedra in F(k). For the computation of the indecomposable polyhedra we have to solve matrix problems and associated bimodule problems which turn out to be of finite representation type. For k>9 the category F(k) is of wild representation type.

Moreover, for k>5 the category A(k) is of wild representation type while for k<4 the category A(k) is of tame representation type. Only the representation types of A(4) and A(5) are unknown.

On some Tame families of modules.

Raymundo Bautista

Take Λ a finite-dimensional k-álgebra, k algebraically closed field.

By $mod\Lambda$ we denote the category of finitely generated left $\Lambda-$modules and by $ind\Lambda$ the full subctegory of indecomposable modules in $mod\Lambda$.

If $\mathcal{F}$ is a family of indecomposable modules closed under isomorphisms and d is a natural number then $\mathcal{F}_{\ d} = \{X \in \mathcal{F} \mid dimX = d\}$.

1. The family $\mathcal{F}$ is called discrete if for every $d, \mathcal{F}_d$ consists of only a finite number of isomorphism classes.

2. The family $\mathcal{F}$ is called tame if there is a finite number of one-parameter families $\mathcal{S}_1, \cdots \mathcal{S}_{m(d)}$ of modules in $\mathcal{F}_d$ such that for almost all isomorphism classes $\{M\}$ with $M \in \mathcal{F}_d$, $M \cong N$ for some $N \in \mathcal{S}_i$ for some i. The minimal possible $m(d)$ is denoted by $\mu_{\mathcal{F}}(d)$.

We will consider the following family of modules. Let

$$U(i) = \{X \in ind\Lambda \mid \dim_k Hom_\Lambda(X, DtrX) - \dim_k \mathcal{S}(X, DtrX) = i\}$$

where $\mathcal{S}(X, DtrX) =$ the vector subspace of $Hom_\Lambda(X, DtrX)$ consisting of those maps which factor through semisimple modules. Then we have:

1. $U(0)$ is discrete,

2. $U(1)$ is tame.

For Λ tame we examine the families $U(i)$, we prove that in certain cases they are of polynomial growth, in other words there is a $n(i)$ such that $\mu_{U(i)}(d) \leq d^{n(i)}$.

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Actions of Algebraic Groups and Quasi-Hereditary Algebras

Thomas Brüstle, Lutz Hille, and Gerhard Röhrle

Part I **Motivation from Algebraic Groups**

Let k be an infinite field and let V be a finite-dimensional k-space. Further let $GL(V)$ be the linear group of V and let P be a stabilizer of a flag F in V. By P_u we denote the unique maximal unipotent normal subgroup of P, the unipotent radical of P. Now P acts on P_u via conjugation and on $\mathfrak{p}_u$, the Lie algebra of P_u via the adjoint action. We describe some recent results classifying all instances when P acts on $\mathfrak{p}_u$ or P_u with a finite number of orbits, and more generally, when P admits the same finiteness property with respect to the action on $\mathfrak{p}_u^{(l)}$, the l-th member of the descending central series of $\mathfrak{p}_u$.

For k algebraically closed, we shall provide a complete description of the partial order given by orbit closures on the set of P-orbits on $\mathfrak{p}_u$ resp. $\mathfrak{p}_u^{(l)}$ in the finite cases.

The classification result for $GL(V)$ envovles a study of the representation theory of a particular quiver with certain relations (seee Part II).

Moreover, we are able to recover Richardson's dense orbit theorem in this instance.

Part II **Representations of Algebras**

In this section we discuss the representation theoretic machinery to solve problems of the type mentioned in Part I. Time permitting we shall discuss other applications some of them are mention in Part III below.

Let Λ be a finite-dimensional algebra and B a Λ–bimodule. We are interested in the category of matrices $mat - B$ over the bimodule B. To analyse the category $mat - B$ we construct a finite dimensional algebra $\mathcal{A}$ together with an embedding $\Lambda \longrightarrow \mathcal{A}$, such that the category $mat - B$ is equivalent to the category of those $\mathcal{A}$–modules which are projective over Λ (here we consider a module over $\mathcal{A}$ as a Λ–module via the embedding above).

IIa **The categorical construction**

First we present the construction of $\mathcal{A}$ from B using combinatorics of quivers with relations. We associate to the bimodule B a bounded biquiver $Q(B)$ and to the embedding of the algebras $\Lambda \to \mathcal{A}$ a bounded biquiver $Q(\Lambda, \mathcal{A})$. We note that $\mathcal{A}$ is quasi-hereditary precisely when B is a simultaneously triangular bimodule (that is Λ is directed and B is triangular with opposite order).

IIb **Actions of algebraic groups**

We discuss the corresponding representation spaces for both problems and present several examples. Let $R(B)(d)$ be the space of all matrices over B with dimension vector d and $P(d)$ the corresponding group of basis transformations, such that the orbits correspond to isomorphism classes of objects in $mat - B$. The group $P(d)$ is reductive precisely when Λ is semi-simple. Using part I, we may replace this action by an action of a certain reductive group on the space of all representations of $\mathcal{A}$ which are projective over Λ.

Part III **Applications**

In this part we present applications concerning both the categorical equivalence considered in part IIa, and the group actions defined in part IIb and specifically those of part I.

1: Classification of orbits for actions of parabolic subgroups P in GL_n on its unipotent radical P_u and $P_u^{(l)}$.

2: (jt. with C. M. Ringel) The Auslander algebra of $k[T]/T^n$ and the classification of its classical tilting modules.

3: (jt. with G. Zwara) Degenerations for Δ-filtered modules over quasi-hereditary algebras, matrices over bimodules and degenerations in AR-aggregates.

4: The characteristic functor of the radical bimodule and the characteristic module over the quasi-hereditary algebra $\mathcal{A}$.

5: Richardson's dense orbit theorem and the action of $P(B)(d)$ on $P_u^{(l)}(B)(d)$ and other unipotent normal subgroups.

6: The bimodule corresponding to an action of a parabolic subgroup in GL_n on a unipotent normal subgroup.

7: Multiple mixed flag varieties of finite type and tame type, separated bimodules, an representations of posets.

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The vanishing of group cohomology and subcategories of the stable category

Jon Carlson

We look at modules over a group algebra kG where G is a finite group and k is a field of characteristic $p > 0$. The question is when two modules extend each other. More specifically, for a module M we consider the subcatgory generated by M by extensions and taking cokernels and kernels of maps to and from projective modules. We want to know when two such subcategories are orthogonal in the sense that no module from one has any nonsplit extension by a module from the other. This means the same as saying that all of the functors $Ext^n_{kG}(M,N)$ vanish, where M and N are the generators for the categories. In some cases, such as when G is a p-group, the answer is particularly nice, and the vanishing of cohomology is largely determined by the support varieties of the modules. In other cases the problem is completely open and seems to depend on some open questions in the homological algebra. However a lot of progress has been made recently. In this lecture I will present a survey of the current state of the area.

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Kac's Theorem and deformed preprojective algebras

William Crawley-Boevey

Let K be an algebraically closed field and let Q be a quiver with vertex set I. The **deformed preprojective algebra** has been introduced by M. P. Holland and the speaker [2] (see also [1]). It is defined for $\lambda \in K^I$ by

$$\Pi^\lambda = K\overline{Q}/(\sum_{a\in Q}[a,a^*] - \sum_{i\in I}\lambda_i e_i).$$

Here $\overline{Q}$ is the double of Q, obtained by adjoining an arrow $a^* : j \to i$ for every arrow $a : i \to j$ in Q, we write e_i for the trivial path at i, and $[a,a^*]$ is the commutator $aa^* - a^*a$.

Kac's Theorem [3] asserts that the dimension vectors of indecomposable representations of a quiver Q are exactly the positive roots for Q.

In this talk I shall:

(1) Discuss how simple representations of deformed preprojective algebras can be used to give an easy proof of most of Kac's Theorem, and for example give an explicit construction of the indecomposables whose dimension vector is a real root.

(2) Discuss the characterization of possible dimension vectors of simple representations of Π^λ.

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[1] W. Crawley-Boevey, Preprojective algebras, differential operators and a Conze embedding for deformations of Kleinian singularities, preprint 1998. Available from the speaker's Leeds homepage.

[2] W. Crawley-Boevey and M. P. Holland, Noncommutative deformations of Kleinian singularities, Duke. Math. J., 92 (1998), 605–635

[3] V. G. Kac, Root systems, representations of graphs and invariant theory. In: Invariant theory, Proc. Montecatini 1982, ed F. Gherardelli, Lec. Notes in Math., 996 (Springer, Berlin, 1983), 74–108.

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Representations of Infinitesimal Group Schemes

R. Farnsteiner

Let H be a finite dimensional cocommutative Hopf algebra, defined over an algebraically closed field k. To H we associate the representable functor $\mathcal{G}$ that assigns to every commutative k-algebra R the group $\mathcal{G}(R)$ of k-algebra homomorphisms $H^* \longrightarrow R$. In this situation we write $H = H(\mathcal{G})$.

By general theory, the finite algebraic k-group $\mathcal{G}$ decomposes into a semidirect product

$$\mathcal{G} = \mathcal{G}^0 \times \mathcal{G}_{red}$$

with an infinitesimal normal subgroup $\mathcal{G}^0$, and a reduced group $\mathcal{G}_{red}$. Accordingly, the representation theory of cocommutative Hopf algebras comprises that of infinitesimal groups and reduced groups. Since $H(\mathcal{G}_{red}) \cong k[\mathcal{G}(k)]$, the group algebra of the finite group of k-rational points, we will study infinitesimal groups, i.e., those for which the dual $H(\mathcal{G})^*$ is local. In view of Cartier's theorem these occur only in positive characteristic.

In this talk, which is partly based on joint work with Detlef Voigt [2, 3], we will employ recent results by Suslin, Friedlander, and Bendel [4, 5] on rank varieties to study the representation type and the structure of the stable Auslander-Reiten quiver $\Gamma_s(\mathcal{G})$ of $H(\mathcal{G})$.

If the principal block $\mathcal{B}_0(\mathcal{G}) \subset H(\mathcal{G})$ has finite representation type, then the group $\mathcal{G}$ is necessarily supersolvable. More precisely, we have

Theorem 1 *Let $\mathcal{G}$ be an infinitesimal group. Then the following statements are equivalent:*

(1) *$\mathcal{B}_0(\mathcal{G})$ has finite representation type.*

(2) *$\mathcal{B}_0(\mathcal{G}) \cong k[X]/(X^{p^n})\#\Lambda$ is the smash product of a truncated polynomial ring with a commutative, semisimple Hopf algebra Λ.*

(3) *The Hopf algebra $H(\mathcal{G}_2)$ of the second Frobenius kernel $\mathcal{G}_2$ of $\mathcal{G}$ is a Nakayama algebra.*

The group algebras of supersolvable infinitesimal groups afford a nice linkage principle. A more detailed analysis of the irreducible representations of these groups then yields the determination of the basic algebras of the representation-finite Hopf algebras.

For Frobenius kernels of reductive groups one has a complete understanding of the structure of the representation-finite and tame blocks.

Theorem 2 *Let $\mathcal{G}_r$ be the r^{th} Frobenius kernel of a reductive group $\mathcal{G}$, $\mathcal{B} \subset H(\mathcal{G}_r)$ a block. Then the following statements hold:*

(1) *If $\mathcal{B}$ is representation-finite, then $\mathcal{B}$ is simple.*

(2) *If $\mathcal{B}$ is tame, then $\mathcal{B}$ is Morita equivalent to a tame block of $H(SL(2)_1)$.*

To each component Θ of the stable Auslander-Reiten quiver $\Gamma_s(\mathcal{G})$ of $H(\mathcal{G})$ we associate its support variety $\mathcal{V}_{\mathcal{G}}(\Theta)$.

Theorem 3 *Let $\Theta \subset \Gamma_s(\mathcal{G})$ be a component. Then the following statements hold:*

(1) *The tree class $\bar{T}_\Theta$ is either a finite or infinite Dynkin diagram, or a Euclidean diagram.*

(2) *If $\dim \mathcal{V}_{\mathcal{G}}(\Theta) \geq 3$, then $\Theta \cong \mathbb{Z}[A_\infty]$.*

(3) *If $\mathcal{G}$ is a Frobenius kernel of a reductive group, then $\mathbb{Z}[\tilde{A}_{12}]$ is the only Euclidean component of $\Gamma_s(\mathcal{G})$, and no components of finite Dynkin type occur.*

References

[1] R. Farnsteiner. *On Auslander-Reiten Components for Lie Alegbras of Reductive Groups.* SFB Preprint No. 98-046, Universität Bielefeld

[2] R. Farnsteiner and D. Voigt. *On Cocommutative Hopf Algebras of Finite Representation Type.* SFB Preprint No. 97-123, Universität Bielefeld

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[5] A. Suslin, E. Friedlander and C. Bendel. *Support Varieties for Infinitesimal Group Schemes.* J. Amer. Math. Soc. **10** (1997), 729-759

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Auslander Reiten components for clannish algebras

Ch. Geiß and J.A. de la Peña

Let k be a field. Clans are a class of matrix problems over k, introduced in [2], see also [1],[3]; the indecomposable representations admit a combinatorial description which is independent of the field, similar to the well-known case of string-algebras.

We provide a combinatorial description of the homomorphisms between representations of clans, similar to the result in [6], this allows us to calculate Auslander-Reiten sequences, and again the result is independent of the field k, [4].

For the particular case of clannish algebras, the skew group algebra constructions from [8] provide in case $char\mathrm{k} \neq 2$ a more convenient way to obtain information about indecomposables and Auslander-Reiten theory from the well-known and more handy results for string algebras. The restriction to $char\mathrm{k} \neq 2$ can be eliminated, since by the above considerations such results are independent of the field. For example, we find in this way, that the stable components of the Auslander-Reiten quiver of a clannish algebra are of the following types: $\mathbb{Z}\mathbb{A}_n, \mathbb{Z}\widetilde{\mathbb{A}}_n, \mathbb{Z}\mathbb{A}_\infty/(\tau^n), \mathbb{Z}\mathbb{A}_\infty^\infty, \mathbb{Z}\mathbb{D}_n, \mathbb{Z}\widetilde{\mathbb{D}}_n, \mathbb{Z}\mathbb{D}_\infty$.

Next, we consider *skewed-gentle algebras*, a class of clannish algebras with the property that the repetitive algebra is up to the socle again clannish (in particular, each gentle algebra is skewed gentle). Again, by reducing to the gentle case by the above argument, we obtain from [9] plus some new results on gentle algebras the following result:

Theorem 1 *Let A be a finite-dimensional connected skewed gentle algebra of finite global dimension, and denote by $\Gamma_{\hat{A},s}$ the stable Auslander-Reiten quiver of the repetitive algebra of A. Then the following are equivalent:*

(*a*) *A is derived equivalent to a hereditary algebra,*

(*b*) *$\Gamma_{\hat{A},s}$ contains a component with finitely many τ-orbits;*

(*c*) *$\Gamma_{\hat{A},s}$ contains no component of type $\mathbb{Z}\mathbb{A}_\infty$*

Moreover, we can calculate for a given skewed gentle algebra the (finite!) number of $\mathbb{Z}\mathbb{A}_\infty$ components in $\Gamma_{\hat{A},s}$; all this material is from [5].

Note finally, that in contrast to the gentle case, there are many interesting algebras which are not skewed gentle, but derived equivalent to a skewed-gentle algebra; all these algebras are of course derived tame [7].

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SYMMETRIC GROUPS AND RELATED ALGEBRAS

Gordon James

There exist many unexpected relations between the representation theory of the symmetric group and the representation theory of other algebras.

One of the most tantalizing problems concerning the symmetric groups is to determine the dimensions of their irreducible modules. Over fields of characteristic zero, the solution is well known, since for each partition λ, there exists a Specht module S^λ, and the Specht modules provide a complete set of irreducible modules. However, a Specht module S^λ can be defined over an arbitrary field, and its dimension is equal to the number of standard λ-tableaux. If one could properly understand the structure of S^λ then the dimensions of the irreducible modules for the symmetric group would be known.

A good way to define Specht modules involves an endomorphism algebra which is called the Schur algebra. It turns out that the problem of determining the dimensions of the irreducible modules for all Schur algebras is equivalent to two other important problems, namely

(i) determining the dimensions of the irreducible modules for all symmetric groups; and

(ii) determining the dimensions of the irreducible modules for all general linear groups, in the describing characteristic.

Remarkably, these problems, in turn, are special cases of a third problem:

(iii) determining the dimensions of the irreducible modules for all general linear groups $\mathrm{GL}_n(q)$, in a non-describing characteristic.

The connection is provided by the q-Schur algebra, which is the same as the usual Schur algebra if $q = 1$.

Recently, a significant advance towards a solution of all these problems has been provided by the proof (by Ariki - see also Grojnowski) of the Lascoux-Leclerc-Thibon Conjecture.

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Applications of derived equivalences to Hall algebras

M. Kapranov

This is a report on a joint work with E. Vasserot. Let G be a finite subgroup in $SL_2(\mathbf{C})$. The variety $\mathbf{C}^2/G$ is known as a Kleinian singularity. It has a canonical desingularization, denoted $\mathbf{C}^2//G$. Our first result is an equivalence between the derived categories of coherent sheaves on $\mathbf{C}^2//G$ and of G-equivariant coherent sheaves on $\mathbf{C}^2$, i.e., of G-equivariant $\mathbf{C}[x, y]$-modules. Such an equivariant module can be described as a "double representation" of the affine quiver corresponding to G.

The equivalence we construct takes, in general, individual equivariant modules to complexes of sheaves. Nevertheless, the irreducible double representations $V(i)$ corresponding to the vertices of the finite part of the Dynkin quiver, are taken into actual coherent sheaves on $\mathbf{C}^2//G$, namely the structure sheaves $\mathcal{O}_{E_i}$ of the components of the exceptional divisor. We consider the Hall algebras of the two categories based on Euler characteristic. We show (by using the derived equivalence above) that the classes of the $\mathcal{O}_{E_i}$ in the Hall algebra satisfy the same relations as the classes of the $V(i)$, i.e., the Serre relations for the corresponding Lie algebra. As a corollary, we get a statement about any configuration of (-2) curves on a complex algebraic surface, since the formal neighborhood of the union of any two (-2)-curves meeting at one point, is isomorphic to the formal neighborhood of the exceptional divisor of $\mathbf{C}^2//G$ where G is of type A_2.

Modular Littlewood-Richardson coefficients

A. Kleshchev (University of Oregon)

We shall give a review of various representation theoretic problems which lead to the Littlewood-Richardson coefficients and admit modular analogues. One of such problems is "decomposing" tensor product of two irreducible GL(n)-representations, which has at least two natural modular counterparts: to decompose a product of (partial) tilting modules or to "decompose" a product of irreducible modules. Let V be the natural n-dimensional GL(n)-module and L is an arbitrary irreducible GL(n)-module. One of our main results, ontained jointly with J. Brundan, is a description of the socle of the tensor product of L and V. Moreover we consider the block components of such tensor `products as special cases of the images of Jantzen's translation functors.`

On characteristic tilting modules

Steffen König

The aim of this talk is to celebrate the tenth anniversary of the following theorem.

Theorem. *(C.M.Ringel) Let A be a quasi–hereditary algebra. Then the category $\mathcal{F}(\Delta) \cap \mathcal{F}(\nabla)$ of modules, which are both Δ–good and ∇–good, equals $add(T)$ for some tilting and cotilting module T. Moreover, $End_A(T)$ again is a quasi–hereditary algebra.*

The algebra $End_A(T)$ now usually is called the 'Ringel–dual' of A.

Many applications of this result are already in the literature. I will list some of them and some new ones, relating tilting modules to the following problems and areas: character formulae for Lie algebras and algebraic groups, quantum groups, quasi–hereditary and cell structures, invariant theory.

In particular, I will discuss recent results on Ringel duality for certain classes of algebras. Soergel has shown that blocks of category $\mathcal{O}$ are Ringel selfdual. Donkin proved that the Ringel duals of classical Schur algebras of type A are again (generalized) Schur algebras. Donkin's result has been generalized to other types by Adamovich and Rybnikov.

An equivalent version of Kazhdan–Lusztig theory deals with the vanishing or non–vanishing of certain Ext–groups between simple modules. One may compare such conditions for an algebra A and its Ringel dual A', thus getting a few more examples where the conditions are satisfied.

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Some applications of tilting modules

O. Mathieu (Strasbourg)

For algebraic groups, tilting modules are stable by tensor products. Therefore, we can define tensor product multiplicities $V_\lambda^{\mu,\nu}$ by:

$$T(\mu) \otimes T(\nu) = \oplus_{\lambda \in P^+} V_\lambda^{\mu,\nu} T(\lambda)$$

where for $\mu \in P^+$, $T(\mu)$ denotes the tilting module with highest weight μ (here the notation $mT(\lambda)$ means m copies of $T(\lambda)$).

Computing all $V_\lambda^{\mu,\nu}$ is an open problem. However we can compute some of them, and they are given by a formula similar to Verlinde's formula in Conformal Field Theory. From this, we can deduce character formulas for some simple representations of S_n and $GL(n)$ over a field of finite characteristic. More recently, we computed some $V_\lambda^{\mu,\nu}$ outside the validity domain of our previous formula, from which new results for S_n can be deduced.

Let us mention that some works were done in collaboration: the modular Verlinde's formula is a joint work with G.Georgiev (1992), the character formula for $GL(n)$ is joint with G.Papadopoulo (1997) and the results about 3–lines representations of S_n is joint with J.Jensen (1998).

Categorical methods in representation theory

Sergei Ovsienko

The aim of this talk is to present former and recent developments of some constructive aspects of representation theory. The techniques discussed here originated in the theory of matrix problems and are based on the possibility to compare and to transform the representation categories by changing objects, which represent these categories.

As the main prototype we use the box representation theory and, generalizing this approach, we try to show, that for the theory of matrix problems an important role is played by the existence and systematic use of the structure of an $A(\infty)-$(co)category on these represented objects and induced structures on the correponding representation category. It allows to expose in some uniform way various problems of representations theory and to construct transformations, connect different kinds of categories of representations, for example, those of associative algebras, free boxes, vector space categories and differential $\mathbf{Z}-$graded categories etc. We discuss here important interactions of the considered techniques with the theory of integral quadratic and nonsymmetric bilinear forms and with group actions on the categories, especially covering theory.

As an application we consider the class of differential $\mathbf{Z}-$graded categories, such that the underlying quiver is a tree and prove in this case a criterion of representation finiteness. We show also, how the developed approach in auniform way can be applied to some problems from associative algebras and boxes representations theory, Lie algebras representations etc.

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Generic extensions and Hall algebras at $q = 0$

Markus Reineke

We show that the operation of taking generic extensions gives the set of isoclasses of representations of a quiver of Dynkin type a monoid structure, whose monoid ring is isomorphic to Ringel's Hall algebra at $q = 0$.

This provides the latter algebra with a multiplicatively closed basis, which is shown to arise from specialization of a crystal basis of an appropriately defined quantum group.

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Some thoughts on problems and directions in the representation theory of algebras

As suggested by the organizers, I will talk about directions and (types of) problems related to the representation theory of artin algebras, which should be interesting to pursue. The talk will also include old problems connected with the work of Maurice auslander, for example coming from the Queen Mary notes.

(Idun Reiten, Trondheim, Norway)

Geometry of modules over tame quasi-tilted algebras

Andrzej Skowroński

Given a finite-dimensional algebra A over algebraically closed field K, it is an interesting task to study the associated affine varieties $\mathrm{mod}_A(\mathbf{d})$ of A-modules of fixed dimension-vectors $\mathbf{d}$ and the actions of the corresponding products $\mathrm{G}(\mathbf{d})$ of general linear groups by conjugations. Then the $\mathrm{G}(\mathbf{d})$-orbits in $\mathrm{mod}_A(\mathbf{d})$ coincide with the isomorphism classes of A-modules of dimension-vector $\mathbf{d}$. In this setting a number of interesting questions arises very naturally: what are the irreducible components, orbit closures, degenerations, singularities, invariants, semi-invariants, ... in the varieties $\mathrm{mod}_A(\mathbf{d})$? An another interesting problem is to understand the relationship between the above questions and the representation type of A?

An important role in recent investigations of finite dimensional algebras is played by quasi-tilted algebras. Following Happel-Reiten-Smalø, an algebra A is said to be quasi-tilted if A is of global dimension at most two and each indecomposable finite dimensional A-module has projective dimension at most one or injective dimension at most one. Important classes of quasi-tilted algebras are provided by all tilted algebras, tubular algebras, canonical algebras and their relatives. The tame quasi-tilted algebras and their categories of finite dimensional modules have been described three years ago by the speaker. We shall discuss the following problems concerning the geometry of the varieties $\mathrm{mod}_A(\mathbf{d})$ over tame quasi-tilted algebras A:

- description of degenerations in $\mathrm{mod}_A(\mathbf{d})$ in terms of the representation theory of A (joint work with G. Zwara).
- when $\mathrm{mod}_A(\mathbf{d})$ is irreducible, complete intersection, normal, ... for the dimension-vectors $\mathbf{d}$ of indecomposable finite dimensional A-modules? (joint work with G. Bobiński)?

- the structure of semi-invariant rings of the $\mathrm{G}(\mathbf{d})$-action on $\mathrm{mod}_A(\mathbf{d})$ for A hereditary or canonical (joint work with J. Weyman).

In particular, some geometric criteria for quasi-tilted algebras to be tame will be presented. We will also show an interaction between the geometry of modules and their behaviour in the module category.

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How special are representations of finite group algebras ?

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Let G be a finite group, and k a field of characteristic p, where p is a prime. Since the group algebra kG of G over k is a symmetric algebra, many devices, developed for symmetric, or more generally, self-injective algebras or else, may be applicable to kG. However, there are some places where group algebras behave in rather special way, partly because kG does not stand itself but goes with related subgroup algebras kH. Here H is a subgroup of G. In the talk, mainly the properties of the stable Auslander-Reiten quivers of finite group algebras will be considered from this point of view. More precisely, the following will be discussed. Here Γ is a connected component of the stable Auslander-Reiten quiver of kG

(1) The tree class T of Γ :

Already it is known by Webb that possibility of the shape of T is restricted for group algebras. In fact, they are special types of Dynkin or Euclidean diagrams. Now, among those possibilities, those which appear and which never do are completely determined. Moreover, for a non-periodic component Γ, we can answer which modules in Γ come from the same p-subgroup algebra. This is important since it gives a device for reducing problems to p-subgroups or their normalizers.

(2) The places of simple kG-modules in Γ :

While this problem does not seem to be difficult for finite or tame block algebras, almost nothing is known for wild block algebras. However, there is a method for reducing the problem to almost simple groups. Then, in principle, if one can answer the question for almost simple groups, then we know the answer for all finite group algebras. At present, by the classification of finite simple groups, we can determine the places of simple modules at least in certain situations.

As long as time permits, we discuss some other topics such as relations between certain group algebras and their subgroup algebras from similar points of view.

The multiple flag varieties and generalized quivers

Jerzy Weyman

In the first part I describe joint results with P. Magyar and A. Zelevinsky on classifying multiple flag varieties of finite type.

For a reductive group G the problem is to classify such k-tuples $(P_1, P_2, \ldots, P_k)$ of parabolic subgroups in G for which the diagonal action of G on $G/P_1 \times G/P_2 \times \cdots \times G/P_k$ has finitely many orbits. We obtained the complete solution of this problem for the general linear and the symplectic groups. The results for orthogonal groups are also discussed.

In the second part (joint with H. Derksen) I describe the notion of generalized quivers associated to reductive groups. In the case of classical groups this notion reduces to the quiver with a contravariant involution.

I also describe the connection between both topics.

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On the number of absolutely indecomposable representations
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of a quiver over a finite field
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An old conjecture of Kac states that the constant term of the
polynomial counting the number of absolutely indecomposable representations,
with a given dimension vector, of a quiver over a finite field is
equal to the multiplicity of the corresponding root in the associated
Kac-Moody Lie algebra.
We will give two reformulations of this conjecture.  One in terms
of the Hall-algebra, and one in terms of the combinatorics of Gaussian
binomial coefficients.
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Michel Van den Bergh
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Liangang Peng and Jie Xiao

Derived Category and Kac-Moody algebra

By using the Ringel-Hall algebra approach, we give a realization of all symmetrizable Kac-Moody Lie algebras via the root categories of the derived categories of all finite dimensional hereditary associative algebras.

Let $\mathcal{C}$ be a triangulated category which is finitary over a central finite field k, period-2 and has the proper Grothendieck group $\mathfrak{h}$.

Given X, Y and $L \in \mathcal{C}$, put

$$W(X,Y;L) = \{(f,g,h) \in \mathrm{Hom}(X,L) \times \mathrm{Hom}(L,Y) \times \mathrm{Hom}(Y,TX) |$$

$$X \xrightarrow{f} L \xrightarrow{g} Y \xrightarrow{h} TX \text{ is a triangle}\}$$

The action of $\mathrm{Aut}X \times \mathrm{Aut}Y$ on $W(X,Y;L)$ is defined as

$$(a,c)(f,g,h) = (af, gc^{-1}, ch(Ta)^{-1})$$

and the orbit space

$$V(X,Y;L) = W(X,Y;L)/\mathrm{Aut}X \times \mathrm{Aut}Y$$

and $F^L_{YX} = |V(X,Y;L)|$. We define a bilinear function $(-|-)$ on $\mathfrak{h} \times \mathfrak{h}$ as: for $X, Y \in \mathcal{C}$, $(h_X|h_Y) = \dim_k \mathrm{Hom}(X,Y) - \dim_k \mathrm{Hom}(X,TY) + \dim_k \mathrm{Hom}(Y,X) - \dim_k \mathrm{Hom}(Y,TX)$. This is just the symmetric Euler form of $\mathcal{C}$. Let $\mathfrak{n}$ be a free abelian group with a basis $\{u_X | X \in \text{ind}\mathcal{C}\}$, $\mathfrak{g} = \mathfrak{h} \oplus \mathfrak{n}$, $\mathfrak{g}_{(q-1)} = \mathfrak{g}/(q-1)\mathfrak{g}$ where $q = |k|$. We define a bilinear operation $[-,-]$ on $\mathfrak{g}_{(q-1)}$ as follows.

(a) For any two indecomposable objects $X, Y \in \mathcal{C}$

$$[u_X, u_Y] = \begin{cases} \sum_{[L], L \in \text{ind}\mathcal{C}} \gamma_{XY}^L u_L & \text{if } Y \not\simeq TX \\ \frac{h_X}{d(X)} & \text{if } Y \simeq TX \end{cases}$$

where $\gamma_{XY}^L = F_{XY}^L - F_{YX}^L$, $d(X) = \dim_k \text{End}X/\text{radEnd}X$.

(b) For any objects $X, Y \in \mathcal{C}$ with Y indecomposable

$$[h_X, u_Y] = -(h_X | h_Y) u_Y \text{ and } [u_Y, h_X] = -[h_X, u_Y]$$

(c) $[\mathfrak{h}, \mathfrak{h}] = 0$

Theorem 1 *Together with the operation $[-,-]$, $\mathfrak{g}_{(q-1)}$ is a Lie algebra over $Z/(q-1)$.*

Let $\mathcal{R}$ be the root category of the derived category of a finite dimensional hereditary algebra A, i.e $\mathcal{R} = D^b(A)/T^2$. Then we have the Lie algebra $\mathfrak{g}_{(q-1)}$ from $\mathcal{R}$ as we did as in Theorem 1. According to a generic trick (by Ringel) we get the generic form $\mathfrak{g}$ of $\mathfrak{g}_{(q-1)}$ which is generated by $\{u_X | X$ is exceptional object in $\mathcal{R}\}$. Then the main result is

Theorem 2 *The generic form $\mathfrak{g}$ of the Lie algebra arising from $\mathcal{R}$ gives a realization of symmetrizable Kac-Moody algebras, i.e, $C \otimes_Z \mathfrak{g}$ is a symmetrizable Kac-Moody algebra, which generalized Cartan matrix is given by the symmetric Euler form and simple modules of A.*

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Degenerations of modules are given by extensions.

Grzegorz Zwara

Let A be a finite dimensional associative algebra over algebraically closed field k. If $a_1 = 1, \ldots, a_\alpha$ is a basis of A over k, we have the structure constants a_{ijk} defined by $a_i a_j = \sum a_{ijk} a_k$. The affine variety $mod_A^d(k)$ of A-module structures on k^d consists of α-tuples $m = (m_1, \ldots, m_\alpha)$ of $d \times d$-matrices with coefficients in k such that m_1 is the identity matrix and

$m_i m_j = \sum a_{ijk} m_k$ holds for all indices i and j. Any such α-tuple m corresponds to a d-dimensional A-module M in the obvious way. The general linear group $Gl_d(k)$ acts on $mod_A^d(k)$ by conjugation, and the orbits correspond to the isomorphism classes of d-dimensional A-modules. By abuse of notation N is a degeneration of M if n belongs to the Zariski closure $\overline{Gl_d(k)(m)}$ of $Gl_d(k)(m)$ in $mod_A^d(k)$, and we denote this fact by $M \leq_{deg} N$.

We shall present the following characterization of degenerations.

Theorem 1 *Let m, n be points in $mod_A^d(k)$, $d \geq 1$. Then the following are equivalent:*

1. *$M \leq_{deg} N$.*
2. *There is a short exact sequence $0 \to N \to M \oplus Z \to Z \to 0$ for some finite dimensional A-module Z.*
3. *There is a short exact sequence $0 \to Z \to Z \oplus M \to N \to 0$ for some finite dimensional A-module Z.*

We may consider another partial order $\leq_{ext}$ defined as follows:

• $M \leq_{ext} N$: $\Leftrightarrow$ there are A-modules M_i, U_i, V_i and short exact sequences $0 \to U_i \to M_i \to V_i \to 0$ such that $M = M_1$, $M_{i+1} = U_i \oplus V_i$, $1 \leq i \leq s$, and $N = M_{s+1}$ for some natural number s.

Then for A-modules M and N, the following implication holds:

$$M \leq_{ext} N \quad \Longrightarrow \quad M \leq_{deg} N$$

Observe that for any A-modules M, N with $M <_{ext} N$, the module N is decomposable. The reverse implication is not true in general.

Our second result is as follows.

Theorem 2 *Let M, N be A-modules such that N is a minimal degeneration of M. Then $M <_{ext} N$ or there are A-modules W, $\overline{M}$, $\overline{N}$ such that $M \simeq W \oplus \overline{M}$, $N \simeq W \oplus \overline{N}$, $\overline{M} <_{deg} \overline{N}$ and the module $\overline{N}$ is indecomposable.*

As a direct consequence of the above theorem we get that the orders $\leq_{ext}$ and $\leq_{deg}$ are equivalent for all A-modules if and only if for any A-modules M, N with $M <_{deg} N$, the module N is decomposable.

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Computer algebra packages for problems in representation theory

Peter Dräxler

The session will be devoted to the presentation of recent developments. The following program is planned:

- E. Green (Blacksburg) will introduce the HOPF project.
- D. Guhe (Bielefeld), H.v. Höhne (Berlin), A. Krause (Bielefeld), R. Lychman (Kiev) and R. Nörenberg (Bielefeld) will demonstrate some new procedures and packages of the CREP system dealing with Hall polynomials, quadratic forms, preprojective components and coverings of zero relation algebras.

In the days after the session we will offer the opportunity for practical exercises at the computer.

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A generalization of quasi-hereditary algebras

István Ágoston

This is a joint work with **Vlastimil Dlab** and **Erzsébet Lukács**.

A is a basic algebra over a field K, $\mathbf{e} = (e_1, e_2, \ldots, e_n)$ a complete ordered sequence of primitive orthogonal idempotents in A and $\varepsilon_i = e_i + \cdots + e_n$ with $\varepsilon_{n+1} = 0$. The *standard* and *proper standard modules* are defined by $\Delta(i) = e_iA/e_iA\varepsilon_{i+1}A$ and $\overline{\Delta}(i) = e_iA/e_i(\operatorname{rad} A)\varepsilon_iA$. The left counterparts are denoted by $\Delta^\circ(i)$ and $\overline{\Delta}^\circ(i)$.

Definition 1 *Let X be a local (right) A-module and e a primitive idempotent in A. The idempotent ideal $I = AeA$ is called (right) X-stratifying if both eA and AeA have an X-filtration (i. e. one, where the quotients are isomorphic to X). The ideal is called standardly stratifying, if X can be chosen to be projective (i. e. the ideal is projective). The algebra $(A, \mathbf{e})$ is called $\mathcal{X}$-stratified with $\mathcal{X} = \big(X(1), \ldots, X(n)\big)$ if the ideals $A\varepsilon_iA/A\varepsilon_{i+1}A$ are $X(i)$-stratifying ideals in $A/A\varepsilon_{i+1}A$. The algebra $(A, \mathbf{e})$ is standardly stratified, if it is $\Delta = \big(\Delta(1), \ldots, \Delta(n)\big)$-stratified.*

Then we get the following results:

– X-stratifying ideals are also stratifying in the sense of Cline, Parshall and Scott (*Stratifying endomorphism algebras*, Memoirs of the AMS **591**, 1996). This gives a method for constructing non-trivial stratifying ideals.

– Sufficient conditions will be given for a stratifying ideal to be X-stratifying.

– $(A, \mathbf{e})$ is standardly stratified on the right side if and only if $(A, \mathbf{e})$ is $\overline{\Delta}^\circ$-stratified on the left side. Further conditions will be given to have suitable filtration on both sides.

– A version of the Bernstein–Gelfand–Gelfand reciprocity can be formulated for standardly stratified algebras.

– Quasi-hereditary algebras are precisely the stratified algebras with finite global dimension.

– Some homological characterizations of quasi-hereditary algebras can be extended to standardly stratified algebras.

Further homological and structural properties of stratified algebras, as well as canonical constructions for some special classes will also be discussed.

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DOUBLE INFINITE CHAIN CONDITION, AND GENERALIZED DEVIATIONS OF POSETS AND ALGEBRAS

TOMA ALBU

The *Krull dimension* (resp. *dual Krull dimension*) of a poset P is a measure of how far P is from being Artinian (resp. Noetherian), i.e. how far P is from containing a chain of order type ω^* (resp. ω), where ω is the order type of the positive integers and ω^* is the order type of the negative integers, both with their usual order.

If we replace ω^* or ω with an arbitrary linear order type γ, or more generally by a nonempty set Γ of linear order types, then we obtain the general notion of the Γ–*deviation* of P, first considered by M. Pouzet and N. Zaguia in *"Dimension de Krull des ensembles ordonnés"*, Discrete Math. **53** (1985), 173-192.

The aim of this talk is to introduce the concept of *balanced Krull dimension*, which measures the deviation of a poset P from being either Noetherian from any point $x \in P$ or Artinian from the same point x (i.e., for any $x \in P$, the interval $[x, \rightarrow)$ is Noetherian or the interval $(\leftarrow, x]$ is Artinian), or more generally, that of *symmetric* Γ–*deviation* of a poset, which gives a balance between the Γ–deviation and the dual Γ–deviation of P. Then we relate these notions with the concept of ζ–deviation, where ζ is the order type of the set of rational integers.

Our symmetric Γ–deviation is a natural extension to posets, of various particular concepts involving the *Double Infinite Chain Condition* (*DICC* for short), such as those considered by Contessa (1986) for commutative rings and modules, and then, in a more general setting, by Osofsky (1987) and Karamzadeh – Mohamedi (1994).

The results which will be presented have been obtained jointly with Mark L. Teply (University of Wisconsin–Milwaukee).

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Tilting and cotilting classes in Mod R

Lidia Angeleri Hügel

Report on joint work with Alberto Tonolo and Jan Trlifaj

Let Λ be an artin algebra and mod Λ the category of finitely generated Λ-modules. Smalø described in [2] the torsion classes in mod Λ which are generated by a tilting module in terms of covariantly finiteness. In my talk, I will present an analogous result for the category Mod R of all modules over an arbitrary ring R.

Theorem The following are equivalent for a torsion class $\mathcal{C} \subset \mathrm{Mod}\, R$.
(1) There is a tilting module T such that $\mathcal{C}$ consists of the modules generated

by T.
(2) For every module M_R there is an exact sequence $0 \longrightarrow M \xrightarrow{f} Y \longrightarrow Z \longrightarrow 0$ such that f is a left $\mathcal{C}$ -approximation and $\mathrm{Ext}^1_R(Z,C)=0$ for all $C \in \mathcal{C}$.
(3) R_R has an exact sequence as in (2).

Here we will consider (possibly not finitely generated) tilting modules in the sense of [1]. Moreover, a map $f : M \to Y$ is called a left $\mathcal{C}$ -approximation if $Y \in \mathcal{C}$ and the map $\mathrm{Hom}_R(f,C) : \mathrm{Hom}_R(Y,C) \to \mathrm{Hom}_R(M,C)$ is surjective for each $C \in \mathcal{C}$. The dual result for the cotilting case will also be discussed.

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Hall polynomials for $\tilde{A}_n$

The purpose of this short note is to present another direct, combinatorial proof for the existence of Hall polynomials of the cyclic quiver $\tilde{A}_n$. The main point is the using of ideas, results from [2] to extend the beautiful proof of Zelevinsky in [5] for the existence of classical Hall polynomials over discrete valuation rings with finite residue fields to $\tilde{A}_n$. There are already two other proofs for Hall polynomials over $\tilde{A}_n$ given in [2] and [4]. However, these proofs are of different nature.

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Koszul-tiltings algebras
Regina Maria de Aquino

Let A be an hereditary finite dimensional over a field k. Let T be a tilting module over A and consider B , the endomorphism ring of T over A. In order to describe the tiltings algebras that are koszul algebras , we defined a class of maps , named "sink-torsion-maps , that relates the maps between the direct summands of the tilting module T and the arrows of the quiver of B. As an aplication we present a description of the Brenner-Butler algebras and a caracterization of the koszul modules of projective dimension one over the algebra B.

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Quantum polynomials and their representations
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Let D be a division ring with automorphisms $\alpha_1, \dots, \alpha_n$, where $n \geq 2$. Suppose that there are given elements $q_{ij} \in D^*$, $i, j = 1, \dots, n$, such that

$$q_{ii} = q_{ij}q_{ji} = Q_{ijr}Q_{jri}Q_{rij} = 1, \quad \alpha_i(\alpha_j(d)) = q_{ij}\alpha_j(\alpha_i(d))q_{ji},$$

where $Q_{ijr} = q_{ij}\alpha_j(q_{ir})$, and $d \in D$. Denote by

$$\Lambda = D_{Q,\alpha}[X_1^{\pm 1}, \dots, X_r^{\pm 1}, X_{r+1}, \dots, X_n]$$

the associative ring generated by elements of D and by elements $X_1, \dots, X_n, X_1^{-1}$, $\dots, X_r^{-1}$, subject to the defining relations

$$\begin{aligned} X_iX_i^{-1} = X_i^{-1}X_i = 1, &\qquad 1 \leq i \leq r; \\ X_id = \alpha_i(d)X_i, &\qquad d \in D, \quad i = 1, \dots, n; \\ X_iX_j = q_{ij}X_jX_i, &\qquad i, j = 1, \dots, n. \end{aligned}$$

The algebra Λ was introduced in the paper [6]. It also appears in non-commutative geometry as the algebra of affine functions on quantum affine plane [5], [4]. Division ring of fractions of Λ is related to the division ring of fractions of a generalized quantum Weyl algebra [3], [4].

Definition 1 *The algebra Λ is a algebra of quantum polynomials.*

Notation 2 *Let N be the subgroup in the multiplicative group D^*, generated by the derived subgroup $[D^*, D^*]$ and by all elements of the form $z^{-1}\alpha_i(z)$ where $z \in D^*$ and $i = 1, \dots n$.*

Definition 3 *The algebra Λ is a general quantum polynomial algebra if the images of all elements $q_{ij}, 1 \leq i < j \leq n$, are independent in the multiplicative abelian group D^*/N.*

Theorem 4 ([1]) *Let Λ ge a general quantum polynomial algebra. and P be a finitely generated projective Λ-module of a rank at least 2. Then P is free.*

Theorem 5 ([2]) *Let Λ ge a general quantum polynomial algebra. Suppose that M is a finitely generated Λ-module. Then the set $t(M)$ of all torsion elements in M form a submodule. Moreover $t(M)$ is a cyclic submodule and $M = t(M) \oplus N$ where N is a projective Λ-module of a rank r. If $r = 1$, then the module N is isomorphic to a left ideal in Λ. If $r > 1$, then N is free.*

Notation 6 *Denote by F the division ring of fractions of Λ.*

Theorem 7 *The algebra Λ is a general quantum polynomial algebra if and only if the multiplicative group*

$$([F^*, F^*] \cap D^*)/([F^*, D^*] \cap D^*)$$

has the maximal possible rank which is equal to $\frac{n(n-1)}{2}$.

Theorem 8 *Let $\Lambda, \hat{\Lambda}$ be algebras of quantum polynomials and Λ is a general quantum polynomial algebra. If Λ and $\hat{\Lambda}$ are Morita-equivalent, then $\Lambda \simeq \hat{\Lambda}$. Moreover the group $Pic(\Lambda)$ is trivial.*

Theorem 9 (Zarisky problem) *Let B be a subring in a general quantum polynomial algebra Λ containing D. Suppose that there exists an automorphism β of B and an element $Y \in \Lambda$ such that $Yb = \beta(b)Y$ for all $b \in B$. If $\Lambda = B[Y, \beta]$, then $Y = \lambda_i X_i$ for some $i > r$, where $\lambda_i \in \Lambda^*$, is a monomial in $X_1, \dots, X_r$. Moreover B coincides with the subalgebra generated by $D, X_1^{\pm 1}, \dots, X_r^{\pm 1}$, and by all X_j, $j > r, j \neq i$.*

Theorem 10 (Zarisky problem for Laurent polynomials) *Let B be a subring in a general quantum polynomial algebra Λ, containing D. Suppose that Y is an invertible element of Λ, and an automorphism β of B such that $Yb = \beta(b)Y$ for all $b \in B$. Assume that $\Lambda = B[Y^{\pm 1}, \beta]$. Then Y is a monomial in $X_1, \dots, X_r$, and performing a change of these variables we can assume that $Y = X_1$. In the case B coincides with the subalgebra generated by $D, X_2^{\pm 1}, \dots, X_r^{\pm 1}$, $X_{r+1}, \dots, X_n$.*

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On the number of crossed homomorphisms

Tsunenobu Asai

I will talk about some congruences in finite group theory, which are concerned with the number of crossed homomorphisms. I study mainly the following conjecture.

Conjecture: Let C be an abelian p-group and H a p-group such that C acts on H. Then

$$|Z^1(C,H)| \equiv 0 \mod \gcd(|C|,|H|),$$

where $Z^1(C,H) := \{\eta : C \to H \mid \eta(cc') = \eta(c) \cdot {}^c\eta(c') \text{ for } c, c' \in C\}$.

The above conjecture is closely related to the congruences about the number of group homomorphisms between finite groups and the number of subgroups of a given finite group.

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From stable equivalences to derived equivalences for some classes of selfinjective algebras

Hideto Asashiba

All algebras considered here are assumed to be finite-dimensional connected algebras over an algebraically closed field. It is well-known that if two selfinjective algebras are derived equivalent, then they are stably equivalent ([3], [4]). We are interested in when the converse holds. In [4] Rickard has shown that this is the case for Brauer tree algebras. In [2] we have also shown that the converse holds for representation-finite selfinjective algebras by giving the classification of these algebras up to derived equivalences.

In our talk we will exhibit some method for solving this problem using a covering technique for derived equivalence developed in [1]. In particular, we will give a simple proof of the following theorem.

For an algebra A, we call an algebra of the form $A^{\mathbb{Z}}/\langle {\nu_A}^n \rangle$ a *finite repetition* of A, where $n \geq 1$ is an integer, $A^{\mathbb{Z}}$ the repetition (=repetitive category) of A and ν_A the Nakayama automorphism of $A^{\mathbb{Z}}$. Note that a finite repetition is always selfinjective.

Theorem 1 *In each of the following classes of algebras, two algebras are stably equivalent iff they are derived equivalent:*

(1) *Representation-finite standard selfinjective algebras; and*

(2) *Finite repetitions of piecewise hereditary algebras.*

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Non-negative unit forms and algebras

M. Barot
Joint work with J. A. de la Peña

The study of the interrelationship between properties of an algebra and properties of the associated quadratic forms (such as the Euler form or the Tits form) has a long history in the Theory of Representations of Algebras.

An integer form $q : \mathbb{Z}^n \to \mathbb{Z}$ is called a *unit* form if for all $i = 1, \dots, n$ we have $q(\mathbf{e}_i) = 1$, where $\mathbf{e}_i$ denotes the i^{th} canonical base vector of $\mathbb{Z}^n$. Moreover, q is called *positive* (resp. *non-negative*) if for all $x \in \mathbb{Z}^n \setminus \{0\}$ we have $q(x) > 0$ (resp. $q(x) \geq 0$). If q is non-negative then $\mathrm{r}ad\, q = q^{-1}(0)$ is a direct summand of $\mathbb{Z}^n$ and we define $\mathrm{c}orank\, q = \mathrm{r}ank\, (\mathrm{r}ad\, q)$.

Theorem 1 [1] *Let $q : \mathbb{Z}^n \to \mathbb{Z}$ be a connected non-negative unit form and let $c = \mathrm{c}orank\, q$. Then there exists a linear $\mathbb{Z}$-invertible transformation $T : \mathbb{Z}^n \to \mathbb{Z}^n$ and a unique Dynkin diagram Δ such that*

$$q \circ T(x_1, \dots, x_n) = q_\Delta(x_1, \dots, x_{n-c}).$$

In the situation of the above Theorem we call Δ the *Dynkin type* of q and denote $\mathrm{D}yn(q) = \Delta$. As a direct corollary we obtain that two non-negative unit forms q and q' are $\mathbb{Z}$-equivalent iff $\mathrm{D}yn(q') = \mathrm{D}yn(q)$ and $\mathrm{c}orank\, q' = \mathrm{c}orank\, q$.

Theorem 2 [1] *Let q be a connected non-negative unit form. Then for any connected restriction q' of q we have $\mathrm{c}orank\, q' \leq \mathrm{c}orank\, q$ and $\mathrm{D}yn(q') \leq \mathrm{D}yn(q)$, where the partial order on Dynkin diagrams is induced by $\mathbb{A}_m \leq \mathbb{A}_n \leq \mathbb{D}_n \leq \mathbb{D}_p \leq \mathbb{E}_p \leq \mathbb{E}_q$ for $m \leq n \leq p \leq q$ whenever this makes sense.*

Equipped with this tools it is possible to show the following two results.

Theorem 3 [2] *Let A be a strongly simply connected algebra such that χ_A is non-negative. If $\mathrm{D}yn(\chi_A) = \mathbb{A}_n$ for some n then A is derived equivalent to a hereditary algebra of type $\mathbb{A}_n$. In particular, χ_A is positive.*

Theorem 4 [2] *Let A be tree algebra or a strongly simply connected poset algebra such that the Euler form χ_A is non-negative of corank 2. Then A is derived equivalent to a tubular or to a pq-critical algebra. Moreover, if A has more that 6 points, then A is derived equivalent to a tubular algebra iff $\mathrm{D}yn(\chi_A) = \mathbb{E}_{n-2}$ and A ids derived equivalent to a pg-critical algebra iff $\mathrm{D}yn(\chi_A) = \mathbb{D}_{n-2}$.*

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Non-regular sincere objects in factorspace categories

Manfred Bauch

For multitubular vector space categories it is known (see [1]) that if an n-tubular vector space category is sincere then $n \leq 2$, and in these cases there are always regular sincere indecomposable objects.

Extending the considerations to the class of generalized multitubular vector space categories, it occurs that all the sincere objects are non-regular.

In our talk, we present inductive arguments on the level of factorspace categories leading to lists presented in papers by Bekkert [2] and Zavadskij [3]. In particular, the detailed knowledge of the categories of representations suggests a different kind of classification of the vector space categories that are considered.

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The Simple Holonomic Modules over Rings of Differential Operators on Surface

Vladimir Bavula

The present talk is based on the joint papers with F. Van Oystaeyen, [1], [2]. Let K be an a.c. field of characteristic 0 and let R be a **regular commutative Noetherian K-affine domain of Krull dimension** $\mathcal{K}(R) = n < \infty$ (i.e. R is the coordinate ring $\mathcal{O}(X)$ of a smooth irreducible affine variety X of dimension n). The **ring of differential operators** $\mathcal{D}(R)$ on R is a simple K-affine Noetherian algebra. Fort any nonzero f. g. $\mathcal{D}(R)$-module M its Gelfand-Kirillov dimension GK $(M) \geq n$. M is **holonomic** iff GK $(M) = n$. An aim of the talk is to describe the simple holonomic $\mathcal{D}(R)$-modules in case when the algebra $R = D_1 \otimes D_2$ is the tensor product of regular commutative Noetherian affine domains of Krull dimension 1. The second Weyl algebra $A_2 = \mathcal{D}(K[X_1, X_2]) = K[X_1, X_2, \partial/\partial X_1, \partial/\partial X_2]$ is an example of the ring $\mathcal{D}(D_1 \otimes D_2)$. Set $\Lambda = \mathcal{D}(R)$, $C = \mathcal{D}(D_1)$ and $A = \mathcal{D}(D_2)$, then $\Lambda = C \otimes A$.

The set of isomorphism classes of simple holonomic Λ-modules can be partitioned in two subsets:

$$\hat{\Lambda}(\text{holonomic}) = \hat{C} \otimes \hat{A} \cup \hat{\Lambda}(\text{holonomic}) \backslash \hat{C} \otimes \hat{A}.$$

The sets $\hat{C}$, $\hat{A}$ are described using the Block's approach.

Denote by k the full quotient ring (in fact a skew field) of C. Choose $c \in D \equiv D_2$ in such a way that the localization A_c of A at the powers of c is an Ore extension $A_c = D_c[X; \delta]$. We have a natural algebra monomorphism

$$\Lambda = C \otimes A \to \mathcal{A} := k \otimes A_c = \mathcal{D}_c[X; \delta], \quad \mathcal{D}_c = k \otimes D_c.$$

Theorem 1 *The map*
$\hat{\Lambda}(\text{holonomic}) \backslash \hat{C} \otimes \hat{A} \to \hat{\mathcal{A}}(k - \text{fin.dim})$,
$[M] \to [\mathcal{A} \otimes_\Lambda M]$, *is bijective with inverse* $[N] \to [\mathrm{Soc}_\Lambda N]$.

Corollary 2 *Let M be a nonzero simple Λ-module and let $\tilde{M} = \mathcal{A} \otimes_\Lambda M$. Then*

1. $\tilde{M} = 0 \Leftrightarrow [M] \in \hat{C} \otimes \hat{A}$*;*
2. $1 \leq \dim_k \tilde{M} < \infty \Leftrightarrow [M] \in \hat{\Lambda}(\text{holonomic}) \backslash \hat{C} \otimes \hat{A}$*;*
3. $\dim_k \tilde{M} = \infty \Leftrightarrow [M] \in \hat{\Lambda}(\text{non} - \text{holonomic})$.

Hence, M is holonomic (resp. non-holonomic) iff $\dim_k \tilde{M} < \infty$ *(resp. M_c contains a free $C \otimes K[X]$-module of rank 1).*

The set $\hat{\mathcal{A}}(k - \text{fin.dim})$ of isomorphism classes of simple left k-finite dimensional $\mathcal{A}$-modules is described. Then, using Theorem 0.1, every element $[M] \in \hat{\Lambda}(\text{holonomic}) \backslash \hat{C} \otimes \hat{A}$ is presented as a factor module $M = \Lambda / I$ for some maximal left ideal I (which is described explicitly).

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Gorenstein Categories

Apostolos Beligiannis

Let $\mathcal{C}$ be an exact category with enough projectives. We consider the stable category $\mathcal{C}/\mathcal{P}$ modulo projectives, as a left triangulated category. Associated to $\mathcal{C}/\mathcal{P}$ is a triangulated category $\mathcal{S}(\mathcal{C}/\mathcal{P})$ and an exact functor $\mathbf{S} : \mathcal{C}/\mathcal{P} \to \mathcal{S}(\mathcal{C}/\mathcal{P})$ which is universal for exact functors from $\mathcal{C}/\mathcal{P}$ to triangulated categories. The category $\mathcal{S}(\mathcal{C}/\mathcal{P})$ is called the stabilization of $\mathcal{C}/\mathcal{P}$. We call the exact category $\mathcal{C}$, $\mathcal{P}-$**Gorenstein** if the stabilization $\mathcal{S}(\mathcal{C}/\mathcal{P})$ can be realized as a full triangulated subcategory of $\mathcal{C}/\mathcal{P}$. One defines dually the concept of an injective Gorenstein category. An associative ring Λ is called **right Gorenstein** if any projective right module has finite injective dimension and any injective module has finite projective dimension. We characterize right Gorenstein rings through: the property that the module category is a (projective or injective) Gorenstein category, the existence of Auslander-Buchweitz contexts [3], [1], the existence of complete (projective or injective) resolutions, the finitude of Gorenstein (projective or injective) resolution dimension, and through various equivalences of triangulated categories. Note that if Λ is Noetherian, then Λ is right Gorenstein iff Λ is left Gorenstein iff Λ has finite self-injective dimension. We present applications of the theory to Gorenstein Artin algebras.

A ring Λ **has a (projective) stable homotopy category** if the stabilization of $\underline{\mathrm{Mod}}(\Lambda)$ is compactly generated. We show that a right Gorenstein ring which is left coherent and right perfect or right Noetherian and the injective envelope of any finitely presented module is finitely presented, has a stable homotopy category. We characterize also when the stable homotopy category of a ring is phantomless and we give sufficient conditions for the stable homotopy category to be a Brown category.

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Geometry of directing modules over tame algebras

Grzegorz Bobiński
(joint work with Andrzej Skowroński)

Let A be a basic algebra over an algebraically closed field K. An indecomposable finite dimensional A-module M is said to be directing if it does not belong to a cycle $M \to M_1 \to \cdots \to M_r \to M$ of nonzero nonisomorphisms between indecomposable finite dimensional A-modules. Directing modules have played an important role in the representation theory of algebras: the preprojective components and the preinjective components in general and the connecting components of tilted algebras consist entirely of directing modules. C. M. Ringel has proved that if M is a directing A-module then its support algebra is a tilted algebra and the dimension-vector of M is a root of the associated Euler (equivalently, Tits) integral quadratic form. Given a nonnegative vector $\mathbf{d}$ in the Grothendieck group $K_0(A)$ of A it is an interesting problem to study the affine variety $\mathrm{mod}_A(\mathbf{d})$ of A-modules of dimension-vector $\mathbf{d}$ and the action of the corresponding product $G(\mathbf{d})$ of general linear groups.

We shall describe the geometry of $\mathrm{mod}_A(\mathbf{d})$ for the dimension-vector $\mathbf{d} = \mathbf{dim}\, M$ of an arbitrary directing module M over a tame algebra A. We will show that such a variety $\mathrm{mod}_A(\mathbf{d})$ is a complete intersection with at most two irreducible components, and describe when $\mathrm{mod}_A(\mathbf{d})$ is irreducible (respectively, normal). In particular, we get that $\mathrm{mod}_A(\mathbf{d})$ is irreducible if and only if $\mathrm{mod}_A(\mathbf{d})$ is normal. Moreover, a complete description of the irreducible components and maximal $G(\mathbf{d})$-orbits in the considered varieties $\mathrm{mod}_A(\mathbf{d})$ will be given.

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On an extension of the problem about representations of bundles of semichains

Vitalij M. Bondarenko

For a (finite) poset A with involution $*$ and a field k, we denote by $\mathrm{mod}_{(A,*)}k$ the category of $(A,*)$-spaces over k, i.e. the category with objects (finite–dimensional) vector k-spaces $U = \oplus_{u\in A} U_a$, where $U_{a^*} = U_a$ for all $a \in A$, and with morphisms $\varphi : U \to V$ those linear maps $\varphi \in \mathrm{Hom}_k(U,V)$ for which $\varphi_{a^*a^*} = \varphi_{aa}$ and $\varphi_{bc} = 0$ if $b \not\leq c$ (φ_{xy} is the map of U_x into V_y induced by φ). We identify $(A,*)$ with A if the involution $*$ is trivial.

Let A and B be (disjoint) posets, $*$ an involution on $A\cup B$, M a lover subset of $A \times B$ (i.e. $(a,b) \in M$, $a \geq a'$, $b \geq b'$ imply $(a',b') \in M$), and ε a map of $M_*^- \cup M_*^+ = \{a \in A \,|\, (a,a^*) \in M\} \cup \{b \in A \,|\, (b^*,b) \in M\}$ into the field k such that $\varepsilon(a) = \varepsilon(b)$ if $(a,b) \in M$. For maps α, $\beta \in \mathrm{Hom}_k(U,V)$, where $U \in \mathrm{mod}_A k$ and $V \in \mathrm{mod}_B k$, we write $\alpha \equiv \beta (\mathrm{mod}\, M)$, if $\alpha_{ab} = \beta_{ab}$ for all $(a,b) \in M$. If U is an A-space, V is a B-space and $A \cup B$-space $U \oplus V = (\oplus_{a\in A} U_a) \oplus (\oplus_{b\in B} V_b)$ lies in $\mathrm{mod}_{(A\cup B,*)}k$, then $[\varepsilon]^-_{U,V}$ (resp. $[\varepsilon]^+_{U,V}$) denotes the following map $\varphi \in \mathrm{Hom}_k(U,V) : \varphi_{xy} = \varepsilon(x)\mathbf{1}_{U_x,U_y}$ if $x \in M_*^-$, $y = x^*$ (resp. $\varphi_{xy} = \varepsilon(y)\mathbf{1}_{U_x,U_y}$ if $y \in M_*^+$, $x = y^*$) and $\varphi_{xy} = 0$ otherwise.

We denote by $\mathcal{R}_k(A,B,*,M,\varepsilon)$ the category with objects the triples (U,V,φ), where $U \in \mathrm{mod}_k A$, $V \in \mathrm{mod}_k B$, $\varphi \in \mathrm{Hom}_k(U,V)$, and $U \oplus V \in \mathrm{mod}_{(A\cup B,*)}k$, and with morphisms $(U, V, \varphi) \to (U', V', \varphi')$ the pairs (α, β) of linear maps $\alpha : U \to U'$, $\beta : V \to V'$ such that $\alpha \in \mathrm{mod}_A k$, $\beta \in \mathrm{mod}_B k, \alpha \oplus \beta \in \mathrm{mod}_{(A \cup B, *)} k$ and (in the right-side notation)

$$(\varphi + [\varepsilon]^+_{U,V})\beta \equiv \alpha(\varphi' + [\varepsilon]^-_{U',V'})(\mathrm{mod}\, M).$$

Recall that a semichain is a poset of the form $X = \cup_{i=1}^m X_i$, where each X_i (called a *link* of X) consists of either one point or two incomparable points and $x_1 < \cdots < x_m$ for all $x_i \in X_{ij}$.

THEOREM. *Let A, B be semichains and the involution $*$ satisfy $x^* = x$ ($x \in A \cup B$) for every x belonging to a two-point link. Then the category $\mathcal{R}_k(A, B, *, M, \varepsilon)$ is tame.*

In the case $M = A \times B$ and $\varepsilon = 0$ we have the problem about representations of bundles of (two) semichains, completely solved in the invariant form by the author. Similar statement is fulfilled in more general situation, when ε is a map of $(M_*^- \times A) \cup (M_*^+ \times B)$ into k. The problem of classifying indecomposable objects is considered.

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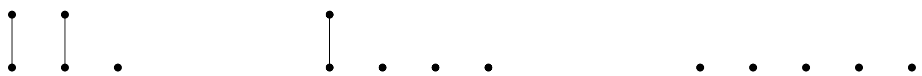
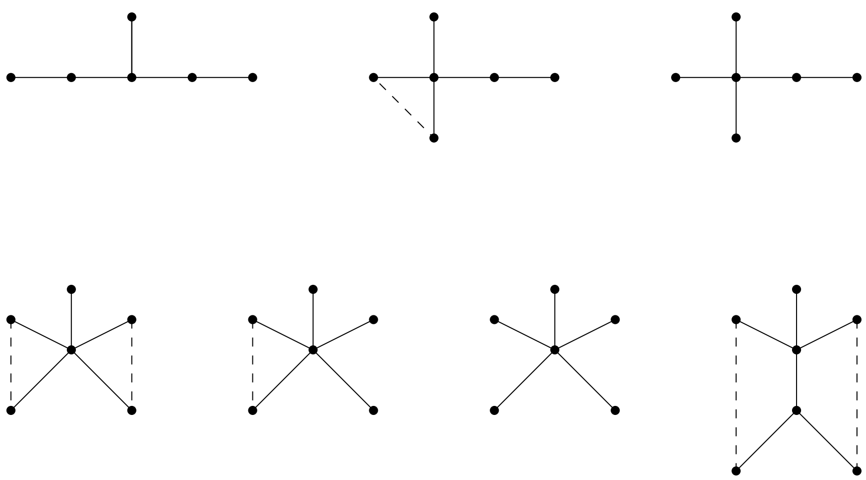
On E_6-free tree algebras

THOMAS BRÜSTLE

This talk deals with E_6-free tree algebras. These are algebras $A = kQ/I$ where the quiver Q is a tree and the following condition holds: For each vertex x of Q, the poset S_x associated to x (see [1]) does not contain one of the following three as a full subposet:



This can be equivalently formulated in terms of "forbidden" subcategories as follows: a tree algebra is E_6-free if it does not contain a convex subcategory of one of the following forms:



The main motivation for studying E_6-free algebras is the hypothesis that they play the same role for tame non-polynomial growth sincere algebras as the coil algebras do in the tame polynomial growth case. More precisely, the conjecture is: If a sincere tree algebra is tame of non-polynomial growth, then it is E_6-free and can be obtained by certain admissible operations.

We plan to give some structure results on sincere E_6-free tree algebras, for example that one can restrict the study to algebras with relations of length 2 or 3 only. Moreover, we discuss basis and consequences of the hypothesis above in more detail.

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Some algebras with periodic projective resolutions

Michael C. R. Butler

Any quadratic algebra Λ over a field k determines, by means of a standard construction, a *Koszul bimodule complex.* Its terms are projective $\Lambda-$ bimodules, its homology in degree 0 is Λ, and it is the minimal projective bimodule resolution of Λ exactly when this algebra is a Koszul algebra in the now well-known sense of [2]. In this talk I shall discuss some quadratic algebras which are certainly not Koszul algebras, but which have periodic projective resolutions whose terms are given correctly by the appropriate initial terms of the Koszul bimodule complex. Examples of such algebras are the preprojective algebras of Dynkin graphs, which have been shown in [1] (following unpublished work of Schofield) to have resolutions of period dividing 6. Another example is that of the trivial extension algebra, Λ_n, of the quiver A_n, with $n \geq 3$ and radical square-zero orientation. This algebra has a periodic resolution of period $n+1$ which is most readily obtained by suitably truncating the projective resolution of a related infinite dimensional Koszul algebra, namely the trivial extension algebra, Λ_∞^∞, of the path algebra of the quiver A_∞^∞ (again with radical-square zero orientation). One would expect similar constructions of periodic resolutions of the trivial extension algebras of the other representation - finite radical-square-zero hereditary algebras, the expected periods for those of Dynkin classes D_n, E_6, E_7, E_8 being $2n-3, 11, 17, 29$, respectively.

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Vectorspace categories imersed in directed components

Gladys Chalom

We know, after [3], that, given a tame algebra B, the Tits form q_B is weakly non negative. Moreover, the converse has been shown for some families of algebras, but it is not true in general.

We are interessed in the one-point extension $B[M]$, where B is a tame algebra and M an indecomposable module such that
$= Hom(M, B-mod) = Hom(M, \mathcal{C})$ is finite and $\mathcal{C}$ is a directed, standard, tree type component of the A-R- quiver of B.

For this purpose, we analize the classification of the polynomial growth categories given by Bekkert [1].

The aim of this talk is to show that for some wild vectorspace categories, we can reduce its study at the categories obtained by Marmaridis [2].

Moreover, if $gldimB \leq 2$, we obtain that $q_{B[M]}$ is strongly indefinite.

The results presented here are mainly contained in the author Ph. D. thesis.

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Algebras with small homological dimensions

Flávio Ulhoa Coelho

Joint work with Marcelo Lanzilotta [1, 2]

We shall consider in this talk the class of algebras Λ satisfying the following property: (S) for each indecomposable Λ-module X, either $\mathrm{pd}_\Lambda X \leq 1$ or

$\mathrm{id}_\Lambda X \leq 1$. As observed by Happel-Reiten-Smalø [1], any algebra satifying condition (S) has global dimension at most 3. Our main motivation was to extend results proven for quasitilted algebras to this general situation.

For an algebra Λ, denote by $\mathcal{L}_\Lambda$ (respectively, $\mathcal{R}_\Lambda$) the subcategory of all indecomposable Λ-modules whose predecessors (respectively, successors) have projective dimension (respectively, injective dimension) at most one. In order to state our main result we need a further concept. Let $(*) X_0 \longrightarrow X_1 \longrightarrow \cdots \longrightarrow X_t$ be a path of irreducible maps in $\mathrm{ind}\Lambda$. A *hook* in $(*)$ is an j, $1 \leq j \leq t-1$ such that $\tau_\Lambda X_{j+1} = X_{j-1}$, where τ_Λ denotes the Auslander-Reiten translate of Λ.

Theorem 1 *The following are equivalent for an algebra Λ:*

(a) for each indecomposable module X, either $pd_\Lambda X \leq 1$ or $id_\Lambda X \leq 1$.

(b) $\mathcal{L}_\Lambda \cup \mathcal{R}_\Lambda = ind\Lambda$.

(c) (add $\mathcal{L}_\Lambda$, add $(\mathcal{R}_\Lambda \setminus \mathcal{L}_\Lambda)$) is a split torsion pair in $mod\Lambda$.

(d) (add $(\mathcal{L}_\Lambda \setminus \mathcal{R}_\Lambda)$, add$\mathcal{R}_\Lambda$) is a split torsion pair in $mod\Lambda$.

(e) there exists a split torsion pair $(\mathcal{X}, \mathcal{Y})$ in $mod\Lambda$ such that $pd_\Lambda Y \leq 1$ for each $Y \in \mathcal{Y}$ and $id_\Lambda X \leq 1$ for each $X \in \mathcal{X}$.

(f) Any path from an indecomposable injective module to an indecomposable projective module can be refined to a path of irreducible maps and any such refinement has at most two hooks, and in case there are two, they are consecutive.

We shall also look at the structure of the Auslander-Reiten quiver Γ_A of an algebra A satisfying condition (S). In case global dimension of A is 3, we shall give a fairly complete description of Γ_A.

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Reflective subcategories

Alberto del Valle (joint work with M. Saorín)

For a full subcategory $\mathcal{F}$ of a category $\mathcal{A}$, the existence of a left $\mathcal{F}$-approximation completing diagrams uniquely for every object of $\mathcal{A}$ is equivalent to the fact that $\mathcal{F}$ is reflective in $\mathcal{A}$ in the classical terminology of category theory. The goal of this talk is twofold. On one side we establish, under several conditions on $\mathcal{A}$, the relationship between reflectivity of $\mathcal{F}$ in $\mathcal{A}$ and covariant finiteness of $\mathcal{F}$ in $\mathcal{A}$; these result generalize Freyd's adjoint functor theorem (for inclusion functors) to not necessarily complete categories. One of them is:

Theorem: Assume that $\mathcal{A}$ is well-powered and equalizers of $\mathcal{F}$-morphisms and intersections of them exist in $\mathcal{A}$. Then $\mathcal{F}$ is reflective in $\mathcal{A}$ if and only if $\mathcal{F}$ is covariantly finite in $\mathcal{A}$ and closed for intersections of equalizers of $\mathcal{F}$-morphisms.

Still in this first part, we remark that uniqueness in the completion of diagrams allows to construct, from a $\mathcal{F}$-reflection of each member of a direct system, a $\mathcal{F}$-reflection for the direct limit of the system. As an application of this we have:

Proposition: Let $\mathcal{C}$ be a skeletally small preadditive category, and let $\mathcal{R}$ (resp. $\mathcal{F}$) be the class of all representable (resp. flat) functors in $(\mathcal{C}^{op}, \mathrm{Ab})$. Then $\mathcal{R}$ is a reflective subcategory of fp $(\mathcal{C}^{op}, \mathrm{Ab})$ if and only if $\mathcal{F}$ is a reflective subcategory of $(\mathcal{C}^{op}, \mathrm{Ab})$, if and only if $\mathcal{C}$ has cokernels.

As a second objective, we give various examples of reflective subcategories. Some of them follow from the next result, where Copres (M) is the class of all objects X that fit in an exact sequence of the form $0 \to X \to M^I \to M^J$ for some sets I, J.

Proposition: Assume that $\mathcal{A}$ is complete and abelian and $M \in \mathcal{A}$. Then Copres (M) is covariantly finite in $\mathcal{A}$. When $\mathcal{A}$ is well-powered, Copres (M) is a reflective subcategory of $\mathcal{A}$ if and only if it is closed for kernels.

An additive category $\mathcal{F}$ is *generalized Krull-Schmidt* when every object in $\mathcal{F}$ is a coproduct of endolocal objects and the isomorphism classes of indecomposable objects of $\mathcal{F}$ form a set. By results of H. Krause and M. Saorín (preprint 97-122, univ. Bielefeld), the covariantly finite, generalized Krull-Schmidt subcategories of Mod_R are exactly the subcategories of the form Add (M) (direct summands of coproducts of copies of the module M) that are covariantly finite in Mod_R.

We study the modules M_R such that Add (M) is reflective in Mod_R in terms of the rings $S = \mathrm{End}\,(M_R)$ and $\bar{R} = \mathrm{End}\,({}_SM)$ and of the modules ${}_SM$ and $M_{\bar{R}}$. Specially good properties are obtained when ${}_SM$ (and hence $M_{\bar{R}}$) is a progenerator.

Finally, we propose the following question: Suppose that Add (M) is reflective in Mod_R; are there only finitely many non-isomorphic indecomposable direct summands of M_R? Note that an affirmative answer would solve the "pure-semisimple conjecture". We give affirmative answers when R is a principal ideal domain or a finite dimensional tame hereditary algebra over a field.

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On good module categories without short cycles

Bangming Deng

This is a joint work with B.Zhu.

Quasi–hereditary algebras were introduced by L.Scott [3] and have been further studied by many authors.

For a given quasi–hereditary algebra A, instead of the complete module category of A , one is mainly interested in the so–called good module category $\mathcal{F}(\Delta)$ which consists of A–modules filtered by standard modules. It is proved by Ringel [2] that $\mathcal{F}(\Delta)$ has almost split sequences. Thus the usual techniques of representation theory can be adapted.

In this talk we consider the notion of a short cycle in $\mathcal{F}(\Delta)$, i.e. a chain of nonzero noninvertible maps $M \to N \to M$ with M and N indecomposable in $\mathcal{F}(\Delta)$. It is shown that two indecomposable modules in $\mathcal{F}(\Delta)$ are isomorphic if they are not on short cycles in $\mathcal{F}(\Delta)$ and have the same composition factors. Moreover, if there is no short cycle in $\mathcal{F}(\Delta)$, we show that $\mathcal{F}(\Delta)$ is finite, that is, there are only finitely many isomorphism classes of indecomposables in $\mathcal{F}(\Delta)$. This is an analogue to a result in a complete module category proved by Happel and Liu [1].

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Invariant theory of canonical algebras

Mátyás Domokos
(joint work with **Helmut Lenzing)**

We work over an algebraically closed zero characteristic base field k. Let X be the affine variety of representations in a fixed dimension vector of a canonical algebra, and G the direct product of general linear groups acting naturally on X such that the orbits are the equivalence classes of representations. Using a simple computational principle we get the following results on relative invariant polynomial functions on X:

– explicit generators of the algebra of relative invariants of all possible weights when the dimension vector is a multiple of the homogeneous root;

– generators of the algebra of relative invariants whose weight is a multiple of the defect for any rank zero dimension vector;

– relations of the above generators for canonical algebras with three arms.

In view of the theory of moduli spaces for representations of finite dimensional algebras developed by King we obtain the following geometric consequence for domestic canonical algebras:

– We give *explicitly* a morphism $F : X \to P^m$ of quasi-projective varieties such that F is regular precisely on X^{reg}, the set of regular representations, the image of F is a projective subvariety of P^m, and two representations lie in the same fiber of F if and only if they have the same regular composition factors. Moreover, $\mathrm{Im}(F)$ is a coarse moduli space for families of regular representations up to S-equivalence.

The category of representations of a tame quiver is tilting equivalent to the category of representations of a domestic canonical algebra. We interprete this equivalence on the level of relative invariants. Thus the above results give an alternative explanation for some results of Ringel and Happel on the invariant theory of tame quivers. The advantage of our approach is that it is uniform, the role of the Kronecker algebra is very explicit. On the other hand, the method does not depend on the tameness of the canonical algebra. Instead we derive from the first fundamental theorem of classical invariant theory that under certain conditions one can reduce the study of relative invariants to the case of a smaller quiver.

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Tame derived circular biextensions of piecewise tame hereditary algebras

Peter Dräxler

In [3] the construction of tame algebras of non-polynomial growth using extension processes which refer to modules lying in tubes in the derived category was introduced and multiple one-point biextensions by indecomposables of length 2 where studied. This work was completed recently in [2] by clarifying the case of length 1. Note, that length$\geq$ 3 will usually yield wild algebras. The observation that indecomposables of length 2 in a stable tube lie on a cycle was used in [1] to introduce the concept of circular extensions. The tame circular biextensions of tame concealed algebras were classified. We will present a common generalisation of the tameness results in [3] and [1]:

Let B be a basic algebra over an algebraically closed field k which is derived equivalent to a hereditary algebra $k\Delta$ where Δ is a quiver with underlying graph $\widetilde{\mathbf{D}}_n$ $(n \geq 4)$. Let $(\mathcal{T}[\nu])_{\nu\in\mathbf{Z}}$ be a shift orbit of tubes of rank $n-2$ in the derived category $D^b(B)$ of bound complexes over the category $B - mod$ of finite-dimensional left B-modules. Let $\mathcal{R} = R_1, \dots, R_t$ and $\mathcal{R}' = R'_1, \dots, R'_s$ be two sequences of mutually non-isomorphic indecomposable modules which are of length 2 in one of the tubes $\mathcal{T}[\nu]$. All the indecomposable objects of length 2 in one of the tubes $\mathcal{T}[\nu]$ form a representation in $D^b(B)$ of the oriented cycle satisafying the relations that the product of two consecutive arrows is 0. By considering only those objects on the cycle which are modules in the sequence $\mathcal{R}$ (resp. $\mathcal{R}'$), we obtain a B-$C(\nu)$-bimodule $R(\nu)$ (resp. a B-$C'(\nu)$-bimodule $R(\nu')$) where $C(\nu)$ (resp. $C'(\nu)$) is the corresponding full subcategory of the cycle. Define C (resp. C') to be the product over $\nu \in \mathbf{Z}$ of all the algebras $C(\nu)$ (resp. $C'(\nu)$) and R (resp. R') to be the direct sum of all $R(\nu)$ (resp. $R'(\nu)$) considered as B-C-bimodule (resp. B-C'-bimodule). We form the matrix algebra

$$A = \begin{pmatrix} C & 0 & 0 \\ R & B & 0 \\ D(R') \otimes_B R & D(R') & C' \end{pmatrix}$$

and its ideal $J = \oplus_{\nu\in\mathbf{Z}} D(R(\nu+1)) \otimes_B R(\nu)$.

Theorem *The algebra A/J is tame provided that for each module R_i in the sequence $\mathcal{R}$ the module $\tau^- R_i[-1]$ does not appear in the sequence $\mathcal{R}'$.*

Remark An analogous result holds if Δ is a quiver of type $\widetilde{\mathbf{A}}_{p,q}$ and one considers shift orbits of non-homogeneous tubes.

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Classification of quadratic modules

Yuriy A. Drozd

A *quadratic module* is, by definition, a quadruple $M = (M_1, M_2, H, P)$, where M_i are Abelian groups and $H : M_1 \to M_2$, $P : M_2 \to M_1$ their homomorphisms such that $HPH = 2H$ and $PHP = 2P$. Quadratic modules arise from quadratic functors playing an important role in algebraic topology and are in one-to-one correspondence with the right exact quadratic functors. Namely, each right exact quadratic functor is isomorphic to the *quadratic tensor product* with a quadratic module as defined in [1].

We give a complete classification of finitely generated quadratic modules (it means that both groups M_i are finitely generated). This description is given in terms of *projective presentations*, i.e. exact sequences $P_1 \to P_0 \to M \to 0$, where P_i are projective modules, and uses the results of [2, 3]. The result matches the frames of "strings and bands," so widespread in the representation theory of algebras.

Here are some consequences of the obtained classification. All quadratic modules are supposed to be finitely generated.

Corollary 1 *The decomposition of quadratic modules into direct sums of undecomposable ones is unique up to isomorphism and permutation of summands.*

Corollary 2 *Two quadratic modules are isomorphic if and only if all their localizations are isomorphic.*

Corollary 3 *The rank of the torsion free part of an indecomposable quadratic module is at most* 2.

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Tilting modules of dual extension algebras and induced torsion theories

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Let B be a finite-dimensional k-algebra over a fixed algebraically closed field k, M be a $B-B$-bimodule. Let $\mathcal{A}(B)$ be the dual extension algebra of B. For a fixed $n \geq 1$, denote by R_B^n the algebra

$$R_B^n = \left\{ \begin{pmatrix} \lambda_1 & x_1 & & & \\ & \lambda_2 & x_2 & & \\ & & \ddots & \ddots & \\ & & & \lambda_n & x_n \\ & & & & \lambda_1 \end{pmatrix} \mid \lambda_i \in B, x_i \in M \right\}.$$

Abbreviate R_B^n by R. Ket T be the functor $- \otimes_B R : mod\ B \to mod\ R$, $F = -\otimes_B \mathcal{A}(B)_{\mathcal{A}(B)} : mod\ B \to mod\ \mathcal{A}(B)$ and $G = -\otimes_{\mathcal{A}(B)} B_B : mod\ \mathcal{A}(B) \to mod\ B$.

In section 1 we show that if $X \in mod\ B$, then TX is a tilting R-module if and only if X is a tilting B-module and the B-modules $Hom_k(M,k)$ and $X \otimes_B M$ belong to the torsion class $\mathcal{T}_B^X$ induced by X. FX is a tilting $\mathcal{A}(B)$-module if and only if X is a tilting B-module and $X \otimes_B \mathcal{A}(B)$ is generated by X as B-modules.

In section 2, we show that tilting modules FX and FX' induce the same torsion theories in $mod\ \mathcal{A}(B)$ if and only if X and X' induce the same torsion theories in $mod\ B$.

keywords: tilting module, dual extension, torsion theory

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On quasitilted algebras

Ole Enge

We will consider some examples of quasitilted algebras (see [1] for definitions and basic properties) and closely connected non-quasitilted algebras. Then we will give some criteria on iterated one point extensions of quasitilted algebras ensuring that the final algebra is no longer quasitilted.

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Stable Categories of Tame Local Symmetric Algebras
Karin Erdmann and Jörg Feldvoss

We would like to know which tame local algebras A occur as blocks of reduced enveloping algebras (or of other self-injective algebras). A main problem is the following:

Suppose A and A' are basic and self-injective such that $A \not\cong A'$, but $A/\mathrm{Soc}(A) \cong A'/\mathrm{Soc}(A')$. How can one distinguish A and A' homologically (without knowing quiver and relations).

Here we study the Ω-action on the stable module category of a tame local symmetric algebra A using the classification due to Ringel and the first author. This gives some criteria towards solving the above problem. We also discuss applications.

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T-Koszul Algebras

E. L. Green, I. Reiten, Ø. Solberg

Presenter: E.L. Green

For the last few years, we have been working on the theory of *T-Koszul algebras.* Let $\Lambda = \Lambda_0 \oplus \Lambda_1 \oplus \Lambda_2 \oplus \cdots$ be a positively Z-graded algebra which satisfies the following properties: (i) Λ_0 is an artin algebra, (ii) each Λ_i has finite length over Λ_0, and (iii) Λ is strongly graded; that is, for each $i \geq 1$, $\Lambda_i = \Lambda_1 \cdot \Lambda_{i-1}$.

We let T be a cotilting module for Λ_0 and view T as a graded Λ-module with support in degree 0. Let $E_T(\Lambda)$ be the Ext-algebra $\coprod_{n\geq 0} \mathrm{Ext}^n_\Lambda(T,T)$. We have a functor $\mathcal{E} : Grmod(\Lambda) \to Grmod(E_T(\Lambda))$ given by $\mathcal{E}(M) = \coprod_{n\geq 0} \mathrm{Ext}^n_\Lambda(M,T)$, where $Grmod(\Lambda)$ denotes the category of graded Λ-modules and degree 0 maps.

We investigate when there is a natural duality between certain subcategories of $Grmod(\Lambda)$ and $Grmod(E_T(\Lambda)$. In particular, we have developed a theory that includes the following two dualities as special cases:

(I): If $\Lambda = \Lambda_0$ then $E_T(\Lambda) = \mathrm{End}_\Lambda(T)$ and the duality in question is usual duality associated to a cotilting module.

(II): If $\Lambda = KQ/I$ is a Koszul algebra with I generated by quadratic elements, then grade Λ by length. Hence Λ_0 is the subalgebra generated by the vertices of Q. Let $T = \Lambda_0$. Hence $E_T(\Lambda)$ is the usual cohomology ring of the graded simple Λ-modules. Then the duality we study specializes to the usual "Koszul duality" on the subcategory of Koszul modules.

The subcategory of $Grmod(\Lambda)$, $\mathcal{H}_\Lambda$, that we are interested in is defined as follows: The objects are graded modules generated in degree 0. The category $\mathcal{H}_\Lambda$ is the largest category that is closed under taking rejects, corejects, images in the T-approximations, submodules of the first syzygy generated in degree 0 and the quotient of the first syzygy by this submodule (shifted -1). We say Λ is a *T-Koszul algebra* if both Λ and T are in $\mathcal{H}_\Lambda$.

Theorem*If Λ is a T-Koszul algebra then*

- *$E_T(\Lambda)$ is a T-Koszul algebra (for $\mathcal{E}(\Lambda)$).*
- *$E_T(E_T(\Lambda))$ is isomorphic to Λ as graded algebras.*
- *$\mathcal{E}$ restricts to a duality from $\mathcal{H}_\Lambda$ to $\mathcal{H}_{E_T(\Lambda)}$.*

We provide numerous new classes of T-Koszul algebras and hence new dualities. We describe the category $\mathcal{H}_\Lambda$ for some examples. We give new examples T-Koszul algebras (and hence new dualities) where both Λ and $E_T(\Lambda)$ are finite dimensional.

Toeplitz operators with symbol having generalized almost–periodic discontinuities

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Let function $a(x) = p(\alpha(x))$, where $p(x)$ is uniform almost–periodic function on the real axis R, and $\alpha(x)$ is continuous and monotonically increasing function on R such that

$$\lim_{x\to\pm\infty} \alpha(x) = \pm\infty.$$

The problem of normal solvability of Toeplitz operator

$$T(a) = P^+ a P^+$$

is considered. Here $T(a)$ acting in Hardy space $H^p(\mathrm{R})$, $1 < p < \infty$, $P^+ : L_p(\mathrm{R}) \to H^p(\mathrm{R})$ is standard singular projector.

Let $\inf_{x\in \mathrm{R}} |p(x)| > 0$, and $\mu_p \in \mathrm{R}$ be an almost–periodic index of function $p(x)$. In present work the sufficient conditions on function $\alpha(x)$ is given such that the following result takes place.

Theorem 1 *In order that operator $T(a)$ was normal solvable it is necessary and sufficient that*

$$\inf_{x\in R} |a(x)| > 0.$$

Where this condition takes place ones have:

1. *if $\mu_p = 0$ then $T(a)$ is Fredholm in $H^p(R)$;*
2. *if $\mu_p > 0$ then $T(a)$ is invertible from the left in $H^p(R)$ moreover* $\dim \operatorname{coker} T(a) = \infty$*;*
3. *if $\mu_p < 0$ then $T(a)$ is invertible from the right in $H^p(R)$ moreover* $\dim \ker T(a) = \infty$.

Mentioned sufficient conditions on function $\alpha(x)$ are very wide. In particular the following odd functions satisfy ones (here $x > 0$):

$$\alpha(x) = c \cdot x^\beta, \quad \beta > 0; \quad \alpha(x) = c \ln^\beta (x+1), \quad \beta > 1;$$

$$\alpha(x) = c \cdot x^\beta \ln^\gamma (x+1), \quad \beta > 0, \quad \gamma \in \mathrm{R};$$

$$\alpha(x) = c \exp(x^\beta), \quad \beta > 0; \quad \alpha(x) = c \exp(\exp x^\beta), \quad \beta > 0,$$

where $c > 0$.

In this work the matrix analog of operator $T(a)$ is considered in the case when $p(x)$ be a periodic matrix–function.

The Hall polynomials for $\mathbb{A}_n$ and $\mathbb{D}_n$

Dietmar Guhe

Let k be a finite field with cardinality q_k and Q a quiver whose underlying graph is a Dynkin diagram $\overline{Q}$. In the following *finite* means always of finite cardinality.

For finite kQ-modules N_1, N_2 and M, we define $\langle N_1 N_2 \phi\, M\rangle$ to be the number of submodule N of M such that $M/N \cong N_1$ and $N \cong N_2$.

According to Gabriel [Ga], we may identify the set of isomorphism classes $[X]$ of indecomposable finite kQ-modules X with the set Φ^+ of positive roots of the quadratic form corresponding to $\overline{Q}$.

For any map $\alpha : \Phi^+ \to \mathbb{N}_0$, we define a kQ-module $M(\alpha) = \bigoplus_i \alpha(i)[X_i]$. In this way we obtain representatives of all isomorphism classes of finite kQ-modules.

If $\alpha, \beta, \gamma : \Phi^+ \to \mathbb{N}_0$ are maps, then there exists a so called *Hall polynomial* $\phi^\beta_{\alpha\gamma}$, such that $\phi^\beta_{\alpha\gamma}(q_k) = \langle M(\alpha) M(\gamma) \phi\, M(\beta)\rangle$ for any finite field k [R1].

These Hall polynomials are analogous to the classical Hall polynomials dealing with finite p-groups and corresponding partitions. In this case Miller Maley [Mi] has shown that all the Hall polynomials have non-negative coefficients in powers of $q - 1$.

In our setting, the same assertion is true in case $\overline{Q}$ is of type $\mathbb{A}_n$, but not in general: already for $\overline{Q} = \mathbb{D}_4$, the polynomial $q - 2$ occurs as one of the Hall polynomials.

Theorem 1 *Let* $\overline{Q} = \mathbb{A}_n$, $\alpha, \beta, \gamma : \Phi^+ \to \mathbb{N}_0$. *Then for* $\phi^{\beta}_{\alpha,\gamma}(q) = \sum_{i \geq 0} c_i (q - 1)^i$ *all coefficients* c_i *are non-negative.*

Theorem 2 *Let* $\overline{Q} = \mathbb{D}_n$, $\alpha, \beta, \gamma : \Phi^+ \to \mathbb{N}_0$. *Then for* $\phi^{\beta}_{\alpha,\gamma}(q) = \sum_{i \geq 0} c_i (q - 2)^i$ *all coefficients* c_i *are non-negative.*

One may conjecture that the same assertion as in the case $\mathbb{D}_n$ is true for all Dynkin diagrams. Ringel has verified this for $\mathbb{E}_6$ and $\mathbb{E}_7$.

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KOSZUL CONE AND GENERALIZED AUSLANDER REGULAR ALGEBRA

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Smith and Mart'inez-Villa prove that there is a Koszul duality between the Koszul selfinjective algebras and the Koszul generalized Auslander algebras [2, 3]. In [1], the case of dimension 2 Koszul generalized Auslander algebras is studied, via the graded selfinjective algebras of vanishing radical cube.

This paper tries to find a method to deal with the case of higher dimensions. We introduce the Koszul cone and Loewy matrix. And prove that Koszul cone is an invariant cone of the Loewy matrix. Use the spectral radius of Loewy matrix we study the Koszul generalized Auslander regular algebra, we find a necessary condition for them to have the Noetherian property and calculate their Gelfand-Kirilov dimension. As a corollary, we see that a Noetherian connected Koszul generalized Auslander regular algebra is Artin-Shelter regular.

The results are also applied to the selfinjective algebra. We find a class of such algebras sharing the properties of Liu-Schultz example.

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Hochschild cohomology of piecewise hereditary algebras

Dieter Happel

A connected finite-dimensional algebra Λ over a field k is called piecewise hereditary if Λ is derived equivalent to a hereditary abelian k-category $\mathcal{H}$. For any natural number i we denote by $H^i(\Lambda)$ the i-th Hochschild cohomology space of Λ with coefficients in the bimodule Λ. We will show that $H^i(\Lambda) = 0$ for each $i \geq 3$ and that $H^0(\Lambda) = k$, if Λ is piecewise hereditary.

The precise computation of $H^1(\Lambda)$ and $H^2(\Lambda)$ will be carried out in the currently known two (derived equivalence) types of hereditary abelian categories. These are the module categories of connected finite-dimensional hereditary k-algebras and the categories of coherent sheaves on weighted projective lines. If Λ is of the first type it follows that $H^2(\Lambda) = 0$ and if Λ is of the second type then $H^1(\Lambda) = 0$ and $\dim H^2(\Lambda) = t-3$, if $t \geq 3$ denotes the number of weights on the given projective line. As a consequence we obtain that a piecewise hereditary algebra Λ is necessarily representation-infinite if Λ is derived equivalent to the category of coherent sheaves on a weighted projective line with $t \geq 4$ weights. This provides an alternative proof of a result by H. Meltzer.

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Moduli Spaces of Quiver Representations

Lutz Hille

Let Q be a quiver without oriented cycles, d a dimension vector of Q, and denote by $\mathcal{M}(Q,d)^\theta$ the moduli space of θ–semistable representations of Q with dimension vector d. According to King's basic result this is a non-empty algebraic variety only for Schur roots d. Thus this moduli space carries only very less of information.

We present several possibilities to avoid the problem above:

1) We denote by $\bar{Q}(d)$ the quiver obtained from Q by adding for each vertex q in Q additional $d_q - 1$ vertices (q,j) for $j = 1, \ldots d_q - 1$ (for simplicity we denote q by (q, d_q) in $\bar{Q}(d)$ and additional arrows $\alpha_{q,j} : (q,j) \longrightarrow (q, j+1)$ for $j = 1, \ldots d_q - 1$. We call $\bar{Q}(d)$ also the subspace flag quiver.

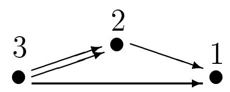
2) If we reverse the direction of the additional arrows in 1), we obtain the quotient flag quiver. Moreover if we add both, subspace vertices and quotient vertices with arrows like above, we obtain the double flag quiver denote by $\bar{\bar{Q}}(d)$. We will consider these quivers only with the fixed dimension vector $d(q,j) = j$ for all $j = 1, \ldots, d_q$.

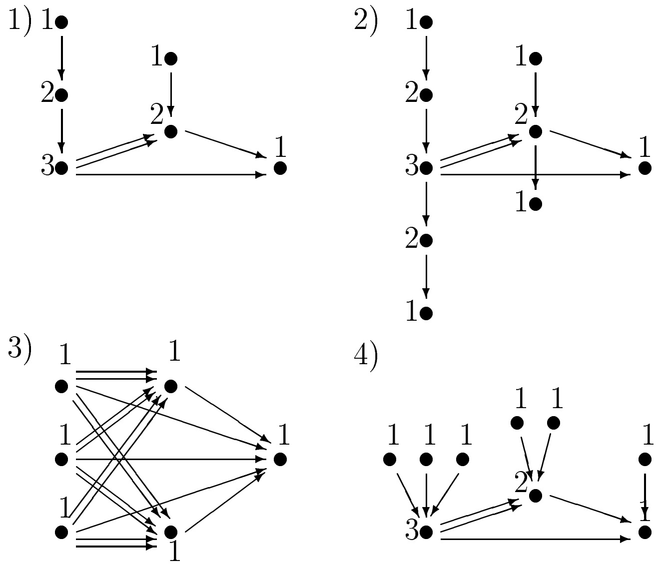
3) Next we consider the d–covering quiver $\widetilde{Q}(d)$ of Q, for any dimension vector d of Q. Its vertices are tuples (q,j) for $j = 1, \ldots, d_q$ and we have arrows $\alpha_{(i,j)} : (q,i) \longrightarrow (p,j)$ for each arrow $\alpha : i \longrightarrow j$ in Q. We consider this quiver with thin sincere representations, that is with dimension vector constant one.

4) The fourth construction is related to modules together with elements or subspaces.

The aim of this talk is to discuss the relations and properties of these moduli spaces. For convenience we present the four constructions above in an example.

Example:





Twisted bimodules and Hochschild cohomology

Thorsten Holm

We discuss Hochschild cohomology of selfinjective algebras of finite type with tree class A_n. These algebras fall into two classes, the wreath-like algebras (containing all Brauer tree algebras) and the Möbius algebras.

Our approach relies on the key observation that certain Ω-translates of such an algebra A (as bimodule over itself) are isomorphic to a twisted bimodule structure on A, i.e., the action on one side is given via an automorphism of finite order. Studying stable homomorphisms on such twisted bimodules makes the computation of Yoneda products accessible and leads to a general description for the subring $HH^{r*}(A)$ of the Hochschild cohomology ring, where $\Omega^r(A)$ is isomorphic to a twisted bimodule.

For wreath-like algebras (where $r = 2$) we obtain presentations for the Hochschild cohomology rings by generators and relations.

For the Möbius algebras $B_{m,k}$ (in the notation of [SW]) we have $r = 2k-1$, and we get results on the subring $HH^{(2k-1)*}(B_{m,k})$.

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Normal extensions and induced characters of 2-groups

Youichi Iida

In Character theory of 2-groups, we know the dihedral groups D_n, the generalized quaternion groups Q_n, the semidihedral groups SD_n and the cyclic groups C_n play an important role. As is wellknown, the 2-groups D_n, Q_n and SD_n have remarkable properties among all nonabelian 2-groups (see [2]). Here we consider a new property which might characterize these three kinds of 2-groups (cf. [1]–[3]).

We have the following

Theorem 1. *Let $H = D_n$ or Q_n or SD_n of order 2^{n+1} and ϕ be a faithful irreducible character of H. Let G be a 2-group containing H as a normal subgroup of index 2^t $(t \geq 1)$. Suppose that the induced character ϕ^G is irreducible. Then $t \leq n-2$. In particular, for any t, such a 2-group G for $H = D_n$ or Q_n is* *uniquely determined. And such a 2-group G for $H = SD_n$ is isomorphic to the 2-group determined for $H = D_n$ or the 2-group determined for $H = Q_n$.*

Indeed, we can determine such 2-groups G from the following Lemma.

Lemma 2. *Let $H = D_n$ or Q_n or SD_n and ϕ be a faithful irreducible character of H. Let G be a 2-group containing H as a normal subgroup. Suppose that the induced character ϕ^G is irreducible. Then G/H is cyclic.*

Furthermore using the theorem we have determined all 2-groups having a faithful irreducible character induced from a faithful irreducible character of a normal cyclic subgroup C_n.

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A characterization of Auslander-Reiten quivers of orders of finite representation type

Osamu Iyama

We shall give a characterization of finite Auslander-Reiten quivers of orders [**I2**], a one-dimensional version of Igusa-Todorov theorem [**IT2**] which treats a zero-dimensional case.

1. Notation Let (Q, τ, d, d') be a locally finite valued translation quiver, where $X \xrightarrow{(d(X,Y),d'(X,Y))} Y$ and the translation $\tau : Q - P(Q) \longrightarrow Q - I(Q)$.

(i) Let $\mathbb{Z}Q$ be the free $\mathbb{Z}$-module generated by the set Q of vertices. For $X \in \mathbb{Z}Q$, let (X, Y) be its coefficient i.e. $X = \sum_{Y \in Q}(X, Y)Y$, and put $X_+ := \sum_{Y \in Q,(X,Y)>0}(X, Y)Y$.

(ii) Define $\theta, \theta^- \in \mathrm{End}_{\mathbb{Z}}(\mathbb{Z}Q)$ by $(\theta X, Y) := d(Y, X)$, $(\theta^- X, Y) := d'(X, Y)$. Let τ also denote its $\mathbb{Z}$-linear extension $\tau \in \mathrm{End}_{\mathbb{Z}}(\mathbb{Z}Q)$, where we have put $\tau X := 0$ if $X \in P(Q)$.

(iii) For a subset S of Q, let $i_S : \mathbb{Z}(Q - S) \longrightarrow \mathbb{Z}Q$, $p_S : \mathbb{Z}Q \longrightarrow \mathbb{Z}(Q - S)$ be the natural injection and projection. For $f \in \mathrm{End}_{\mathbb{Z}}(\mathbb{Z}Q)$, put $f_{Q/S} := p_S \circ f \circ i_S \in \mathrm{End}_{\mathbb{Z}}(\mathbb{Z}(Q - S))$.

2. Main Theorem *For a valued translation quiver Q with a finite number of vertices, the following condition (A) is equivalent to the condition (B).*

(A) There is a complete discrete valuation ring R and an R-order Λ such that Q is isomorphic to the Auslander-Reiten quiver of Λ.

(B)(1) For any $A \in I(Q)$, put $X_1 := \theta^- A$, $X_2 := \theta\theta^- A - A$ and $X_i := \theta X_{i-1} - \tau X_{i-2}$ for $i \geq 3$. Then

(i) $(X_i, Y) \geq 0$ for any $Y \in Q$ and $i \in \mathbb{N}$.

(ii) There exists $n \geq 1$ such that $X_n \in P(Q)$, $X_{n+1} = 0$ and $(X_i, Y) = 0$ for any $Y \in P(Q)$ and i $(1 \leq i \leq n-1)$.

(iii) Let $s(A)$ be the union of the support of X_i $(1 \leq i \leq n)$ i.e. $s(A) := \{Y \in Q \mid (X_i, Y) \neq 0$ by some i $(1 \leq i \leq n)\}$. Then $Q = \bigcup_{A \in I(Q)} s(A)$.

(2) For any $A \in I(Q)$, put $Y_0 := 0$, $Y_1 := A$ and $Y_i := (\theta_{Q/\,P(Q)} Y_{i-1} - \tau_{Q/\,P(Q)} Y_{i-2})_+$ for $i \geq 2$. Then there exists $m \geq 1$ such that $Y_m = 0$.

3. Remarks

(0) Assuming that Λ (in (A)) is a Gorenstein order, the above conditions (B) is equivalent to the ones given by Wiedemann [**W**].

(1) To prove (B)$\Longrightarrow$(A), we shall, as in [**IT2**] or [**W**], construct a (complete) mesh category $\widehat{M}_Q$ form Q, then show that $\widehat{M}_Q$ plays the role of the Auslander order of Λ in (A).

However, $\widehat{M}_Q$ is an object constructed over a residual field, and sheer difficulty in our case lies in showing that $\widehat{M}_Q$ can be an order over a complete discrete valuation ring. This difficulty does not exists in [**IT2**] case, and bypassed by half-explicit construction in [**W**], aided strongly by the Gorenstein assumption.

(2) What we have done is, among other things, to establish *Radical layers Theorem* of Igusa-Todorov [**IT**] and *Rejection Lemma* of Drozd-Kirichenko ([**DK**] and [**I**]), for somewhat wider class of categories which we call *τ-category* (=Krull-Schmidt category with almost split sequences, so to speak). By Rejection Lemma (for a lattice categoty $\operatorname{lat}\Lambda$), we mean to give a criterion for a full subcategory $\mathcal{C}'$ of $\operatorname{lat}\Lambda$ to be a lattice category $\mathcal{C}' = \operatorname{lat}\Lambda'$ by some overorder Λ' of Λ. Recall that a very useful Rejection Lemma of [**DK**] is a very special case where Λ' loses only one indecomposable lattice from Λ.

(3) Once, this general theory is granted, we can apply to $\widehat{M}_Q$, the most effective method to study orders, namely to reduce the theory of Λ to that of some overorder Λ' of Λ. Eventually, we can reduce the proof of (B)$\Longrightarrow$(A) to the simple enough case where $\widehat{M}_Q$ is maximalorderlike.

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Otto Kerner
Almost serial regular Auslander-Reiten components for wild hereditary algebras

Let H be the path-algebra of some connected wild quiver $\mathcal{Q}$ and denote by $H-\mathrm{reg}$ the category of regular H-modules. If M is a regular H-module, a filtration

$$M = M_0 \supset M_1 \supset \cdots \supset M_{r-1} \supset M_r = 0$$

is called a *regular composition series* of M, if all the subquotients M_i/M_{i+1} are nonzero and regular for $0 \leq i < r$ and there is no proper refinement of this filtration with regular subquotients.

Normally neither the length r nor the factors M_i/M_{i+1} of a regular composition series are unique.

If X is a quasi-simple regular module, $X(i)$ denotes the indecomposable regular with quasi-socle X and quasi-length i.

A regular component $\mathcal{C}$ in the Auslander-Reiten quiver $\Gamma(H)$ is called *almost serial*, provided for any indecomposable $X(n) \in \mathcal{C}$, with quasi-socle X and quasi-length n, the filtration

$$X(n) \supset X(n-1) \supset \cdots X(2) \supset X(1) = X \supset 0$$

given by the components $\mathcal{C}$ is the *unique* regular composition series of $X(n)$.

Theorem. *Let $H = k\mathcal{Q}$ be a connected wild hereditary algebra. If the quiver $\mathcal{Q}$ is not a wild star $\mathrm{T}_{a,b,c}$ with 3 arms, and if the growth number ρ_H of H is at least 2, then $\Gamma(H)$ contains almost serial components.*

The path-algebras of quivers of type $\mathrm{T}_{2,3,7}$ do not have almost serial components. But I guess, the assumption $\rho_H > 2$ only is technical, caused by the proof.

The proof is done by first considering another class of distinguished regular components in $\Gamma(H)$, called *filtration closed*: If $\mathcal{C}$ is a regular component in $\Gamma(H)$ and for any $X \in \mathrm{add}\,\mathcal{C}$ all short exact exact sequences $0 \to U \to X \to V \to 0$ with U, V regular, split, then $\mathcal{C}$ is called filtration closed. The theorem is shown, by proving that filtration closed regular components are almost serial, provided $\rho_H > 2$.

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Orthogonal subcategories and computation of almost split sequences for relatively projective and prinjective modules

Mark Kleiner

This is joint work with Efrén Pérez. Let Λ be an artin algebra with A-R translation τ. For $\mathcal{W} \subset mod\,\Lambda$, put ${}_\perp\mathcal{W} = \{X \in mod\,\Lambda \mid Hom\,(X,Y) = 0\ \forall Y \in \mathcal{W}\}$ and ${}^\perp\mathcal{W} = \{X \in mod\,\Lambda \mid Ext^1(X,Y) = 0\ \forall Y \in \mathcal{W}\}$; $\mathcal{W}_\perp$ and $\mathcal{W}^\perp$ are defined analogously. For $M \in mod\,\Lambda$, $r_\mathcal{W}M$ is a minimal right $\mathcal{W}$-approximation of M, $t_\mathcal{W}M$ ($ct_\mathcal{W}M$) is the trace (cotrace) of $\mathcal{W}$ in M. We apply the main result of [1].

Theorem 1. *Let ${}_\perp(\mathcal{W}_\perp) \subset ({}^\perp\mathcal{W})^\perp$. Then ${}^\perp\mathcal{W}$ has almost split sequences on the left. If $0 \to L \to M \to N \to 0$ is almost split in ${}^\perp\mathcal{W}$, then $N \cong \tau^{-1}ct_{\mathcal{W}_\perp}L$ and $L \cong r_{{}^\perp\mathcal{W}}\tau N$.*

Corollary 2. *Let Γ be an hereditary subalgebra of Λ with Λ_Γ projective. If the category $p(\Lambda,\Gamma)$ of left relatively projective modules is extension closed, it has almost split sequences. If $0 \to L \to M \to N \to 0$ is almost split in $p(\Lambda,\Gamma)$, then $N \cong \tau^{-1}ct_{p(\Lambda,\Gamma)^\perp}L$ and $L \cong r_{p(\Lambda,\Gamma)}\tau N$.*

Theorem 3. *Let $\mathcal{V},\mathcal{W} \subset mod\,\Lambda$ satisfy $Ext^1(\mathcal{V},\mathcal{W}) = 0$, ${}_\perp(\mathcal{W}_\perp) \subset ({}^\perp\mathcal{W})^\perp$, and $({}_\perp\mathcal{V})_\perp \subset {}^\perp(\mathcal{V}^\perp)$, with $\mathcal{W}$ covariantly finite extension closed. If $Ext^2(({}_\perp\mathcal{V})_\perp,\mathcal{W}) = 0$, for example, if ${}^\perp\mathcal{W}$ is closed under kernels of surjections, then $\mathcal{V}^\perp \cap {}^\perp\mathcal{W}$ has almost split sequences on the right and, for an almost split sequence $0 \to L \to M \to N \to 0$ in $\mathcal{V}^\perp \cap {}^\perp\mathcal{W}$, we have $L \cong r_{{}^\perp\mathcal{W}}\tau t_{{}_\perp\mathcal{V}}N$.*

As a special case, Corollary 2 applies to representations of free bocses. Theorems 1 and 3 extend the results of [2].

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On algebras with complete preprojective components

Peter Dräxler and **Klara Kögerler**

Let A be a basic finite-dimensional algebra over an algebraically closed field such that the quiver Q of A is directed but not necessarily connected. We consider the category $mod - A$ of finite-dimensional right A-modules and the associated Auslander-Reiten quiver Γ_A. The algebra A is said to have *complete preprojective components* if for each vertex x of Q the corresponding indecomposable projective module P_x belongs to a preprojective component of Γ_A.

We denote by T the subset of Q given by all vertices x such that P_x has the property that for any natural number n there exists a path $X_n \to X_{n-1} \to \cdots \to X_0 = P_x$ of non-zero radical morphisms between indecomposables X_i in $mod - A$. Clearly, T is successor closed and consequently the complement Q^* of T in Q is predecessor closed in Q. Moreover, Q^* contains all sources of Q. We denote by A^* the full subcategory of A given by Q^*.

Theorem *The algebra A^* has complete preprojective components.*

The proof rests on an algorithm which produces in a suitably ordered way the subcategory A^* together with initial pieces of the preprojective components. To make sure that the algorithm really stops, one needs the results of [1] on occurrance of indecomposables in preprojective components.

Corollary *Let R be a connected component of Q^* and B the associated subcategory of A. Let $\mathcal{C}$ be the complete preprojective component of Γ_B. Then $\mathcal{C}$ is a preprojective component of Γ_A if and only if for each direct neighbour x of R in Q none of the indecomposable direct summands of the radical of P_x appears in $\mathcal{C}$.*

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Sincere posets of finite prinjective type

Justyna Kosakowska

Let K be a field and let $(I, \preceq)$ be a finite poset (i.e. partially ordered set) with respect to the partial order $\preceq$. The poset I will be called an r-**peak poset** if $|\max I| = r$, where $|X|$ denotes the cardinality of the set X. We denote by KI the **incidence K-algebra** of the poset I, that is, KI is the K-subalgebra of the full $I \times I$ matrix algebra ${}_I(K)$ consisting of all matrices $[\lambda_{ij}]$ in ${}_I(K)$ such that $\lambda_{ij} = 0$ if $i \not\preceq j$ in $(I, \preceq)$. Given $j \in I$ we denote by $e_j \in KI$ the standard primitive idempotent corresponding to j. Right KI-modules will be identified with the systems $X = (X_i;\ {}_jh_i)_{i,j \in I}$ where $X_i = Xe_i$ is a finite dimensional vector space over K and ${}_jh_i : X_i \to X_j$, $i \prec j$, are K-linear maps such that ${}_ih_i =$id and ${}_th_j \cdot {}_jh_i = {}_th_i$ for all $i \prec j \prec t$ in I. The right KI-module X is said to be **prinjective** if X is finitely generated and the right module Xe_- over the algebra $KI^- \cong e_-(KI)e_-$ is projective, where $I^- = I \setminus \max I$ and $e_- = \sum_{j \in I^-} e_j$. We define the poset I to be of **finite prinjective type** if the number of isomorphism classes of indecomposable prinjective KI-modules is finite. The poset I is called **sincere** if there exists an indecomposable sincere prinjective KI-module.

We shall present the following results.

THEOREM 1. *For any integer r there are only finitely many sincere r-peak posets of finite prinjective type.*

THEOREM 2. *Let I be a sincere poset of finite prinjective type. Then there exists $j \in I$ such that the poset $J = I \setminus \{j\}$ is a sincere poset.*

We shall present an algorithm for a construction of sincere r-peak posets of finite prinjective type. Applying this algorithm we can construct sincere r-peak posets I of finite prinjective type, such that $|I| = n+1$, from sincere r-peak posets J of finite prinjective type, such that $|J| = n$. Moreover we give the complete list of all sincere two-peak posets I of finite prinjective type and their sincere prinjective modules.

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Broué conjecture on p-blocks of finite groups with abelian defect groups

Shigeo Koshitani

M. Broué has given the following conjecture which is on modular representation theory of finite groups. Namely,

Conjecture 1. (M. Broué) *Let p be a prime number. Let A and B be the principal p-blocks of finite groups G and H, where $H = N_G(P)$ is the normalizer of a Sylow p-subgroup P in G. Then, is it true that these p-blocks A and B are derived equivalent provided P is abelian?*

I am going to talk on some positive partial answers to the conjecture for the case that $p = 3$ and the Sylow 3-subgroup P is elementary abelian of order 9. This is a joint work with colleagues of mine.

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Derived Dynkin Extensions

Peter Dräxler and **Amrey Krause**

The finite dimensional tame hereditary algebras are associated with the extended Dynkin diagrams. An indecomposable module over such an algebra is either preprojective or preinjective or lies in a family of tubes whose tubular type is the corresponding Dynkin diagram. The study of one-point extensions by simple regular modules in such tubes was initiated in [3].

We generalize this approach by starting out with algebras which are derived equivalent to a tame hereditary algebra and considering one-point extensions by modules which are simple regular in tubes in the derived category. If the obtained tubular type is again a Dynkin diagram these algebras are called *derived Dynkin extensions.*

Theorem 1. *A representation infinite algebra is derived equivalent to a tame hereditary algebra if and only if it is an iterated derived Dynkin extension of a tame concealed algebra.*

As application we get a new proof of a theorem in [1].

Corollary 2. *The representation infinite piecewise tame hereditary algebras are precisely the domestic tubular branch enlargements of the tame concealed algebras.*

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Decomposing thick subcategories of the stable module category

Henning Krause

Let $\underline{\mathrm{mod}}\, kG$ be the stable category of finitely generated modular representations of a finite group G over a field k. In my talk, I shall discuss a Krull-Remak-Schmidt theorem for thick subcategories of $\underline{\mathrm{mod}}\, kG$. It is shown that every thick tensor-ideal $\mathcal{C}$ of $\underline{\mathrm{mod}}\, kG$ (i.e. a thick subcategory which is a tensor ideal) has a (usually infinite) unique decomposition $\mathcal{C} = \coprod_{i \in I} \mathcal{C}_i$ into indecomposable thick tensor-ideals. This decomposition follows from a decomposition of the corresponding idempotent kG-module $E_{\mathcal{C}}$ into indecomposable modules. If $\mathcal{C} = \mathcal{C}_W$ is the thick tensor-ideal corresponding to a closed homogeneous subvariety W of the maximal ideal spectrum of the cohomology ring $H^*(G,k)$, then the decomposition of $\mathcal{C}$ reflects the decomposition $W = \bigcup_{i=1}^n W_i$ of W into connected components.

The results which are presented in this talk are based on ideas from stable homotopy theory. In a number of papers, Benson, Carlson, and Rickard exploited the analogy between the stable homotopy category and the stable module category of a finite group. However, there is a crucial difference: The centre of the stable module category $\underline{\mathrm{Mod}}\, kG$ is the underlying field k whereas the centre of the stable homotopy category is the ring of integers which is *not* a local ring. Any Krull-Remak-Schmidt theorem requires local endomorphism rings for the indecomposable objects and, therefore our results present a genuinely new phenomenon.

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$\mathbf{Z}\mathcal{S}_n$ and modular morphisms

Matthias Künzer

Let n be a natural number. The integral group ring of the symmetric group on n letters, $\mathbf{Z}\mathcal{S}_n$, can be embedded into a direct product of integral matrix rings $\Gamma := \prod_\lambda (\mathbf{Z})_{n_\lambda}$ via a tuple of integrally realized irreducible ordinary representations, such as e.g. the Specht lattices. We identify $\mathbf{Z}\mathcal{S}_n$ with the image of the embedding. The index of $\mathbf{Z}\mathcal{S}_n$ in Γ as abelian groups being $\sqrt{n!^{n!}/\prod_\lambda n_\lambda^{\binom{n_\lambda}{2}}}$ [S 6.2 prop. 11 + 10%], the question of its description via ties, i.e. via congruences, arises.

Working locally, we filter the indecomposable projectives of $\mathbf{Z}_{(p)}\mathcal{S}_n$, which are contained in a certain direct sum of Specht lattices, $P_j \xrightarrow{\iota_j} \bigoplus_\gamma S^{\lambda_{j,\gamma}}$, via intersection with partial direct sums. Since an element of $\Gamma_{(p)}$ is in $\mathbf{Z}_{(p)}\mathcal{S}_n$ iff its left multiplication restricts to the indecomposable summands of $\mathbf{Z}_{(p)}\mathcal{S}_n$, conjugations with the rationally invertible ι_j give a complete system of ties.

Writing the matrix of ι_j in a basis respecting the filtration of P_j yields block lower triangular matrices M_j interpretable as consisting of inclusions of simple lattices on the block main diagonal and of $(\mathbf{Z}/n!)\mathcal{S}_n$-morphisms in the block columns, the main diagonal block excepted.

Conversely, any such morphism and any such inclusion yield ties, being part of some M_j or not. Due to the explicit tableau combinatorics of the Specht lattices, it is straightforward to transform this idea into generic partial results for the initial question.

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Non-commutative Factorial Domains and associated Curves

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If the base field is algebraically closed the parametrizing space of a separating tubular family over a canonical algebra is determined by a *commutative* graded factorial domain, as shown by Geigle and Lenzing. For non-algebraically closed base field such a "projective coordinate algebra" will in general not be commutative.

We introduce the concept of a *non-commutative* graded factorial domain of Krull dimension two in order to describe the parametrizing space $\mathbb{X}$ for a large class of canonical algebras over non-algebraically closed base fields. Our main defining property for factoriality is that each (two-sided) homogeneous prime ideal of height one is of the form $R\pi = \pi R$, where π is homogeneous. Such elements π are called prime elements. Each prime element is a product of irreducible elements, hence contrary to the commutative case the notions of irreducible and prime elements differ.

We consider the quotient category $coh(\mathbb{X}) := \frac{\text{mod}^H(R)}{\text{mod}_0^H(R)}$, modulo the H-graded finite length modules, and let $\mathcal{O}$ be the image of R. If we further assume $Ext^1(\mathcal{O}, \mathcal{O}) = 0$, then the parametrizing space $\mathbb{X}$ is an exceptional curve in the sense of Lenzing, that is, $coh(\mathbb{X})$ contains a tilting bundle whose endomorphism ring is a canonical algebra. By graded factoriality we get

Proposition 1. *Each line bundle is isomorphic to some $\mathcal{O}(h)$ ($h \in H$). Moreover, each morphism between line bundles is induced by multiplication with homogeneous elements in R.*

We establish a bijection between prime ideals (of height one) in R and (Auslander-Reiten orbits of) simple sheaves via universal exact sequences. For simplicity we formulate the result only for the homogeneous case (that is, each Auslander-Reiten orbit of a simple sheaf consists of precisely one object):

Theorem 2. *Let R be graded factorial and homogeneous.*

(a) *Let $\pi \in R$ be a prime element of degree h, and assume that π has e irreducible factors. Then multiplication by π induces an exact sequence*

$$0 \longrightarrow \mathcal{O} \xrightarrow{\cdot\pi} \mathcal{O}(h) \longrightarrow \mathcal{S}^e \longrightarrow 0$$

where $\mathcal{S}$ is a simple sheaf, and it is the $\mathcal{S}$-universal exact sequence of $\mathcal{O}$.

(b) *Conversely, if $\mathcal{S}$ is a simple sheaf, then the universal exact sequence of $\mathcal{S}$ is induced by multiplication with a prime element.*

By so-called insertion of weights in prime elements it is possible to treat also the non-homogeneous situations. The result also gives new insight in the structure of preprojective algebras.

Two-orbifolds of tubular type

Helmut Lenzing

We work over the base field $\mathbb{R}$ of real numbers. A two-orbifold $\mathbb{X}$ in the sense of W. Thurston admits the structure of a non-singular projective, possibly weighted, curve. In more detail, the category $\mathcal{H} = \mathrm{coh}\,(\mathbb{X})$ of coherent sheaves on $\mathbb{X}$ satisfies among others the following properties: (1) $\mathcal{H}$ is an abelian $\mathbb{R}$-category with finite dimensional morphism and extension spaces; (2) $\mathcal{H}$ is hereditary and noetherian; (3) $\mathcal{H}$ satisfies Serre duality in the form $D\mathrm{Ext}^1\,(X,Y) = \mathrm{Hom}\,(Y,\tau X)$ for an automorphism $\tau : \mathcal{H} \to \mathcal{H}$, in particular has almost-split sequences with Auslander-Reiten translation τ.

Extending the concept we say that each category $\mathcal{H}$ with the properties (1), (2), (3) and some additional technical conditions defines a *generalized two-orbifold.*

Proposition 1. *Let $\mathcal{H}$ be a generalized two-orbifold, and denote by $\mathcal{H}_0$ its full subcategory of objects of finite length. Then the quotient category $\mathcal{H}/\mathcal{H}_0$ is equivalent to the category* $\mathrm{mod}\,(D)$ *of finite dimensional vector spaces over a skew field D which is module-finite over its centre K which is a function field in one variable over $\mathbb{R}$.*

We distinguish two cases: generalized 2-orbifolds, where D is commutative, which are called *generically commutative* and the remaining ones which are *generically non-commutative.* Two-orbifolds in Thurston's sense are generically commutative.

A generalized 2-orbifold $\mathcal{H}$ is called *tubular* if each AR-orbit of $\mathcal{H}$ is a (standard, stable) tube of finite rank. The aim of the talk is to classify these:

Theorem 2. *Each tubular two-orbifold belongs to one of the following three classes*

(a) the generically commutative ones. These form 17 classes corresponding bijectively to the wallpaper patterns.

(b) those with a tilting object. These form 31 classes corresponding bijectively to the canonical algebras of tubular type.

(c) the generically non-commutative ones without a tilting object. This class is non-empty. More specifically there is a tubular orbifold $\mathbb{Y}$ of type (c) related to the Klein bottle $\mathbb{X}$, which is a tubular orbifold of type (a), such that $\mathrm{coh}\,(\mathbb{X})$ *and* $\mathrm{coh}\,(\mathbb{Y})$ *are derived-equivalent.*

The classes (a) and (b) are not disjoint; their intersection yields 13 classes.

Hochschild Cohomology of Truncated Quiver Algebras

Ana Claudia Locateli

Consider a truncated quiver algebra $A = kQ/J^N$, where k is a field, Q is a finite quiver, and J is the two-sided ideal of kQ generated by the arrows.

We compute the dimensions as k-vector spaces of the Hochschild Cohomology groups of A in terms of the quiver Q when k is a field with characteristic zero. We also prove that in this case the Hochschild cohomology ring of A is finite dimensional if and only if Q has no oriented cycles.

In the case where Q is a single cycle, we give a description of the product on the Hochschild Cohomology ring of A, H(A), and compute the dimensions of the Hochschild groups with no restrictions on the characteristic of k. In this case we also prove that the ring H(A) is always finitely generated and commutative.

Dag Madsen

Homological properties of almost split sequences

Let Λ be an artin algebra and denote by $\operatorname{mod}\Lambda$ the finitely generated Λ-modules. When $0 \to A \to B \to C \to 0$ is an exact sequence in $\operatorname{mod}\Lambda$, we have $\operatorname{pd} B \leq \max\{\operatorname{pd} A, \operatorname{pd} C\}$. If the sequence is split, equality holds. In this talk we discuss under which conditions the equality holds for almost split sequences.

If Λ is a Nakayama algebra, we have a complete description of when equality doesn't hold. Part of the answer is given by the following proposition:

Proposition 1. *Let Λ be an indecomposable Nakayama algebra with finite global dimension. Let $0 \to A \to B \to C \to 0$ be an almost split sequence. Then $\operatorname{pd} B < \max\{\operatorname{pd} A, \operatorname{pd} C\}$ if and only if $A \simeq \Omega^{-(n-1)}\Omega^n S$ or $A \simeq \Omega^{-n}\Omega^n S$ where S is a simple module with $\operatorname{pd} S = n$ an even number.*

We also give some similar results for arbitrary artin algebras.

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Coherent almost cyclic components of an Auslander-Reiten quiver

Piotr Malicki
(joint work with Andrzej Skowroński)

Let A be an artin algebra over a commutative artin ring R. The Auslander-Reiten quiver Γ_A of A is an important combinatorial and homological invariant of the category $\operatorname{mod} A$ of finitely generated right A-modules, although in general it describes only the quotient category $\operatorname{mod} A/\operatorname{rad}^\infty(\operatorname{mod} A)$. We shall discuss the shape of connected components of Γ_A containing oriented cycles.

A vertex of Γ_A lying on an orieted cycle in Γ_A is said to be cyclic. A connected component Γ of Γ_A is said to be **almost cyclic** (respectively, **cyclic**) if all but finitely many (respectively, all) vertices of Γ are cyclic. Moreover, Γ is said to be **coherent** if each projective vertex (respectively, injective vertex) of Γ is the source (respectively, sink) of an infinite sectional path of Γ_A. We shall introduce a new type of translation quiver called a **generalized multicoil**. Roughly speaking a generalized multicoil is a connected translation quiver obtained from a finite family of stable tubes by a finite sequence of admissible operations. All generalized multicoils are coherent and almost cyclic. Special cases of generalized coils are tubes, quasi-tubes and coils, playing an important role in the representation theory of tame algebras. We note also that for any generalized multicoil Γ there exists an artin algebra Λ whose Auslander-Reiten quiver admits a connected component isomorphic to Γ.

Our main result describes the shape of an arbitrary coherent and almost cyclic component of Γ_A.

Theorem 1. *Let Γ be a connected component of Γ_A. Then Γ is coherent and almost cyclic if and only if Γ is a generalized multicoil. In particular, Γ is coherent and cyclic if and only if Γ is a cyclic generalized multicoil.*

We shall derive also some consequences of the above theorem on the neighbourhood of cyclic vertices in coherent almost cyclic components of Γ_A.

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INTEGRAL REPRESENTATIONS OF SOME FINITE GROUPS OVER DEDEKIND RINGS.

D. A. Malinin

We consider different problems of integral representations of finite groups arising under the ground ring extensions.

In [1] representations of arbitrary non-commutative p-groups over the maximal orders of local fields are considered.An explicit construction of an infinite number of pairwise non-equivalent (realized over the ring of integers of the field $Q(\xi_{p^\infty})$) absolutely irreducible representations for any non-commutative p-group is given.

Let K/Q_p be a finite extension of rational p-adic number field, let O_K, e, and J be its ring of integers, ramification index, and the prime ideal of O_K respectively. Let $J = \pi O_K$, $\pi \in O_K$. Let $G = G_l \supset G_{l-1} \supset \cdots \supset G_0 = \{I_n\}$ be the lower central series of a p-group $G \subset GL_n(O_K)$, and for all $g \in G$ the identity matrix $I_n \equiv g\,(mod J^w)$ for some positive integer w. Then $h \equiv I_n(mod J^{iw})$ for all matrices $h \in G_{l-i+1}$. The condition $ew^{-1} < i(p-1)$ implies that the group $G_{l-i+1}, i = 1, 2, \ldots, l$ is trivial. It appears that this condition can not be improved. Let G^0 be a group generated by

$$a, b_1, \ldots, b_l,\ b_i^p = 1,\ b_i b_j = b_j b_i, \quad i, j = 1, 2, \ldots, l,$$

$$ab_1 = b_1 a,\ b_{i-1} = b_i a b_i^{-1} a^{-1}, \quad i = 2, 3, \ldots, l, a^n = 1; \quad n = p^\lambda \geq l > p^{\lambda-1}.$$

Let ξ_p denote a primitive p-root of 1. The character $\Delta(b_1) = \xi_p$, $\Delta(b_i) = 1, i = 2, 3, \ldots, l$ of an abelian subgroup H generated by $b_1, b_2, \ldots, b_l$ and the decomposition $G^0 = 1\cdot H + a^{n-1}\cdot H + \cdots + a\cdot H$ define the induced representation $R = Ind_H^{G^0} \Delta$ of G^0. Let

$$C = \sum_{n \geq i \geq j \geq 1} (-1)^{i-j} C_{n-j}^{i-j} e_{ij} \ (e_{ij} \text{ are matrix units}),$$

$$D_\pi = diag(1, \pi, \pi^2, \ldots, \pi^{n-1}).$$

Let O be the ring of integers of $\Lambda = Q_p(\bigcup_{m=1}^\infty \xi_{p^m})$, $K \subset \Lambda$, and $e > e_0 = e_0(l)$.

Theorem. The representation $R_\pi = D_\pi^{-1} C^{-1} R C D_\pi$ of the group G^0 is a faithful absolutely irreducible representation $G^0 \to GL_n(O_K)$ by matrices contained in the kernel of reduction $(mod J)$ and these representations are pairwise non-equivalent in $GL_n(O)$.

J.G.Thompson in 1976 considered finitely generated torsion free ZG-modules M of dimension $n > 1$, for a finite group G, such that M/pM are irreducible for all primes $p \in Z$.There are very few examples of pairs G, M, and they can exist only for n divisible by 8.We study what happens if Z is replaced by the ring of integers O_E of E and all reductions modulo prime ideals of O_E are (absolutely) irreducible. We consider possible realization fields for appropriate representations of G, admittable dimensions and clarify the situation, in particular, for quadratic extensions $E = Q(\sqrt{d})$.

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Gradings and Derivations

D. Farkas (Virginia Tech, USA), C. Geiss (Unam, Mexico), E. Green (Virginia Tech, USA), E. N. Marcos, (IME-USP, Brazil)

Presented by E. N. Marcos.

Given a k- algebra Λ and a graduation of Λ over the underlying abelian group k^+ of k, then one can automatically define a derivation on Λ by declaring, $d(a) = ia$, for homogeneous elements of degree i. Such derivation is called diagonalizable derivation. The derivations on the k vector space generated by all the diagonalizable derivations, will be called Eulerian. The set of Eulerian derivations has some interesting properties. We call an algebra Euleurian when every derivation is Euleurian. We show that some classes of algebras consist of Eulerian algebras. We cite some:

1. The finite dimensional quotients of quiver algebras by monomial ideal over field of characteristic zero.

2. The finite dimensional algebras which are of the form $k[x_1, \cdots, x_n]/I$, were $k[x_1, \cdots, x_n]$ is the ring of polynomials in n unknowns, k is a field of characteristic zero and I is generated by monomials.

We know some examples showing that the theorems above are not valid in arbitrary characteristic. We also have some hypothesis on the ideals which would make them valid in finite characteristic.

The Green Algebras of $k[X]$

Alex Martsinkovsky

This is a preliminary report on joint work in progress with A. Vlassov.

The original goal of the current project was to find the Jordan normal form of the Kronecker product of two Jordan blocks (over the field of complex numbers) using function-theoretic methods. While this goal remains elusive, we have found a remarkably simple algebraic solution.

We begin by making, in an obvious way, a monoidal category from the category of modules over a bialgebra. The Kronecker product fits this framework. We classify all bialgebra structures on $k[X]$ and then concentrate on $k[X]$-modules of finite length. For each bialgebra structure on $k[X]$ we compute the

Jordan normal form of the corresponding tensor product of Jordan blocks. The proof takes only a few lines and requires no more than freshman linear algebra (the definitions of a vector space and of the Jordan normal form).

Knowing the decomposition for each tensor product, we look at the corresponding Green algebra. Using freshman calculus, we investigate its ideal structure.

Besides being very simple, our method seems to work, at least partially, in positive characteristics.

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Simple Harish-Chandra modules over $\mathbb{Q}$ - Virasoro algebra

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Let $\mathbb{Q}$ ($\mathbb{C}$) denote the field of rational (complex) numbers and let $\mathfrak{G}$ be a Lie algebra with the basis $\{e(x)\,|\,x \in \mathbb{Q}\} \cup \{c\}$ and the Lie brackets given by

$$[e(x), e(y)] = (y-x)e(x+y) + \delta_{x,-y}\frac{x^3-x}{12}c; \quad x, y \in \mathbb{Q};$$

$$[c, e(x)] = 0, \quad x \in \mathbb{Q}$$

(here $\delta_{x,-y}$ is the Kronecker symbol). This algebra is a natural generalization of the Virasoro algebras and will is called $\mathbb{Q}$ - Virasoro algebra. A $\mathfrak{G}$-module V is called weight module if it is $e(0)$-diagonalizable. An eigenspace of the operator $e(0)$ will be called a weight space.

An intermediate series for $\mathfrak{G}$ is a family of modules $V(a,b)$, $a, b \in \mathbb{C}$ defined as follows: $V(a,b)$ has the basis v_x, $x \in \mathbb{Q}$ and the action of the basis of $\mathfrak{G}$ is defined by

$$e(x)v(y) = (ax + y + b)v(x+y), \quad x, y \in \mathbb{Q}. \tag{1}$$

$V(a,b)$ is a simple module if $a \neq 0, 1$ or $b \notin \mathbb{Q}$. In the remaining case it has two simple subquotients, and, moreover, one of them is a trivial $\mathfrak{G}$-module. We will say that the intermediate series also contains all simple subquotients of $V(a,b)$.

Our main result is the following:

Theorem 1. *Each simple weight module with finite-dimensional weight spaces over $\mathfrak{G}$ is a module from the intermediate series.*

Exceptional sequences determined by their Cartan matrix

Hagen Meltzer

This talk is a report on a joint work with Helmut Lenzing.

We study whether a derived equivalent to a hereditary or a canonical algebra is determined in its derived class up to isomorphism by its Cartan matrix. More generally, we investigate whether two complete exceptional sequences in the derived category with the same Cartan matrix do have isomorphic full endomorphism rings.

It turns out that this is true in the hereditary case. On the other hand we give an example of two wild algebras which are derived equivalent to the same canonical algebra, have the same Cartan matrix but which are not isomorphic. However, we can show the following result.

Theorem 1. *Let Λ be a canonical algebra over an algebraically closed field. Then there are only finitely many (up to isomorphism) algebras which are derived equivalent to Λ and have identical Cartan matrices.*

The proof is based on the transitivity of the braid group action on the set of complete exceptional sequences in the category of coherent sheaves for the associated weighted projective line and uses, moreover, information on the automorphism group of the derived category. We also obtain bounds for the number of algebras, in particular we get the following corollary.

Corollary 2. *Let $\Lambda = \Lambda(\mathbf{p}, \lambda)$ be a canonical algebra over an algebraically closed field where $\mathbf{p} = (p_1, p_2, \ldots, p_t)$ and the p_i are pairwise different. Then an algebra which is derived equivalent to Λ is uniquely determined by its Cartan matrix.*

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Generators of the Grothendieck group

Héctor Merklen

The results we present were obtained in joint work with Eduardo N. Marcos and María Inés Platzeck.

Let Λ be an R-Artin algebra, where R is a commutative artininian ring, and let modΛ the category of finitely generated Λ-modules.

M.C.R. Butler proved in [**B**] that, if Λ is of finite representation type, then the Auslander-Reiten sequences give a generating set for the defining relations of the Grothendieck Group of modΛ.

A little time afterwards, em [**A**], M. Auslander proved the converse of that proposition.

We are going to show results that extend that Auslander-Butler theorem to certain subcategories, even to certain connected components, of modΛ.

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On a classification problem and Hochschild cohomology of local extensions

Sandra Michelena

All the algebras considered are finite dimensional algebras over an algebraically closed field K.

Following J. A de la Peña and M. Martins, for two K - algebras A, R with R a local ring and M an $A-R-$ bimodule the triangular matrix ring $B = \begin{pmatrix} R & 0 \\ M & A \end{pmatrix}$ is called the local extension of A by M. Artin algebras with the property that all their idempotent two sided ideals are projective left Λ - modules, which we will call (IIP) algebras, can be gotten from a local ring by means of succesive local extensions. One of our main results is the **complete classification of the (IIP) algebras of finite representation type**. It is given by a list of quivers with relations.

On the other hand, we study the Hochschild cohomology of local extensions. For a local extension B and $P_e = B.\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ we prove:

Theorem 1 *There is a long exact sequence of K - vector spaces*

$$\begin{aligned} 0 \to \quad & Ann\, M_R \cap Center(R) \longrightarrow H^0(B) \longrightarrow H^0(A) \longrightarrow Ext^1_{B\otimes_K R^{op}}(R, P_e) \\ \to \quad & \ldots\ldots \longrightarrow H^i(B) \longrightarrow H^i(A) \longrightarrow Ext^{i+1}_{B\otimes_K R^{op}}(R, P_e) \longrightarrow \ldots \end{aligned}$$

Theorem 2 *There is a long exact sequence of K - vector spaces*

$$\begin{aligned} 0 \to \quad & Ann\, M_R \cap Center(R) \longrightarrow H^0(R) \longrightarrow Ext^1_{B\otimes_K R^{op}}(R, M) \longrightarrow \ldots \\ \to \quad & H^i(R) \longrightarrow Ext^{i+1}_{B\otimes_K R^{op}}(R, M) \longrightarrow Ext^{i+1}_{B\otimes_K R^{op}}(R, P_e) \longrightarrow \ldots \end{aligned}$$

The first theorem generalizes D. Happel's result for one point extensions. The second theorem helps to compute the groups $Ext^i_{B\otimes R^{op}}(R, P_e)$.

As an application of these results we assume that $R = K[\alpha]/ < \alpha^i >$ and we consider local extensions $B = KQ/I$ such that I is generated by relations not involving arrows starting at the new vertex e which are no loops. In this case the above results yield the sequence

$$\begin{aligned} 0 \to \quad & H^0(B) \quad \longrightarrow H^0(A) \longrightarrow Ext^1_{B\otimes_K R^{op}}(R, P_e) \longrightarrow H^1(B) \longrightarrow H^1(A) \\ \to \quad & H^2(R) \quad \longrightarrow \ldots \longrightarrow H^j(B) \longrightarrow H^j(A) \longrightarrow H^{j+1}(R) \longrightarrow \ldots \end{aligned}$$

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Quasi-serial components of strictly wild algebras

Hiroshi Nagase

In [1] Crawley-Boevey conjectured that if A is wild, then there exists a natural number d such that infinitely many isoclasses of d-dimensional indecomposable A-module do not lie in homogeneous tubes. In [3] de la Peña has proved that the conjecture is true in the case that A is strictly wild. An alternative proof of this is immediately given by the following proposition. Note that A is **strictly wild** iff there exists a fully faithful exact functor F from $\mathrm{mod}k\langle x, y\rangle$ to $\mathrm{mod}A$. In this abstruct, we consider the functor F and an AR-component $\mathcal{C}$ of $\mathrm{mod}A$.

Proposition 1 *Let $\mathcal{C}$ be a homogeneous tube. Then*

the number of $F(\mathrm{ind}k\langle x, y\rangle) \cap \mathcal{C}$ is zero.

Now, if A is hereditary, then using a result of Kerner and Lukas [2, 1.8], we see that *the number of $F(\mathrm{ind}k\langle x, y\rangle) \cap \mathcal{C}$ is less than 1* for any component $\mathcal{C}$.

These equality and inequality suggest us the following problem:

What is the number of $F(\mathrm{ind}k\langle x, y\rangle) \cap \mathcal{C}$?

Theorem 2 *Let $\mathcal{C}$ be a quasi-serial component of the form ZA_∞/τ^t. Then*

the number of $F(\mathrm{ind}k\langle x, y\rangle) \cap \mathcal{C}$ is less than $t/2$.

The proof of the **theorem** uses the following two lemmas, but we can prove the **proposition** without the latter lemma.

Lemma 3 $\mathrm{Ext}_{k\langle x,y\rangle}(X, Y) \neq 0$ *for* $X \neq 0$ *and* $Y \neq 0$ *in* $\mathrm{mod}k\langle x, y\rangle$.

Lemma 4 $\mathrm{rad}_{k\langle x,y\rangle} = \mathrm{rad}^2_{k\langle x,y\rangle}$.

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ON GORENSTEIN R-ALGEBRAS

Kenji Nishida, Shinshu University

(joint work with Shiro Goto, Meiji University)

Let R be a commutative Noetherian ring and Λ an R-algebra which is a finitely generated R-module.

Definition 1. Let M be a finitely generated Λ-module. Then M is said to be a *Gorenstein* Λ*-module* if the Cousin complex $C_R^{\cdot}(M)$ of M provides a minimal injective resolution for M. We say that Λ is a *Gorenstein* R*-algebra* if Λ is a Gorenstein module over itself. (For the Cousin complex, see [1] or [2].)

In this talk, we explain Gorenstein R-algebras along our forthcoming paper [2] where we will faithfully follow the commutative cases. Among others, we define 'Bass numbers' and characterize Gorensteiness using it.

Theorem 2. The following are equivalent.

(1) M is a Gorenstein module.

(2) M is a Cohen-Macaulay R-module such that $\mathrm{id}_{\Lambda_p} M_p = \dim_{R_p} M_p$ for $p \in \mathrm{Supp}_R M$.

Corollary. If R is a local ring, then M is Gorenstein iff M is a Cohen-Macaulay R-module and $\mathrm{id}_\Lambda M = \dim_R M$.

Let $C_i(\Lambda)$ denote for each i the full subcategory of all f.g. left Λ-modules such that M is a Cohen-Macaulay R-module of $\dim_R M = i$.

Theorem 3. Let R be a local ring. Assume that Λ is a Cohen-Macaulay R-module of $\dim \Lambda = n$. Then the following are equivalent.

(1) Λ is Gorenstein.

(2) $\mathrm{Ext}^p_\Lambda(M,\Lambda)=0$ for all $0\le i\le n$, $M\in C_i(\Lambda)$, and $p\neq n-i$.

When this is the case, $\mathrm{Ext}^{n-i}_\Lambda(M,\Lambda)\in C_i(\Lambda^{op})$ for all $M\in C_i(\Lambda)$ and, we have a natural isomorphism; $M\cong \mathrm{Ext}^{n-i}_\Lambda(\mathrm{Ext}^{n-i}_\Lambda(M,\Lambda),\Lambda)$.

Let $P\in \mathrm{Spec}\Lambda$. Then $E_\Lambda(\Lambda/P)\cong I(P)^{m(P)}$ for an integer $m(P)$ and an indecomposable injective Λ-module $I(P)$, where $E_\Lambda(M)$ is an injective hull of M. For a f.g. Λ-module M, let $0\to M\to E^0_\Lambda(M)\to\ldots\to E^i_\Lambda(M)\to\ldots$ be a minimal injective resolution of M and write $E^i_\Lambda(M)=\oplus_{P\in\mathrm{Spec}\Lambda}I(P)^{m^i(P,M)}$.

DEFINITION 4. Let $\mu^i(P,M)=m^i(P,M)/m(P)$ and call it the i-th *Bass number* of M with respect to P. Then we have the symbolic direct sum decompositon;

$$E^i_\Lambda(M)=\oplus_{P\in\mathrm{Spec}\Lambda}E_\Lambda(\Lambda/P)^{\mu^i(P,M)}.$$

THEOREM 5. The following are equivalent.

(1) Λ is Gorenstein.

(2) (a) $\dim\Lambda_{P\cap R}=\mathrm{ht}_\Lambda P$ for every $P\in\mathrm{Spec}\Lambda$,

(b) $\mu^i(P,\Lambda)>0$ iff $i=\mathrm{ht}_\Lambda P$ for $P\in\mathrm{Spec}\Lambda$.

(3) Replacing $\mathrm{Spec}\Lambda$ by $\mathrm{Max}\Lambda$ in (2).

(4) Ext_Λ-$\dim\Lambda/P=\mathrm{Ext}_\Lambda$-$\dim\Lambda/Q$ for all $P,Q\in\mathrm{Max}\Lambda$ with $P\cap R=Q\cap R$.

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Modules of second kind via coverings

Rainer Nörenberg

Let $A=kQ/I$ be a finite-dimensional algebra over an algebraically closed field k given by a quiver Q and an admissible ideal I. We fix a vertex s of Q and denote by P_s the corresponding indecomposable projective A-module. By an *s-startmodule* we understand an A-module M satisfying the conditions $\mathrm{Hom}_A(P(s),\tau M)=0$ and $\mathrm{Hom}_A(P(s),M)\neq 0$. Let $\mathcal{K}_s$ be the full subcategory of A-mod formed by all s-startmodules.

For a *string algebra* A and given vertex s the category $\mathcal{K}_s$ can easily be described from [2]. Under a suitable directedness condition (mainly $\dim_k End(P_s)=1$) one obtains a vector space category $(\mathcal{K}_s, Hom(P_s,\_))$.

The *fiber sum functor* F_s from the subspace category $\check{\mathcal{U}}(\mathcal{K}_s, Hom(P_s, \_))$ to A-mod is known to be dense on the hammock of P_s (cf [4]).

The vector space category $(\mathcal{K}_s, Hom(P_s, \_))$ then can be described as follows: Starting form a vertex s' of the quiver $\hat{Q}$ of the universal covering $\hat{A}$ of A covering the vertex s we consider the s'-startmodules of $\hat{A}$. These form a linear vector space category (thus a poset), consisting of at most two chains (of possibly infinite length). The vector space category $(\mathcal{K}_s, Hom(P_s, \_))$ is obtained form this poset by taking those s'-startmodules which under the pushdown functor to A-mod map to s-startmodules and by joining such $\hat{A}$-modules as map to the same A-module to form one object. Also, for a given indecomposable s-startmodule M there are at most two s'-startmodules which are preimages of M under the pushdown functor. Such structures as result from this construction were studied e.g. in [1] and [3].

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Defect groups of the blocks of Mackey algebra

Fumihito Oda

The theory of Mackey functors for a finite group G over a commutative ring R provides a single framework for the various representation theories of G and its subgroups. A Mackey functor is identified as the left module over the **Mackey algebra** $\mu_R(G)$. Let k be a field with characteristic $p > 0$. Then there is the one to one correspondence between the blocks of the group algebra kG and those of the p-part $f_1\mu_k(G)$ of the Mackey algebra [3] (17.1), [1] (12.3.2). I defined the defect group of a block of Mackey algebra [2]. Let B be a p-block of kG and D the defect group of B. Let b be a block of

$f_1\mu_k(G)$ corresponding to B and P the defect group of b. In [2] I showed the following proposition:

Proposition 1

$$D \leq_G P.$$

I shall present the result as follows:

Theorem 2

$$D \cong_G P.$$

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TRISECTIONS OF MODULE CATEGORIES

J. A. de la Peña, UNAM, México

Joint work with I. Reiten.

Let A be an Artin algebra and $\operatorname{mod} A$ be the category of finitely generated left A-modules. A *trisection* $(\operatorname{mod}_+ A, \operatorname{mod}_0 A, \operatorname{mod}_- A)$ of the module category $\operatorname{mod} A$ satisfies:

(1) Each $\operatorname{mod}_+ A$, $\operatorname{mod}_0 A$ and $\operatorname{mod}_- A$ is a full subcategory of $\operatorname{mod} A$.

(2) The indecomposable modules in $\operatorname{mod}_0 A$ form a family of standard pairwise orthogonal components of the AR-quiver Γ_A.

(3) There are no maps from $\operatorname{mod}_{-}A$ to $\operatorname{mod}_0 A$ or from $\operatorname{mod}_0 A \cup \operatorname{mod}_{-}A$ to $\operatorname{mod}_{+}A$.

(4) There are no projective modules in $\operatorname{mod}_{-}A$ and no injective modules in $\operatorname{mod}_{+}A$.

Special situations of trisections have been considered in several papers. We study general properties of trisections and intend to describe the algebras accepting such constructions.

A component $\mathcal{C}$ of Γ_A satisfies the *sectionality condition* if each path $I \to \cdots \to P$ from an injective to a projective in $\mathcal{C}$ is sectional.

Theorem. *Assume* $\operatorname{mod} A$ *accepts a trisection such that the components of* $\operatorname{mod}_0 A$ *satisfy the sectionality condition. Then A is tilted or quasitilted of canonical type.*

Among other problems we study the following:

(a) Uniqueness of trisections.

(b) Given an algebra A accepting a trisection, conditions for a one-point extension $A[M]$ to accept a trisection.

(c) Structure of the AR-components of the middle part $\operatorname{mod}_0 A$ of a trisection.

A note on algebras with every tilting module splitting

Liangang Peng and **Shanping Tang**

Let A be an Artin algebra. In [**1**] I. Assem proved that if A is triangular and every separating tilting A-module is splitting, then A is hereditary. Therefore one can see that, in the case of A triangular, every tilting A-module is splitting if and only if A is hereditary. In this note we consider an arbitrary Artin algebra and prove

Theorem 1 *Let A be an Artin algebra. If every tilting A-module is splitting, then for any indecomposable A-module X, $pdimX \leq 1$ or $pdimX = \infty$.*

Unfortunately there exists some example to show that the converse is not true.

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Computation of almost split sequences in homologically finite subcategories

Efrén Pérez

This is a joint work with Mark Kleiner. Let Λ be an artin algebra and $\mathcal{X}$ a contravariantly finite subcategory of $mod\Lambda$. For $M \in mod\Lambda$, we will denote by $g_M : r_{\mathcal{X}}M \to M$ its minimal right $\mathcal{X}$-approximation.

Theorem 1 *a) $\mathcal{X}$ is extension closed if and only if for all $M \in mod\Lambda$ $Ext^1_\Lambda(\_, g_M) : Ext^1_\Lambda(\_, r_{\mathcal{X}}M)_{|\mathcal{X}} \to Ext^1_\Lambda(\_, M)_{|\mathcal{X}}$ is a monomorphism of functors.*
b) If $\mathcal{X}$ is extension closed then $Ext^1_\Lambda(\_, Kerg_M)_{|\mathcal{X}} = 0$, for all $M \in mod\Lambda$, but the converse is false.

Theorem 2 *Let $\mathcal{X}$ be extension closed and $C \in \mathcal{X}$ not Ext-projective, then;*
a) There exists a unique up to isomorphism exact commutative diagram

$$\begin{array}{lccccccccc}
\alpha : & 0 & \to & r_{\mathcal{X}}DTrC & \to & r_{\mathcal{X}}E & \to & C & \to & 0 \\
 & & & {\scriptstyle g_{DTrC}}\downarrow & & {\scriptstyle g_E}\downarrow & & \| & & \\
\gamma : & 0 & \to & DTrC & \to & E & \to & C & \to & 0
\end{array}$$

where γ is an almost split sequence in $mod\Lambda$
b) The exact sequence α is a direct sum of an almost split sequence $0 \to A \to B \to C \to 0$ in $\mathcal{X}$ and the split exact sequence $0 \to Y \to Y \to 0 \to 0$ with $Y \in \mathcal{X}$ Ext-injective.

The dual statements of the theorems hold for covariantly finite subcategories. In response to a question raised in [1], Theorem 1 shows that the condition on Ext^1_Λ of part b), given by Wakamatsu's Lemma [2], is weaker than that of part a). Theorem 2 is part of a generalization and refinement of the main results in [1].

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V. M. Petrogradsky

Schreier Formulae for free Lie Algebras

Let X be at most countable set with weight function $\mathrm{wt} : X \to \mathbb{N}$ such that

$$X = \bigcup_{i=1}^{\infty} X_i, \quad X_i = \{x \in X \mid \mathrm{wt}\, x = i\}; \quad |X_i| < \infty,\ i \in \mathbb{N}.$$

We call such a set *finitely graded*. For any monomial $y = x_{i_1} \dots x_{i_n}$, $x_j \in X$, we set $\mathrm{wt}\, y = \mathrm{wt}\, x_{i_1} + \dots + \mathrm{wt}\, x_{i_n}$. Suppose that Y is a set of monomials in X then we denote $Y_i = \{y \in Y \mid \mathrm{wt}\, y = i\}$, $i \in \mathbb{N}$, and define a Hilbert-Poincaré series of Y as $\mathcal{H}(Y,t) = \mathcal{H}_X(Y,t) = \sum_{i=1}^{\infty} |Y_i| t^i$. Analogous definitions we give also for homogeneous subspaces $V \subset A$ of an algebra $A = A(X)$ generated by X. We consider the operator $\mathcal{E}$ on power series as

$$\mathcal{E}\left(\sum_{n=1}^{\infty} b_n t^n\right) = \prod_{n=1}^{\infty} \frac{1}{(1-t^n)^{b_n}} = \sum_{n=0}^{\infty} a_n t^n.$$

If L is a Lie algebra generated by X and $U(L)$ is its universal enveloping algebra then $\mathcal{H}_X(U(L)) = \mathcal{E}(\mathcal{H}_X(L))$. The following is an analogue of well-known Schreier formula for free groups.

Theorem. *Let L be a free Lie algebra freely generated by a finitely graded set X. Suppose that H is a homogeneous subalgebra with respect to the induced grading. Then for any homogeneous set Y of free generators of H holds*

$$\mathcal{H}(Y,t) - 1 = (\mathcal{H}(X,t) - 1) \cdot \mathcal{E}(\mathcal{H}(L/H,t)).$$

The same formula is also true for homogeneous subalgebras of free Lie p-algebras, superalgebras and p-superalgebras; one should only use the operator $\mathcal{E}$, which corresponds to the transition to (restricted) enveloping algebra.

This formula easily yields formulae due to G. P. Kukin and A. A. Mikhalev on ranks of subalgebras in these cases.

If in a subset (subspace) Y the size (dimension) of multilinear elements depending on distinct elements $x_{i_1}, \ldots, x_{i_m}$ does not depend on letters but depends only on $n = \mathrm{wt}\, x_{i_1} + \ldots + \mathrm{wt}\, x_{i_m}$ then we denote this number as $c_n(Y)$ and consider *complexity function* (or *exponential generating function*)

$$\mathcal{C}(Y, z) = \sum_{n=0}^{\infty} \frac{c_n(Y)}{n!}\, z^n.$$

Also, there is an exponential version of the Schreier formula (provided that respective functions exist)

$$\mathcal{C}(Y, z) - 1 = (\mathcal{C}(X, z) - 1) \cdot \exp(\mathcal{C}(L/H, z)).$$

These results enables us to find exact formulae for Hilbert-Poincaré series and complexity functions for solvable, or more generally, polynilpotent varieties of Lie algebras.

TRIVIAL EXTENSIONS OF ARTIN ALGEBRAS OF FINITE REPRESENTATION TYPE

Elsa Fernández - María Inés Platzeck

Let A be a Schurian algebra over an algebraically closed field, and $T(A) = A \propto D(A)$ the trivial extension of A. We describe the quiver and relations of A. This description is rather simple and turns out to be very useful, allowing us in particular to describe all the algebras A^1 with the same trivial extension $T(A)$.

It is known for an algebra A that $T(A)$ is of finite representation type if and only if A is an iterated tilted algebra of Dynkin type, and that this happens if and only if there is a tilted algebra B of Dynkin type such that $T(A) = T(B)$. Moreover, the Cartan type of $T(A)$ coincides with the Dynkin type of A.

This interplay between iterated tilted algebras and trivial extensions allows us both to classify the trivial extensions of finite type and the iterated tilted algebras of Dynkin type. To decide if a given trivial extension T is of finite representation type we either find a factor algebra of infinite type, or a tilted algebra A of Dynkin type such that $T(A) = T$. Surprisingly, this can always be done in an easy way.

Once we get the classification of the trivial extensions and their Cartan type we describe a method to decide inmediately wether a given algebra is tilted iterated of Dynkin type Δ. In particular we obtain, with a unified approach, the classifications given for the cases A_n, D_n and E_p by different authors.

Similar methods can be used to study iterated simply connected tilted algebras of Euclidean type $\tilde{D}_n$ and $\tilde{E}_p$.

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Stable equivalences of polynomial growth self-injective algebras

Zygmunt Pogorzaly

Let K be an algebraically closed field. Consider any standard self-injective finite-dimensional K-algebra A which is basic and connected. We are interested in algebras A which are representation-infinite of polynomial growth.
Recall that two algebras A and B are stably equivalent if their stable categories $\underline{\mathrm{mod}}(A)$ and $\underline{\mathrm{mod}}(B)$ are equivalent. Algebras B stably equivalent to standard self-injective algebras A which are representation-infinite of polynomial growth were studied in [1], [2]. There is given a characterization of B in [1] in the case when A have only generalized standard components in their Auslander-Reiten quivers. There is givn a characterization of B in [2] in the case when A have no generalized standard components in their Auslander-Reiten quivers. Now the main aim is to characterize B in the case when A have components which are generalized standard and components which are not generalized standard in their Auslander-Reiten quivers.

Theorem 1 *Let A be a basic, connected, self-injective, standard K-algebra which is representation-infinite of polynomial growth. Let the Auslander-Reiten quiver Γ_A of A contains at least one generalized standard component. If B is a basic and connected K-algebra which is stably equivalent to A then*

B is a standard, self-injective K-algebra which is representation-infinite of polynomial growth whose Auslander-Reiten quiver Γ_B contains at least one generalized standard component.
Moreover, A and B have the same number of pairwise non-isomorphic simple modules.

As a consequence of the above result and those in [1], [2] one gets the following.

Corollary 2 *Let A and B be stably equivalent self-injective, standard K-algebras. Then A is of polynomial growth (resp. domestic) if and only if B is of polynomial growth (resp. domestic).*

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Steps on the classification of Cohen–Macaulay modules on some hypersurfaces

Dorin Popescu

A Knoerrer's result allows to describe the maximal Cohen-Macaulay modules over a singularity of type $f(x)+y^2$ if we know the maximal Cohen-Macaulay modules over singularity $f(x)$, $x=(x_1,...,x_n)$. We study the description of MCM modules over $f(x)+y_1^{s_1}+...+y_r^{s_r}$ if we know the description of MCM modules over $f(x)$. For this purpose we must study deformation of MCM modules. Examples are given for $n=1$, $f(x)=x^t$, and $s_1=...=s_r=3$.

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Perpendicular categories of exceptional vectorbundles on wild weighted projective lines

Jens Reinhold

Let $\mathbb{X}$ be a wild weighted projective line in the sense of Geigle and Lenzing. We denote by $\mathrm{coh}(\mathbb{X})$ the category of coherent sheaves on $\mathbb{X}$. Each indecomposable $M \in \mathrm{coh}(\mathbb{X})$ is either a vectorbundle or a sheaf of finite length. Let $\mathrm{vect}(\mathbb{X})$ be the full subcategory of vectorbundles in $\mathrm{coh}(\mathbb{X})$. The Auslander-Reiten translation $\tau_{\mathbb{X}} : \mathrm{coh}(\mathbb{X}) \longrightarrow \mathrm{coh}(\mathbb{X})$ is an equivalence. In particular there is an induced equivalence $\tau_{\mathbb{X}} : \mathrm{vect}(\mathbb{X}) \longrightarrow \mathrm{vect}(\mathbb{X})$.

Let $Y \in \mathrm{vect}(\mathbb{X})$ be a quasi-simple exceptional vectorbundle. We denote by $Y^{\perp}$ the right perpendicular category of Y. Then we have the equivalence $Y^{\perp} \cong C-\mathrm{mod}$, where C is connected wild hereditary. The Auslander-Reiten translation in $Y^{\perp}$ is also denoted by τ_C, since we can identify $C-\mathrm{mod}$ with $Y^{\perp}$. If $0 \longrightarrow \tau_{\mathbb{X}}Y \longrightarrow Z \longrightarrow Y \longrightarrow 0$ is the Auslander-Reiten sequence in $\mathrm{vect}(\mathbb{X})$, then Z is quasi-simple regular in $Y^{\perp}$. We denote by $Y^{\perp}-\mathrm{reg}$ the full subcategory of $Y^{\perp}$ that consists of $Y^{\perp}$-regular sheaves. Then we have $Y^{\perp}-\mathrm{reg} \subset \mathrm{vect}(\mathbb{X})$.

There is a full and dense functor $F : Y^{\perp}-\mathrm{reg} \longrightarrow \mathrm{vect}(\mathbb{X})$ with $\tau_{\mathbb{X}}F = F\tau_C$. The kernel of F are the maps factoring through the τ_C-orbit of Z. In particular we get the equivalence $Y^{\perp}-\mathrm{reg}/\langle Z\rangle \longrightarrow \mathrm{vect}(\mathbb{X})$, where $\langle Z\rangle$ denotes the kernel of F. So the categories $Y^{\perp}$ and $\mathrm{vect}(\mathbb{X})$ are very similar.

As a consequence, the categories $\mathrm{coh}(\mathbb{X})$ and $\mathrm{coh}(\mathbb{X}')$ for two arbitrary wild weighted projective lines are also very similar. There are vectorbundles $V \in \mathrm{vect}(\mathbb{X})$ and $V' \in \mathrm{vect}(\mathbb{X}')$ such that there is an equivalence $\mathrm{vect}(\mathbb{X})/\langle V\rangle \longrightarrow \mathrm{vect}(\mathbb{X}')/\langle V'\rangle$.

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Strata for the variety of modules

Nicola Richmond

Very little is know about the geometry of the variety $Mod(A, n)$ of n-dimensional modules over a finite dimensional algebra A. We present a result which allows one to compute the dimension of $Mod(A,n)$ and bounds on the number of irreducible components for a large class of algebras, which we call subfinite.

Definition 1 *A finite dimensional algebra A is said to be subfinite if every projective A-module has a finite number of isomorphism classes of submodules.*

Examples include hereditary algebras, finite representation type algebras, radical square zero algebras, string algebras and algebras that are a 1-point extension of a tame concealed algebra by a module without preinjective direct summands.

Definition 2 *$Mod(A,n)_L$ is the set of n-dimensional modules X such that there is a surjection $A^n \longrightarrow X$ with kernel isomorphic to L.*

For any algebra A, $Mod(A,n)$ is the disjoint union of these $Mod(A,n)_L$ taken over the set of isomorphism classes of submodules of A^n of dimension $n(dimA-1)$. In particular, if A is subfinite, then this union is finite.

Theorem 3 *$Mod(A,n)_L$ is a smooth, irreducible, affine variety of dimension $dimHom(L,A^n) - dimEnd(L)$.*

We also generalise results of Kac and Schofield concerning general representations of quivers, see [1] and [2].

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The special biserial algebras have a sufficient supply of algebraically compact indecomposable representations

Claus Michael Ringel

Let k be field and A a finite-dimensional k-algebra. Jenson and Lenzing have asked whether there is a coincidence between the assertions (i) *A is tame* and (ii) *A has a sufficient supply of algebraically compact indecomposable modules* (see the book *Model Theoretic Algebra,* 1989, but also a rather similar discussion in Prest's book *Model Theory and Modules*, 1988). The second conditions means that any non-zero algebraically compact module has an indecomposable direct summand. The (positive) answer is known only for very special classes of algebras, such as the hereditary ones. Even in case A is domestic, it is in general not yet known whether A has a sufficient supply of algebraically compact indecomposable representations. The aim of this lecture is to deal with the special biserial algebras. We are going to show that they have a sufficient supply of algebraically compact indecomposable representations, and we will describe an index set for the algebraically compact indecomposable representations. Many algebraically compact indecomposable representations have been known for a long time. Recall that the finite-dimensional indecomposable representations are *strings* $S(w)$ (indexed by a word w using as letters arrows and their formal inverses) and *bands* $M(w; V, \phi)$ (indexed by a 'primitive' word w and a finite-dimensional vector space V with an indecomposable invertible linear transformation $\phi V \to V$). The usual construction of a band module can be carried out also in case V is infinite-dimensional, and starting with an algebraically compact indecomposable $k[T, T^{-1}]$-module (V, ϕ), one obtains an algebraically compact indecomposable A-module $M(w; V, \phi)$. The remaining algebraically compact indecomposable A-modules $C(w)$ are indexed by $\mathbb{N}$-words w, and by non-periodic $\mathbb{Z}$-words w; there is a chain of inclusions $S(w) \subseteq C(w) \subseteq P(w)$, where $S(w)$ is the infinite analogue of a string module and $P(w)$ a corresponding product module. The proof proceeds as follows: As usual, we will embed the category of all A-modules M as the category of all tensor functors $-\otimes M$ into the category $\mathcal{F}$ of all additive functors from the category of all finitely presented A^{op}-modules to the category of abelian groups. Under this embedding, the category of algebraically compact A-modules is equivalent to the category of all injective objects in $\mathcal{F}$. We are going to describe sufficiently many functors of the form $-\otimes M$ which are uniform, or even $\mathbb{N}$-uniform. On the other hand, we extend the filtration method of

Gelfand and Ponomarev in order to be able to deal with infinite-dimensional representations.

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The cell structure, the Brauer tree structure and extensions of cell-modules for Hecke orders of dihedral groups

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We shall give an inductive description of the Hecke order $\mathcal{H}_{D_n}$ of the dihedral group D_n of order $2 \cdot p^n$ over $\mathbb{Z}[q, q^{-1}]$ for an odd prime p. Since $\mathbb{Q}$ is not splitting field for D_n, the notion of *cellular* in the sense of Graham-Lehrer and in its equivalent formulation by König-Xi can not be used.

We extend the definition of cellular to orders in non split algebras:

Definition 1 *Let R be a noetherian integral domain with field of fractions K, and let Λ be an R–free R–order in a separable finite dimensional K–algebra A. The map ι is an R–linear anti–involution on Λ. A two–sided ideal J of Λ is called a* CELL–IDEAL *provided*

- *J is R–free with R–free quotient,*
- *$K \otimes_R J = A \cdot e_J$ is a simple K–algebra, i. e. e_J is a central primitive idempotent of A.*
- *$\iota(J) = J$ and*
- *there exists an R-free left Λ–ideal P in $\Lambda \cdot e_J$ such that*
    - *P is irreducible, i. e. $K \otimes_R P$ is a simple A–module,*
    - *$E(P) := End_\Lambda(P)$ is R-free,*
    - *P is free as right $E(P)$-module.*

– *$E(P) \subset \Lambda \cdot e_J$ under the natural diagonal inclusion $End_A(K \otimes_R P) \subset A \cdot e_J$,*
– *$\iota(E(P)) = E(P)$;*
these condition guarantee that ${}^{\iota}P$ is an $(E(P), \Lambda)$-bi–module.
– *We furthermore require that there is an isomorphism of bi-modules $\alpha: J \to P \otimes_{E(P)} {}^{\iota}P$, making the following diagram commute*

$$\begin{array}{ccc} J & \xrightarrow{\alpha} & P \otimes_{E(P)} {}^{\iota}P \\ \iota \downarrow & & \downarrow x\otimes y \mapsto \iota(y) \otimes \iota(x) \\ J & \xrightarrow{\alpha} & P \otimes_{E(P)} {}^{\iota}P \,. \end{array}$$

The left Λ-ideal P is said to be the CELL-MODULE TO J.
The cell ideal J of Λ is said to be PROJECTIVELY CELLULAR *provided J is projective as left module over $\Lambda \cdot e_J$ or equivalently, P is a projective $\Lambda \cdot e_J$-module.*
The R–order Λ is said to be a CELLULAR ORDER *provided, there is a chain of 2–sided ideals*

$$0 = J_0 \subset J_1 \subseteq \cdots \subseteq J_i \subseteq J_n = \Lambda$$

such that for $1 \leq i \leq n$ the ideal J_i/J_{i-1} is a cell ideal in Λ/J_{i-1}.
If for the cellular order Λ all the cell ideals are projectively cellular, then Λ is called a PROJECTIVELY CELLULAR ORDER.

In case K split A this definition reduces to the one of Graham-Lehrer.
With our definition we are able to prove:

- The Hecke order $\mathcal{H}_{D_n}$ of the dihedral group of order p^n for an odd prime p is a projectively cellular $\mathbb{Z}^q$-order.
- We explicitly describe the cell ideals and the corresponding cell-modules $M_0, M_1, \cdots, M_n, M_{n+1}$, where M_0 corresponds to the index representation and M_{n+1} to the sign representation; moreover, for $1 \leq i \leq n$ the module M_i is a "faithful" $\mathcal{H}_{D_n}$ module.
- We compute the extension groups between the cell-modules:
  1. For $i > j \in \{1, \ldots, n\}$ we have
     $Ext^1_{\mathcal{H}_{D_n}}(M_i, M_j) = Ext^1_{\mathcal{H}_{D_n}}(M_j, M_i) = \mathbb{F}_p[X]/ < X^{\mu(j)} > [q, q^{-1}]$,
     with $\mu(n) := \frac{p^{n-1} \cdot (p-1)}{2}$. The latter module can be identified with the ring of Laurent-polynomials over a quotient of the dual of the modular augmentation ideal of the cyclic group of order p^n.

2. For $i \in \{1, \ldots, n\}$ we have
   $Ext^1_{\mathcal{H}_{D_n}}(M_i, M_0) = \mathbb{Z}[\theta_i] = Ext^1_{\mathcal{H}_{D_n}}(M_{n+1}, M_i) = \mathbb{Z}[\theta_i]$ and
   $Ext^1_{\mathcal{H}_{D_n}}(M_i, M_{n+1}) = 0 = Ext^1_{\mathcal{H}_{D_n}}(M_0, M_i)\,.$
3. Finally

$$Ext^1_{\mathcal{H}_{D_n}}(M_0, M_{n+1}) = Ext^1_{\mathcal{H}_{D_n}}(M_{n+1}, M_0) = \mathbb{Z}\,.$$

- Finally we prove that the Hecke order $\mathbb{Z}^q_{<p,q-1>} \otimes_{\mathbb{Z}^q} \mathcal{H}_{D_n}$ is the Brauer tree order to the Brauer tree $\bullet - \bullet - \bullet$ with the exceptional vertex in the centre of multiplicity $\mu(n) := \frac{p^{n-1}\cdot(p-1)}{2}$. Here $\mathbb{Z}^q_{<q-1,p>}$ is the completion of $\mathbb{Z}^q$ at the maximal ideal $< q-1, p >$, which specializes to the completion of $\mathbb{Z}$ at $< p >$.

Duality arrising from commutative ring

Masahisa Sato

In the old paper [3] it is known that a commutative local noetherian ring $(R, J))$ is complete if and only if the natural homomorphism $R \to \mathrm{End}_R(\mathrm{E}(R/J))$ is an isomorphism. Based on the Matris result, Matris duality over a commutative semi-local noeterian ring R with the Jacobson radical J was defined. In [1, 2], some chain conditions between module and its Matris dual module were discussed and the following results were proved;

(1) An R-module M is artinian if and only if there is an exact sequence $0 \to M \to E_0 \to E_1 \to \ldots \to E_i \to \ldots$ such that each E_i is a direct sums of finitely many copies of $\mathrm{E}(R/J)$.

(2) Particularly $\mathrm{E}(R/J)$ is artinian.

(3) $\mathrm{E}(R/J)$ is noetherian if and only if R is artinian.

We define the E-**dual modules** N_S^* and ${}_RM^\sharp$ of modules ${}_RM$ and N_S with respect to E by $\mathrm{Hom}_R({}_RM, {}_RE_S)_S$ and ${}_R\mathrm{Hom}_S(N_S, {}_RE_S)$ respectively, and define E-daulity which coincides with the Matris duality in the case of a commutative ring and we will know that the assumption "complete semi-local noether" is not necessary to get the above results even in the case of non-commutative rings.

Corollary 1 *Assume that R is a commutative ring and ${}_RE$ is a cogenerator, then ${}_RE$ is noetherian if and only if ${}_RE$ has a composition series. In this case, R is an artinian ring.*

Furthermore we will know that noetherian condition implies "semi-local". The chain conditions between a module and its dual module are discussed and we give simple and more general results by studying in the stand point view of non-commutative ring theory. One of these is the following theorem.

Theorem 2 *Assume ${}_RE_S$ is a cogenerator both as R-module and as S-module. For an R-module ${}_RM$ and S-module N_S, it holds that ${}_RM^{*\sharp}$ is noetherian (resp. artinian) if and only if M_S^* is artinian (resp. noetherian).*

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Quivers of crossed products

Mary Schaps

Let G be a finite group, k a field of characteristic prime to $|G|$, and A a local k-algebra with $J = RadA$ on which G acts as algebra automorphisms. The quiver of the skew group algebra $A[G]$ is McKay's representation graph of the action of G on J/J^2. If we let γ be a cocyle for G into k^*, the crossed product $A^\gamma[G]$ is a union of connected components of the McKay graph for a central extension of G. If we allow $chark$ to divide the order of G then the quiver of $A[G]$ is the join of the quiver of $k[G]$ with the modular McKay graph of J/J^2.

The finite length modules over thin $\mathbb{Z}$-graded rings

Markus Schmidmeier

Let's call a $\mathbb{Z}$-graded ring $R = \bigoplus_{i\in\mathbb{Z}} R_i$ *thin* provided $R_0 = K$ is a skew field and each R_i is a K-vector space of dimension at most one. The class of these rings contains all (skew) (Laurant) polynomial rings and those subrings which are generated by monomials. Such rings occur as factor rings of generalized Weyl algebras; so their simple modules, which are determined in [1, Theorem 2.4], contribute to the list of all simple modules over the generalized Weyl algebra. As a consequence of the quoted result we obtain the following description of the category $\mathrm{mod}R$ of all finite length right R-modules.

Theorem 1 *Let $R = \bigoplus_{i\in\mathbb{Z}} R_i$ be a thin $\mathbb{Z}$-graded ring with no nonzero nilpotent elements. With $S_\pm = \bigcup_{i\in\pm\mathbb{N}} R_i$ the following situations are possible.*

1. *If $S_+ = S_- = 0$ then $\mathrm{mod}R = \mathrm{mod}K$.*
2. *If $S_+ \neq 0$ and $S_- = 0$ then $\mathrm{mod}R = \mathcal{C}_0 \times \mathcal{C}_+$.*
3. *If $S_+ = 0$ and $S_- \neq 0$ then $\mathrm{mod}R = \mathcal{C}_- \times \mathcal{C}_0$.*
4. *If $S_+ \neq 0$, $S_- \neq 0$ and R is a domain then $\mathrm{mod}R = \mathcal{C}_+$.*
5. *If $S_+ \neq 0$, $S_- \neq 0$ and R is not a domain then $\mathrm{mod}R = \mathcal{C}_- \times \mathcal{C}_0 \times \mathcal{C}_+$.*

Those modules which are S_+-torsion and S_--torsion form the abelian category $\mathcal{C}_0 = \{M \in \mathrm{mod}R : \exists t \in \mathbb{N} : MS_+^t = 0 = MS_-^t\}$ with only one simple object K. The $S_\pm$-torsionfree modules form the category $\mathcal{C}_\pm = \{M \in \mathrm{mod}R : MS_\pm = M\}$. This category is abelian of global dimension zero or one and is equivalent to the category of finite length modules over some skew Laurant polynomial ring $K[X, X^{-1}; \sigma_\pm]$ with $\sigma_\pm$ a field automorphism of K.

In the talk we also present several components of Auslander–Reiten quivers for thin $\mathbb{Z}$-graded rings and indicate some consequences.

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Krull-Gabriel dimension and infinite radical of a module category

Jan Schröer

Let k be an algebraically closed field, and let A be a finite-dimensional k-algebra. We denote by mod-A the category of finite-dimensional right modules over A. Let $\mathcal{C}$ be the category of finitely presented covariant functors from mod-A to mod-k. If $\mathcal{T}$ is a Serre subcategory of an abelian category $\mathcal{A}$, then we denote by $\mathcal{A}/\mathcal{T}$ the corresponding abelian quotient category which was introduced by Grothendieck and Serre. Define $\mathcal{C}_{-1} = 0$. For each $n \in N_0$ let $\mathcal{C}_n$ be the subcategory of all functors $F \in \mathcal{C}$ such that $q_{n-1}(F)$ is of finite length, where q_{n-1} denotes the quotient functor from $\mathcal{C}$ to $\mathcal{C}/\mathcal{C}_{n-1}$. Note that $\mathcal{C}_n$ is a Serre subcategory of $\mathcal{C}$ for all n. Following Geigle define $\mathrm{KG}(A) = \min\{n \mid \mathcal{C}_n = \mathcal{C}\}$ if such a minimum exists, and $\mathrm{KG}(A) = \infty$, else. We call $\mathrm{KG}(A)$ the *Krull-Gabriel dimension* of A. This is a finitely presented version of a definition due to Gabriel. It was an open problem whether 0, 2 and ∞ are the only values that may occur as Krull-Gabriel dimensions.

Theorem 1. *For each $n \in N_0 \setminus \{1\}$ there exists a finite-dimensional k-algebra A with $\mathrm{KG}(A) = n$.*

Burke and Prest proved this independently by using different methods.

Let rad_A be the radical of mod-A. There exists a minimal ordinal $\mathrm{st}(A)$ such that $\mathrm{rad}_A^{\mathrm{st}(A)} = \mathrm{rad}_A^{\alpha}$ for all ordinals $\alpha \geq \mathrm{st}(A)$. The dimension $\dim(F)$ of a functor F in $\mathcal{C}$ is by definition the dimension of its lattice of finitely presented subfunctors. These definitions are due to Prest. If $\mathrm{KG}(A) = n$, then $n = \max\{\dim(F) \mid F \in \mathcal{C}\}$.

Theorem 2 (joint with Prest). *Let A be a finite-dimensional k-algebra. If $\mathrm{rad}_A^{\mathrm{st}(A)} = 0$, then $\dim(F) < \mathrm{st}(A)$ for each uniserial functor F in $\mathcal{C}$.*

Theorem 3. *Let A be a string algebra. Then the following are equivalent:*

(i) *A is domestic;*

(ii) $\mathrm{rad}_A^{\mathrm{st}(A)} = 0$*;*

(iii) *There exists an $n \in N_0$ such that $q_n(F) = 0$ for each uniserial functor F in $\mathcal{C}$.*

A topologist's approach to tilting theory

Stefan Schwede

We describe a framework for constructing derived equivalences between certain kinds of triangulated categories. The main algebraic examples are derived categories of abelian categories and stable module categories of Frobenius algebras. A second source of numerous examples is algebraic topology. In stable homotopy theory there is a notion of a 'ring spectrum' which generalizes that of a ring or a differential graded algebra. A ring spectrum has a category of modules, which in turn has a triangulated 'derived category'.

We describe a 'Morita theorem' in this context which gives a necessary and sufficient condition for identifying module categories over a ring spectrum. For classical rings this criterion specializes to Rickard's theory of tilting complexes and Keller's tilting theory for differential graded categories.

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Canonical Matrices for Linear Matrix Problems

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Belitskiĭ [1] proposed an algorithm for reducing each matrix A to a canonical form under transformations $A \mapsto S^{-1}AS$, where S is an invertible matrix in a certain algebra $\Lambda \subset \mathbb{C}^{n\times n}$. He shows that two matrices are equivalent if and only if they have the same canonical form. I extend Belitskiĭ's algorithm to a large class of matrix problems, which includes representations of modules over aggregates [2], and reprove Drozd's "Tame and Wild Theorem" in the spirit of [2]: *Every matrix problem satisfies one and only one of the following two conditions:*

- *(a) For every pair (m,n), the set of indecomposable canonical $m\times n$ matrices consists of a finite number of points and straight lines in the affine space $\mathbb{C}^{m\times n}$. Every straight line has the form $\{P(J_r(\lambda)) \,|\, \lambda \in \mathbb{C}\}$, where $P(x)$ is a matrix in which one entry is replaced with x and $J_r(\lambda)$ is a Jordan cell. The number of intersections of the line $P(J_r(\lambda))$ with other lines is bounded by a number $t(P)$ that does not depend on r.*

(b) *There exists an affine space* $\mathbb{C}^{m\times n}$ *that contains a reliable plane (in the sense of [2]) consisting of indecomposable canonical matrices.*

Brüstle and the author proved that *the number of points and lines in (a) is bounded by* 4^{mn}. This estimate is based on the explicit form of the canonical matrices and is an essential improvement of the estimate from [3], which was based on the construction of straight lines in [2].

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Computer evidence for some conjectures of Kac

Bert Sevenhant

Let k be a finite field with q elements and Q a quiver. Denote with $a(Q, \alpha, q)$ the number of orbits of absolute indecomposable representations of Q over k, with dimension vector α. Victor Kac showed that you can write $a(Q, \alpha, q)$ as a polynomial in q with integer coefficients which are independent of q and the orientation of Q.

He also conjectured that these coefficients are all positive, and that the constant term would be the multiplicity of α, whenever Q has no loops. He did this allmost without evidence, since there is no obvious way to calculate $a(Q, \alpha, q)$ by hand.

In this talk we show a way to collect more evidence (i.e to test the conjectures by computer), and give some results.

Hereditary Artinian rings with two connected components in the Auslander-Reiten quiver

Daniel Simson
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We shall present a characterization of hereditary right artinian rings R for which the Auslander-Reiten quiver Γ_R of R is a union of two connected components. The characterization will be given in terms of the the Coxeter valued diagram associated with R, by means of a set of infinite dimension-sequences and by the vanishing properties of the infinite radical $\mathrm{rad}^\infty(\mathrm{mod}(R))$ of the category $\mathrm{mod}(R)$ of finitely generated right R-modules.

The existence of such rings R will be discussed in relation with a class of generalized Artin problems for division field extensions (see [1],[2], [3]) and in relation with the pure semisimplicity conjecture asserting that a ring Λ is of finite representation type if every right Λ-module is algebraically compact, or equivalently, if every right Λ-module is a direct sum of finitely presented modules (see [3],[4]).

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One-point extensions of hereditary algebras and quasitilted algebras

Inger Heidi Slungård

The results in the first part of this talk are joint work with Dieter Happel. Let k be an algebraically closed field, H an indecomposable hereditary artin algebra of infinite representation type and let M be a quasisimple regular H-module. For the hereditary artin path algebra $k\mathbb{A}_n$, let P be an indecomposable projective $k\mathbb{A}_n$-module that corresponds to a vertex with only one neighbour. In [HS] we proved that a one-point extension Λ of a decomposable hereditary artin algebra is quasitilted but not tilted, only if Λ is of the form $\Lambda_n = (H \times k\mathbb{A}_n)[M \amalg P]$, $n \geq 1$.

In this talk we will discuss when an algebra of the form Λ_n, $n \geq 1$, is quasitilted. We will show the following.

Theorem 1 *Let H be a wild hereditary algebra.*

(a) *If $H[M]$ is quasitilted but not tilted, then Λ_i is not quasitilted for $i \geq 1$.*

(b) *If Λ_{n-1} is tilted and Λ_n is quasitilted but not tilted, then Λ_{n+1} is not quasitilted.*

In the last part of the talk we will discuss the concept of dominating. Let A be an artin algebra and M an indecomposable A-module. We say that M dominates a module X if for any right minimal map $f : M^t \to X$ we have that $\operatorname{Ker} f$ is projective.

Let H be a hereditary algebra and M an indecomposable regular H-module with $\operatorname{End}_H(M) \simeq k$. It is known that $H[M]$ is a tilted algebra if M dominates all indecomposable injective H-modules.

The canonical algebras are the main source of examples of quasitilted algebras which are not tilted. The wild canonical algebras can be viewed as a one-point extension $H[M]$ of a wild hereditary algebra H with a qasisimple module M, where $\operatorname{End}_H(M) \simeq k$. For these examples M dominates $\mathrm{DTr}(\mathrm{D}H_H)$, [HRS].

We will discuss the relationship between the concept of dominating $\mathrm{DTr}(\mathrm{D}H_H)$ and $H[M]$ being quasitilted for hereditary algebras H. Examples that show that neither property will imply the other will be given. These examples are related to the canonical algebras.

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t-structures generated by a complex of R-modules.

M. J. Souto Salorio

Summary: Let R be a ring, $\mathbf{K}(R)$ and $\mathbf{D}(R)$ its homotopic and derived category, respectively. For a complex E, we denote by $\mathcal{U}$ the smallest suspended subcategory stable for arbitrary coproducts and such that $E \in \mathcal{U}$. Its orthogonal, $\mathcal{U}^{\perp}$, consists of all the complexes Z, such that, $\mathrm{Hom}(E[j], Z) = 0$, $\forall j \geq 0$. Following Beilinson, Bernstein and Deligne ([2]), a t-structure on a triangulated category $\mathcal{T}$ is a pair of strictly full subcategories ($\mathcal{T}^{\leq 0}$, $\mathcal{T}^{\geq 0}$) orthogonal to each other, such that, the first is stable by positive translations and the second by negative ones and also, for every object M there exists a triangle $Y \to M \to Z \to Y[1]$ with $Y \in \mathcal{T}^{\leq 0}$ and $Z \in \mathcal{T}^{\geq 0}[-1]$. Keller and Vossieck have shown that the subcategory $\mathcal{T}^{\leq 0}$ is not necessarily triangulated but suspended, and also that the third condition of t-structure is equivalent to the existence of a right adjoint for the inclusion $\mathcal{T}^{\leq 0} \hookrightarrow \mathcal{T}$ ([3]). The subcategory $\mathcal{T}^{\geq 0}$ is in this case $(\mathcal{T}^{\leq 0})^{\perp}[1]$.

This above results and our study of Bousfield's localization in the derived category ([1]) led us to prove that the pair $(\mathcal{U}, \mathcal{U}^{\perp}[1])$ defines a t-structure in $\mathbf{D}(R)$. Previously, we obtain the analogous result in $\mathbf{K}(R)$ and then we consider a q-projective resolution of E, and transport the result to the derived category.

In order to associate the triangle at M we construct by induction a directed system $\{B_i, \mu_{ij} : B_i \to B_j\}_{i,j \in I}$ indicated by the set of all ordinals less than γ (with $\gamma =$ the smallest ordinal such that its cardinal is bigger than the cardinal of E). We take $B_0 = M$. If $s \in I$ has a predecessor $s-1$, we take B_s the cone of the natural map $\bigoplus_{\Omega_{s-1}} E[k] \longrightarrow B_{s-1}$ where $\Omega_{s-1} = \cup_{k \in N} \mathrm{Hom}_{\mathbf{C}(R)}(E[k], B_{s-1})$. For a limit ordinal s, we define $B_s := \varinjlim_{r<s} B_r$. The triangle $Y \to M \xrightarrow{\mu_{0\gamma}} B_\gamma \to Y[1]$ verifies that $Y \in \mathcal{U}$ and $B_\gamma \in \mathcal{U}^{\perp}$.

Finally, we show that if E is small ($\mathrm{Hom}(E, -)$ conmutes with arbitrary coproducts) and tilting ($\mathrm{Hom}(E, E[j]) = 0$, $\forall j \neq 0$) then E is projective and generator in the abelian subcategory $\mathcal{C} = \mathcal{U} \cap \mathcal{U}^{\perp}[1]$, that is, the heart of the t-structure is equivalent to the category $\mathbf{End}(E)$–modules.

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Standardly stratified algebras and tilting

Luise Unger

This is a report on joint work with I. Ágoston, D. Happel and E. Lukács. Let A be a finite dimensional k- algebra with n simple modules $S(1),...,S(n)$. For $1 \leq i \leq n$, let $P(i)$ be the projective cover of $S(i)$, and $Q(i)$ the injective envelope of $S(i)$. Denote by $U(i)$ the sum of all images of maps from $P(j)$ to $P(i)$ for $j > i$, and by $\Delta(i) = P(i)/U(i)$. Let $\bar{\nabla}(i)$ be the maximal submodule of $Q(i)$ without composition factors $S(j)$ for $j < i$, and with $\mathrm{End}\bar{\nabla}(i) = D$ a division ring. We denote by $\mathcal{F}(\Delta)$ (resp. $\mathcal{F}(\bar{\nabla})$) the full subcategory of $\mathrm{mod}\, A$ consisting of those modules having a filtration with factors in $\Delta = \{\Delta(1), \ldots, \Delta(n)\}$ (resp. in $\bar{\nabla} = \{\bar{\nabla}(1), \ldots, \bar{\nabla}(n)\}$).

The algebra A is called standardly stratified, if $A \in \mathcal{F}(\Delta)$. Note that this generalizes the notion of a quasi-hereditary algebra where in addition it is required that the modules $\Delta(i)$ have trivial endomorphism rings.

In a fundamental article it was shown by C.M. Ringel, that the categories $\mathcal{F}(\Delta)$ and $\mathcal{F}(\bar{\nabla})$ are functorially finite in $\mathrm{mod}\, A$, and $\mathcal{F}(\Delta) \cap \mathcal{F}(\bar{\nabla}) = \mathrm{add}\, T$ for a tilting and cotilting module T, provided A is quasi-hereditary.

This situation is generalized to standardly strafified algebras by the following

Theorem: Let A be a standardly stratified algebra. Then $\mathcal{F}(\Delta) \cap \mathcal{F}(\bar{\nabla}) = addT$ for a tilting module T. The module T is a cotilting module iff A is quasi-hereditary.

The construction of the Ringel-dual of a quasi-hereditary algebra can also be generalized into the context of standardly stratified algebras.

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On the Importance of Being Non-Hausdorff for Certain Orbit Spaces, and a Representation-Theoretic setting for such Phenomena.

[Abstract for the Bielefeld Conf (July 31 - Sep 5, 1998) on Representation Theory of Algebras.]

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This work arose in an effort to understand the 'space' of all LABELLED n-point 'Configurations' in a Projective Space - say $\mathbb{P}^{m-1}$ (of dimension $m-1$): As regards the 'topology' there is no problem if one works (let's say, to be on the safe side) over Complex Numbers $\mathbb{C}$; of course we must allow the 'points' to be repeated (in order to come closer to some 'space' which is Compact). But that's not enough: one further requires, on one hand some restriction, and also a (little bit of) enlargement (in our set of Configurations to be studied) – to be able to begin at the 'right' (compactified) object (in the most liberal setting): On one hand we may want to assume that our (multi-)set of n points span all of $\mathbb{P}^{m-1}$ (for otherwise we may as well work with smaller value of m); but also, if one denotes the resulting set of Configurations by $Z^* = Z^*n, m$, one should also include further degeneracies by incorporating (as limit points) all 'smaller spaces' (for same m but all lower values of n – remembering we're not permitted to make n lower than m itself).

It has been pointed out in the work of Gelfand et al (see e.g. the paper in Advances/Math 1987, with Goresky, MacPherson and Serganova) that the resulting union (of Z^*r, m's for r ranging from m to n) – call it $Z! = Z^!n, m$ – arises (somewhat more) naturally as an orbit space X/T, where $X := Gras_m(n)$ is the Grassmann (projective) variety of m-dimensional linear subspaces of 'the' n-dimensional vectorspace, and T is essentially the maximal torus of the group $G = PGL_n$ that acts naturally on X; however, there is no harm in working with the subgroup T of the diagonal matrices of determinant 1 inside GL_n. That makes $Z! = X/T$ surely compact; it has a dense subset with a natural algebraic structure as a variety of dimension $d = d(n, m) = dimX - dimT = (l-1)(m-1)$ where $l = n - m$. However, as is widely known, putting further structure [say, algebric OR (complex-)analytic] on any such set of ALL 'orbits' is considered a close-to-hopeless task. [The search for a most satisfactory solution of such a general problem ought to be included in the list of 'post-Hilert' Problems for the Next Century.] The well-known G.I.T. (Geometric Invariant Theory) of Mumford provides an approximate solution to this question of Algebraic Structure on $Z! = X/T$, on replacing it by $Z = X//T := Proj(Q)$ for the ring Q of all T-invariant functions in the full ring R of functions on the affine cone above X (in the standard Plücker embedding). But it has very complicated structure: Last year some breakthrough for the $m = 2$ case was reported in my work with S. R. Ghorpade [only a Long Abstract available yet]; in particular, Q is genarally (perhaps for all values of $l\&m > 2$) NOT generated by its lowest proper nonzero component (in other words, Z is just a 'weighted Projective Variety' - without a god-given Projective Embedding).

Let me describe, in this (longish) Abstract, only the first half of this title, via my recent attempt to go beyond the G.I.T.; which was reinforced on my noticing an isolated sentence in the above-cited paper of Gelfand et al, to the effect that the natural topology on the compact space X/T is NOT Hausdorff. What I shall do is to show how a certain "better approximation" $Z\tilde{}$ – that lies 'midway' to Z and $Z!$ also fails to be Hausdorff (except when $n, m = 4, 2$). Yet, we shall see that $Z\tilde{}$ has a natural structure of (non-projective) Algebraic (hence also $\mathbb{C}$-analytic) Variety. While the study of $Z!$ is a hard task – to be postponed til we acquire better technology, it is a natural suggestion that the space $\tilde{Z}$ (described below) is better suited for immediate independent study; one major reason for this is the fact that the Weyl group $W = S_n$ acts (not only on the space $Z!$ but also) on this $Z\tilde{}$.

Since X is commonly viewed as obtained by glueing together a (natural) 'atlas' (or patch-up) of Affine Open Spaces – denoted X_w – which are actually d-dimensional Affine Spaces (the W-images of the so-called 'Big Cell' X_1 where the last subscript refers to the identity element of W) in the Bruhat Decomposition), we should ask as to what happens when we try to glue together the $n - choose - m$ images of the Mumford type quotients $Z_w = X_w//T$, the said gluing being canonical along the "T-quotients" of various intersections of X_w's (studied for long by the Lie-theorists).

Our main finding is that Z_1 (and hence each Z_w) is a Toric Projective Variety; its homogeneous coordinate ring $Q_1 = {Q_1}^{l,m}$ is the ring of $T\tilde{}$ invariants in the affine ring of the said Big Cell X_1 given by $S_1 = \mathbb{C}[Y]$ where $Y = [y_{i,j}]$ is the lxm 'generic matrix', and $T\tilde{} = T'xT$" is the product of the 2 maximal tori T' and T" inside SL_l and SL_m resp (acting on lxm matrices as usual by left and right multiplications). This ring Q_1 is explicitly described as the span of monomials $y^(a)$ where a runs over the set A of all Magic Rectangles of size lxm [the symbolic power $y^(a)$ means the product of all $(y_{i,j})^{a_{i,j}}$

for $a = [a_{i,j}]$, AND the set A consists of matrices a of non-negative integer entries with all row-sums equal as well as all column-sums equal too]; that Z_1 is a (normal) toric variety is then obvious from definitions. The common 'geometry' (needed for the said glueing) of various Z_w's is related to the fact that S_w (= the Affine Ring of X_w) is explicitly realized as the localization of the earlier ring R at the Plücker coordinate corresponding to (the coset of) w in $W/\tilde{W}$ (where $\tilde{W}$ is the Young subgroup $S_l x S_m$ inside $W = S_n$; then the w-conjugate of $\tilde{W}$ acts on X_w and Z_w.

Some specific examples (for low values of l and m) have been computed (and will be shown in the talk); there are many problems at two levels: to understand the Z_1's on their own (together with the given action of $\tilde{W}$), and to understand their patch-up $\tilde{Z}$ in contrast with the Mumford quotient Z.

So the moral is that the seeming 'pathology' of Non-Hausdorffness arises quite naturally (and hence ought not to be treated as a pathology) for $\tilde{Z}$ - which is a Complete and "Non-Separated" (hence Non-Projective) Variety. Since each individual Z_w is already complete, one can find many 'curves' in the intersection whose limits in diverse Z_w's are distinct (and hence NOT identified – which is the reason for failure to be Hausdorff).

The cell structure of diagram algebras

Changchang Xi

This talk will present some developments on cellular algebras, especially on the Morita-equivalence of cellular algebras and on the cell structure of the socalled diagram algebras: Temperley-Lieb algebras, Brauer algebras, partition algebras and Birman-Wenzl algebras; all these algebras can be defined by certain kind of diagrams. Recall that cellular algebras were introduced by Graham and Lehrer to handle a large class of important algebras. We shall show certain relations of the cell structures between an algebra and its basic algebra. Moreover, we shall show that

(1) Temperley-Lieb algebras are cellular,
(2) Brauer algebras are cellular,
(3) Partition algebras are cellular,
(4) Birman-Wenzl algebras are cellular.

Note that we also can determine the quasi-heredity of these diagram algebras from their cell structure.

Let us remark that the first two statements are well-known, and the last two seem to be new.

Some of the work reported here are done jointly with S. König.

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A Construction of Symmetric Algebras

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This is a report on joint work with Yosuke Ohnuki and Kaoru Takeda.

All algebras are finite dimensional over a field.

"It seems that the notion of the symmetric algebra depends on the base field, which is the reason why any structural characterization of symmetric algebras is not known." (Azumaya and Nakayama, 1956) In this talk we give a construction of symmetric algebras by making use of a property of a finite field extension of the base field.

Let L be a finite field extension of a field K and let $\alpha : L \times L \to L$ be 2-cocycle of the K-algebra L. For an arbitray finite quiver Q without oriented cycles, define a 2-cocycle $\hat{\alpha} : LQ \times LQ \to D(LQ)$ of the K-algebra LQ as follows:

$$\hat{\alpha}(x, y) = \sum_i \alpha(e(i)xe(i), e(i)ye(i)),$$

where D denotes a standard duality and $e(i)$ is the idempotent of LQ corresponding to the vertex i of Q. By $T(\Lambda, \beta)$ we denote the (Hochschild) extension algebra of a K-algebra Λ by the module $D(\Lambda)$ and a 2-cocycle β. Then the main theorem is stated as follows:

Theorem *The following properties are equivalent.*
(1) *Every $T(L, \alpha)$ is symmetric.*
(2) *The extension algebra $T(LQ, \hat{\alpha})$ is symmetric for some finite quiver Q without oriented cycles.*
(3) *The extension algebra $T(LQ, \hat{\alpha})$ is symmetric for any finite quiver Q without oriented cycles.*

Moreover we show a concrete example of a non-symmetric extension algebra corresponding to a non-zero element of $H^2(A,A)$ for a simple algebra A. (It is proved by Eilenberg, Rosenberg and Zelinski that $H^n(A,A) \neq 0$ for any inseparable simple algebra A and $n \geq 0$.) A self-injective algebra stably equivalent to a self-injective algebra of tilted type is characterized by a property of its Auslander-Reiten quiver. The example also gives a counterexample for the following question: *Does a stably equivalence between a self-injective algebra and a self-injective algebra of tilted type imply a derived equivalence?*

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A.Yakovlev (St.Petersburg, Russia). *Artinian modules over one matrix ring.*

Let $\mathbb{Z}$, $\mathbb{Q}$ be the ring of integers and the field of rational numbers. Denote by Λ the matrix ring

$$\begin{pmatrix} \mathbb{Q} & 0 \\ \mathbb{Q} & \mathbb{Z} \end{pmatrix} = \left\{ \begin{pmatrix} \alpha & 0 \\ \beta & m \end{pmatrix} \middle| \ \alpha, \beta \in \mathbb{Q},\ m \in \mathbb{Z} \right\}.$$

The set I of matrices with $\alpha = 0$ is a left ideal of Λ; denote by Q the Λ-module Λ/I. We call an Artinian Λ-module X *reduced* if X does not contain any direct summand isomorphic to Q and $mX = X$ for every integer $m \neq 0$.

Proposition. *Every Artinian Λ-module decomposes into the direct sum of finite module, reduced module and several copies of the module Q, and this decomposition is unique up to isomorphism.*

Let $T(X)$ be the torsion subgroup of a Λ-module X considered as an abelian group. Then $X/T(X)$ is a linear space over $\mathbb{Q}$; we call *rank* of X and denote $\operatorname{rk} X$ the dimension of this space.

For an abelian torsion free group A we denote by $E(A)$ its rational hull. Define a full subcategory $\mathfrak{A}'$ of the category of reduced torsion free abelian groups of finite rank: a group A is contained in $\mathfrak{A}'$ if and only if $E(A)/A$ is an Artinian group.

Theorem 1. *The category of reduced Artinian Λ-modules ${}_\Lambda\mathfrak{M}'$ is canonically equivalent to the category $\mathfrak{A}'$.*

Sketch of the Proof. Let A be a group from the category $\mathfrak{A}'$. Denote by π the canonical epimorphism $E(A) \to E(A)/A$ and make the group of columns $G(A) = \{(a, \bar{b})^{\mathrm{T}} \mid a \in E(A), \bar{b} \in E(A)/A\}$ into a left Λ-module in the following way:

$$\begin{pmatrix} \alpha & 0 \\ \beta & m \end{pmatrix} \begin{pmatrix} a \\ \bar{b} \end{pmatrix} = \begin{pmatrix} \alpha a \\ m\bar{b} + \pi(\beta a) \end{pmatrix} \quad \text{for any } \begin{pmatrix} a \\ \bar{b} \end{pmatrix} \in G(A),\ \alpha, \beta \in \mathbb{Q},\ m \in \mathbb{Z}\,.$$

The mapping $A \to G(A)$ produces a functor $\mathfrak{A}' \to {}_\Lambda\mathfrak{M}'$, and it is easy to check that this functor is an equivalence of categories.

Many examples of torsion free abelian groups of finite rank for which the Krull-Schmidt theorem fails are known; it follows from the above theorem that it fails in the category of Artinian Λ-modules as well. Morover, the following is true.

Theorem 2. *Let $n = r_1 + \cdots + r_s = l_1 + \cdots + l_t$ be two partition of a positive integer n into sums of positive integers, and let s_0 and t_0 be the numbers of the summands r_i (respectively, l_j) which are equal to 1. There exists an Artinian Λ-module X of rank n for which there are two decompositions $X = Y_1 \oplus \cdots \oplus Y_s = Z_1 \oplus \cdots \oplus Z_t$ into the direct sums of indecomposable modules, such that $\operatorname{rk} Y_i = r_i$, $\operatorname{rk} Z_j = l_j$ for every i, j, if and only if the following conditions are fulfilled: (i) $1 < s, t < n$; (ii) $r_i \leq n - t_0$ for all i, $1 \leq i \leq s$, $l_j \leq n - s_0$ for all j, $1 \leq j \leq t$; (iii) if $r_i = n - t_0$ for an index i, then only one of the integers l_j differs from 1, and, similarly, if $l_j = n - s_0$ for an index j, then only one of the integers r_i differs from 1.*

The existence of Artinian modules for which the Krull-Schmidt theorem fails was proved by A.Facchini, D.Herbera, L.S.Levy, P.Vamos in their paper in Proc. Amer. Math. Soc., vol. 123 (1995), issue 12. But they did not give any explicit example, and the fact that the Krull-Schmidt theorem does not hold for Artinian modules over so small ring as the ring Λ is rather surprising.

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX

On the Auslander-Reiten Quiver with Oriented Cycles of Representation-finite Algebras

Hailou YAO

Let A be a basic, connected finite-dimensional algebra over an algebraically closed field with finite representation type, $\alpha(A)$ be the maximum of the set consisting of numbers of indecomposable summands in the middle term of Auslander-Reiten sequences in modA. In this paper I show that if $\alpha(A)$=1, then Γ_A contains oriented cycles if and only if Γ_A contains DTr-periodic modules. When $\alpha(A) \geq 2$ I give counterexamples to the assertion.

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Categorical aspects of generating functions.

Tomoyuki Yoshida

Let $\mathcal{E}$ be a skeletally small category with finite hom-sets. Then the **generating function** of $\mathcal{E}$ is a formal summation

$$\mathcal{E}(t) := \sum_{X \in \mathcal{E}/\cong} \frac{1}{|\mathrm{Aut}(X)|}\, t^X.$$

A (strict) **KS-category** $\mathcal{E}$ is a category with finite coproducts in which each object X has a strictly unique decomposition $X = \coprod I_\alpha$ into a finite coproduct of connected objects of $\mathcal{E}$. We denote by $\mathrm{Con}(\mathcal{E})$ the full subcategory of connected objects. Then the **exponential formula** holds:

Theorem. $\mathcal{E}(t) = \exp(\mathrm{Con}(\mathcal{E})(t))$ *for a KS-category* $\mathcal{E}$.

There are many applications of this exponential formula, e.g., an enumeration of group homomorphisms from a finite group to a symmetirc group S_n, an enumeration of rooted trees.

The our theory is closely related to Joyal's species: a species is bijectively correspondent with a faithful functor with finite fibers from a groupoids to Set_f. We can generalize operations on species to categories and faithful functors, for example, summation, multiplication, derivation, substitution, and so on. In particular, the substitution $S(\mathcal{F})$, where $\mathcal{F}$ is a category and

$S : \mathcal{E} \longrightarrow \mathrm{Set}_f$ is a faithful functor, is given by the wreathed product of categories.

In general, neither the composition $\mathcal{F}(\mathcal{E}(t))$ nor the derivation $d\mathcal{E}(t)/dt$ can be defined. To solve this inconvenience, we need to extend the definition of power series. For a locally finite topos $\mathcal{E}$, the required ring of a power series with coefficients in a Mackey functor with multiplicative transfer is defined by $\varprojlim \boldsymbol{T}(\mathcal{E}/\Omega^N)$.

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EQUIPPED POSETS OF FINITE GROWTH

A.G.Zavadskij

An equipped poset is a poset with some simple additional structure. Namely, its points are divided into single and double, and the order relations between points are divided into weak and strong in such a way, that if a¡b is a weak relation, then both points a,b are double and, moreover, in the case $a < c < b$ the relations $a < c$ and $c < b$ are weak too. Each equipped poset naturally defines a matrix problem of mixed type over the fields R and C of real and complex numbers. In the limiting case (when there is no double points) one obtains the ordinary problem on representations of posets in the sense of [NR] over the field R. Note, that the representation-finite equipped posets were described in fact in [KS], and the one-parameter ones in [ZZ]. The aim of the talk is to present a finite-growthness criterion for equipped posets, extending the corresponding criterion for ordinary posets [ZN]. The proof of the criterion is based on eleboration and application of a new differentiation algorithm generalizing to the equipped situation the differentiation with respect to a pair of points [Z].

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Twelve theorem for weakly positive unit quadratic forms

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A vector $x = (x_1, \ldots, x_n) \in \mathbb{R}^n$ is called: positive if $x \neq 0$ and $x_1 \geq 0, \ldots, x_n \geq 0$; sincere if $x_1 \neq 0, \ldots, x_n \neq 0$. A quadratic form $f(x) = \sum_{i,j=1}^n a_{ij}x_ix_j$ $(a_{ij} = a_{ji} \in \mathbb{R})$ is said to be: weakly positive if $f(x) > 0$ for every positive vector x; unit if $a_{ii} = 1$, $2a_{ij} \in \mathbb{Z}$ $(i, j = 1, \ldots, n)$. A vector $z \in \mathbb{Z}^n$ is called a root of a form f if $f(z) = 1$. A unit form f is said to be sincere if there exists sincere positive root of the form f.

Such forms and their roots play an important role in modern representatin theory (cf. [1]).

Theorem *If $z = (z_1, \ldots, z_n)$ is a positive root of weakly positive unit quadratic form f, then for all $i = 1, \ldots, n$ either $z_i = 0$ or $\sum_{j=1}^n |a_{ij}|z_j \leq 12$ /the evaluation 12 is exact and is realized on Tits form of poset (1, 2, 4)/.*

Corollary 1 *The all $z_i \leq 6$ (thix theorem of S. Ovsienko).*

Really, if $z_i \neq 0$ and $\bar{z}_i = -z_i - \sum_{j\neq i} 2a_{ij}z_j$ is i-th coordinate of reflected at i positive root z, then $z_i \leq \bar{z}_i + 1$ since f is weakly positive. Therefore $4z_i \leq 3z_i + \bar{z}_i + 1 \leq 2 \cdot 12 + 1 = 25$, hence $z_i \leq [\frac{25}{4}] = 6$.

Corollary 2 *If f is a sincere weakly positive unit quadratic form and Γ is corresponding to f bigraph (for example, let Γ is a bigraph of Shurian free exact box $\mathfrak{U}$ of finite type and f is its Tits form), then for any vertex i of Γ its degree $s_i = \sum_{j\neq i} |2a_{ij}|$ is bounded by fixed constant $C = 22$ ($s_i \leq 22$ is exact evaluation).*

The proof of the Theorem is reduced to the case of special class of sincere weakly positive unit forms containing Tits forms of exact posets of finite type (and, moreover, graphical ones [1]) as well as Tits forms of exact pairs of posets of finite type (and, moreover, “graphical pairs” ones). This class is inductively calculated by computer as finite list of 512 forms in at most 12 variables. So we obtain new proof for Kleiners lists of exact posets and exact pairs of posets of finite type as well as for Ringel's list of graphical sincere weakly positive unit forms (and also, we obtain a new list of "graphical pairs" ones).

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Representations as elements in affine composition algebras and quantum groups

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Let A be a finite-dimensional hereditary algebra over a finite field. C. M. Ringel and J. A. Green have shown that the generic version of the twisted composition algebra $\mathcal{C}_*(\mathcal{A})$ is isomorphic to the positive quantized enveloping algebra U^+.

Quite different from the case of A being representation-finite, for A being of infinite type only some special A-modules lie in $\mathcal{C}_*(\mathcal{A})$. Indeed, it is known that some important modules have this property, e.g. Ringel observed that a stone lies in $\mathcal{C}_*(\mathcal{A})$; we have proved the following: if A is tame and M is indecomposable, then $[M] \in \mathcal{C}_*(\mathcal{A})$ if and only if M is a stone.

The aim of this paper is to classify general modules M over a tame hereditary algebra such that $[M] \in \mathcal{C}_*(\mathcal{A})$. The main result is as follows

(For an Auslander-Reiten component C and a module M, by $M \in C$ we mean that each indecomposable direct summand of M is in C. Let T be a non-homogeneous tube with quasi-simples $E(1), \cdots, E(m)$, and let $M \in T$. Then $\mathbf{dim}M$ can be uniquely written as $\mathbf{dim}M = c_1\mathbf{dim}E(1) + \cdots + c_m\mathbf{dim}E(m)$. We call $(c_1, \cdots, c_m) \in \mathbb{N}_0^m$ the quasi-dimension vector of M, and is denoted by $\mathbf{q}.\mathbf{dim}M = (c_1, \cdots, c_m)$.)

Theorem *Let A be a finite-dimensional tame hereditary algebra over a finite field, and let $M = P \oplus R \oplus I$ be a A-module with P, R, and I being preprojective, regular, and preinjective, respectively. Let $R = R_1 \oplus \cdots \oplus R_m$ with $R_i \in T_i$, where T_i is a tube of rank n_i, $\forall 1 \le i \le m$, and $T_i \ne T_j$ $(i \ne j)$. Then $[M] \in \mathcal{C}_*(\mathcal{A})$ if and only if $n_i \ge 2$, and $\mathbf{q}.\mathbf{dim}R_i$ are non-sincere, $1 \le i \le m$.*

The proof will use the following structure theorem

Theorem *Let* $r_{\mathbf{d}} = \sum[M]$, *where* M *runs over regular modules with* $\mathbf{dim}M = \mathbf{d}$, *and let* $\mathcal{T}$, $\mathcal{P}$, *and* $\mathcal{I}$ *be the subalgebras generated by all* $r_{\mathbf{d}}$, $\mathbf{d} \in \mathbb{N}_0^n$, *by preprojectives, and by preinjectives, respectively. Then* $\mathcal{C}_*(\mathcal{A}) = \mathcal{P} \cdot \mathcal{T} \cdot \mathcal{I}$.

And also use derivations δ_E by separating quasi-semisimples E inside $\mathcal{T}$. For example, we have the following (Call a direct sum of quasi-simples a quasi-semisimple. A quasi-semisimple $E = E_1 \oplus \cdots \oplus E_t$ is said to be separating provided that if $t \geq 2$ then $Ext_A^1(E_i, E_j) = 0$ for $i > j$.)

Theorem *(i) Let* S *be an arbitrary quasi-simple module. Then* $\delta_S \in Hom_{\mathbb{Q}}(\mathcal{T}, \mathcal{T})$. *More precisely, we have*

$\delta_S(r_{\mathbf{d}_1} \cdots r_{\mathbf{d}_t}) =$

$$\frac{1}{a_S} \sum_{0 \leq i \leq t-1} q^{-\langle \mathbf{d}_1 + \cdots + \mathbf{d}_{i+1} - \mathbf{dim}S, \mathbf{dim}S \rangle} r_{\mathbf{d}_1} \cdots r_{\mathbf{d}_i} \cdot r_{\mathbf{d}_{i+1} - \mathbf{dim}S} r_{\mathbf{d}_{i+2}} \cdots r_{\mathbf{d}_t}.$$

(ii) Let $E = E_1 \oplus \cdots \oplus E_t$ *be a separating quasi-semisimple module with* $t \geq 2$ *and* $Ext_A^1(E_i, E_j) = 0$ *for* $i > j$. *Then* $\delta_M \in Hom_{\mathbb{Q}}(\mathcal{T}, \mathcal{T})$.

(iii) Let M *be a module with* $[M] \in \mathcal{T}$. *Then* $\delta_M \in Hom_{\mathbb{Q}}(\mathcal{T}, \mathcal{T})$.

Corollary *Let* T *be a tube.*

(i) If $M \in T$, *then* $[M] \in \mathcal{C}_*(\mathcal{A})$ *if and only if* T *is non-homogeneous and* $[M] \in \mathcal{C}_*(\mathcal{T})$, *where* $\mathcal{C}_*(\mathcal{T})$ *is the composition algebra of* T.

(ii) Let $[M] \in \mathcal{C}_*(\mathcal{A})$ *and let* M_1 *and* M_2 *be arbitrary two indecomposable direct summands of* M *with* $M_1, M_2 \in T$. *Then at least one of* (M_1, M_2) *and* (M_2, M_1) *is a separating pair.*

(The pair (U, V) *is called a separating pair provided that both* U *and* V *are indecomposable and* $Ext_A^1(V, U) = Ext_A^1(U, U) = Ext_A^1(V, V) = 0$.*)*

Some Results About Tame Ringel-Hall Algebras

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Let $\mathcal{H}(A)$ and $\mathcal{C}(A)$ denote respectively the Ringel-Hall algebra and the composition algebra of a finite dimensional algebra A over a finite field k. By definition, $\mathcal{H}(A)$ is the $\mathbf{Q}$-space with basis of the set of isomorphism classes $\{u_M\}$ of finite A-modules with the following multiplication

$$u_{N_1} u_{N_2} = \sum_M C_{N_1 N_2}^M u_M$$

where $C^M_{N_1N_2}$ is the number of the submodules N of M with $N \simeq N_2$ and $M/N \simeq N_1$, $\mathcal{C}(A)$ is the subalgebra of $\mathcal{H}(A)$ generated by all simple A-modules.

First, we consider the degenerate Ringel-Hall algebra of a finite conected valued Auslander-Reiten quivers. Let $\Gamma = (\Gamma_0, \Gamma_1, \tau)$ be a connected valued Auslander-Reiten quiver with valuation $(1, d)$ or $(d, 1)$ and $d \leq 3$. For any field k , there exists a standard algebra $\Lambda(k, \Gamma)$ with $\Gamma_{\Lambda(k,\Gamma)} = \Gamma$, where $\Gamma_{\Lambda(k,\Gamma)}$ is the Auslander-Reiten quiver of $\Lambda(k, \Gamma)$

Let $\mathcal{K}$ be an infinite set of some finite fields up to isomorphism. We consider the product $\prod\limits_{k\in\mathcal{K}} \mathcal{H}(\Lambda(k))$ of Hall algebras , where $\Lambda(k) = \Lambda(k, \Gamma)$ is the standard algebra over k , it is an associative ring. Denots by $\mathcal{H}(\Gamma, q)$ the subring of $\prod\limits_{k\in\mathcal{K}} \mathcal{H}(\Lambda(k))$ generated by the elements $u_a = (u_a^{(k)})_{k\in\mathcal{K}}$, $a \in \mathcal{B}(\Gamma) = \{\text{all finite functions } \Gamma_0 \to \mathbf{N_0}\}$, and $q = (q^{(k)})_{k\in\mathcal{K}}$, where $q^{(k)} = |k|u_0^{(k)}$, and $u_a^{(k)}$ denotes the $\Lambda(k)$-module $\bigoplus\limits_{x\in\Gamma_0} a(x)M(k, x)$, where $M(k, x)$ is the indecomposable $\Lambda(k)$-module corresponding to vertex $x \in \Gamma_0$. $\mathcal{H}(\Gamma, q)$ is called the generic Ringel-Hall algebra of Γ on $\mathcal{K}$. Note that q is a center element of $\mathcal{H}(\Gamma, q)$, the quotient ring $\mathcal{H}(\Gamma, q)/(q-1)\mathcal{H}(\Gamma, q)$, denoted by $\mathcal{H}(\Gamma)_1$, is called the degenerate Hall algebra of Γ on $\mathcal{K}$.

Let $F : \tilde{\Gamma} \longrightarrow \Gamma$ be the universal covering with the foundamental group G , where Γ is a finite conneted valued Auslander-Reiten quiver. Notice that G is a free group, Γ and $\tilde{\Gamma}$ have valuation $(1, d)$ or $(d, 1)$ with $d \leq 3$.

Lemma 1. For any $x \in \Gamma_0, a \in \mathcal{B}(\Gamma)$, there exists an infinite set $\mathcal{K}$ of some finite fields up to isomorphism, such that, for any $k \in \mathcal{K}$ and any $z \in \Gamma_0, |k| - 1$ divides

$$C^{M(k,z)}_{M(k,x),M(k,a)} - \sum_{g,h\in G} C^{hM(k,\tilde{z})}_{gM(k,\tilde{x}),M(k,\tilde{a})}$$

$$C^{M(k,z)}_{M(k,a),M(k,x)} - \sum_{g,h\in G} C^{hM(k,\tilde{z})}_{M(k,\tilde{a}),gM(k,\tilde{x})}$$

where $F : \tilde{\Gamma} \longrightarrow \Gamma$ is the universal covering with the foundamental group G , $\tilde{x}, \tilde{z} \in \tilde{\Gamma}_0, \tilde{a} \in \mathcal{B}(\tilde{\Gamma}), F\tilde{x} = x, F\tilde{z} = z, F\tilde{a} = a$. In particularly , there exist integers Υ^z_{xa} and Υ^z_{ax} such that for any field $k \in \mathcal{K}$, $|k| - 1$ divides

$$C^{M(k,z)}_{M(k,x),M(k,a)} - \Upsilon^z_{xa}$$

$$C^{M(k,z)}_{M(k,a),M(k,x)} - \Upsilon^z_{ax}$$

Theorem 2. Let $\mathcal{K}$ be an infinite set of some finite fields up to isomorphism, and $\mathcal{H}(\Gamma)_1$ the degenerated Ringel-Hall algebra of Γ on $\mathcal{K}$. Then the structural coefficient of $\mathcal{H}(\Gamma)_1$ is rational numbers which are independent of $\mathcal{K}$.

Let Γ be a finite connected valued Auslander-Reiten quiver and $F : \tilde{\Gamma} \longrightarrow \Gamma$ the universal covering with the foundamemtal group G . We denote by $\mathcal{L}(\Gamma) = \bigoplus_{x\in\Gamma_0} \mathbf{Z}u_x$ and $\mathcal{L}(\tilde{\Gamma}) = \bigoplus_{\tilde{x}\in\tilde{\Gamma}_0} \mathbf{Z}u_{\tilde{x}}$ the free abelian groups respectively with the basis $\{u_x \mid x \in \Gamma_0\}$ and $\{u_{\tilde{x}} \mid \tilde{x} \in \tilde{\Gamma}_0\}$.

Theorem 3. For any finite connected valued Aslander-Reiten quiver Γ, $\mathcal{L}(\Gamma)$ is a Lie subalgebra of $\mathcal{H}(\Gamma)_1$ and is isomorphic to the "orbit" Lie algebra $\mathcal{L}(\tilde{\Gamma})/G$, where $\mathcal{H}(\Gamma)_1$ is the degenerate Ringel-Hall algebra of Γ on $\mathcal{K}$ and $\mathcal{K}$ is an infinite set of finite fields up to ismorphism. Furthermore, $\mathcal{H}(\Gamma)_1 \otimes_{\mathbf{Z}} \mathbf{Q}$ is the universal enveloping algebra of $\mathcal{L}(\Gamma) \otimes_{\mathbf{Z}} \mathbf{Q}$.

Finally, we consider the tame composition algebras of non-simple-laced types. Let A be a finite dimensional hereditary algebra over a finite field k, and $\mathcal{C}(A)$ the composition algebra of A. Let $\mathcal{P}$(resp. $\mathcal{I}$) be the subalgebra of $\mathcal{C}(A)$ generated by the preprojective (resp. preinjective) A-modules , and $\mathcal{T}$ the subalgebra generated by $r_{\mathbf{d}}$, where $r_{\mathbf{d}} = \sum[M] \in \mathcal{H}(A), [M]$ runs over the isomorphism classes of the regular A-modules with dimension vector $\mathbf{d}$.

Theorem. 4. Let A be one of the type $\tilde{B}_n$, $\tilde{C}_n$, $\widetilde{BC}_n$, $\widetilde{BD}_n$, $\widetilde{CD}_n$, $\tilde{F}_{41}$, $\tilde{F}_{42}$, $\tilde{G}_{21}$, $\tilde{G}_{22}$ or $\tilde{A}_{11}$. Then $\mathcal{C}(A)$ has triangular decomposition, i.e, $\mathcal{C}(A) = \mathcal{P} \cdot \mathcal{T} \cdot \mathcal{I}$.

Number of parameters of module varieties in a wild category

Yingbo Zhang

Let $\Lambda = k[\varepsilon]/(\varepsilon^2)$, $R = \Lambda[t]$, $\mathcal{C} = \{M \in \operatorname{mod} R \mid {}_\Lambda M$ is projective$\}$, a subcategory of finite-dimensional module category of R, which is of wild type. If $\mathcal{V}$ is a variety, with an algebraic group G action. $\mathcal{V}(s) = \{M \in \mathcal{V} \mid \dim X = s\}$, then the number of parameters of $\mathcal{V}$ is defined by $\mu(\mathcal{V}) = \max_s(\dim \mathcal{V}(s) - s)$. Around 1980, Kac proved his remarkable result on finite-dimensional representations of finite quivers, namely if α is a root of the quiver, then $\mu(R(Q,\alpha)_{\text{ind}}) = 1 - q(\alpha)$. Afterwards, there are only very few results on the calculation of the number of parameters of wild categories.

In the present paper, we show that for each dimension $n \in \mathbb{Z}^+$, $\mathcal{V}(n) = \left\{ \begin{pmatrix} A & B \\ 0 & A \end{pmatrix} \,\middle|\, A, B \in M_n(k) \right\}$, $G(n) = \left\{ \begin{pmatrix} X & Y \\ 0 & X \end{pmatrix} \,\middle|\, X \in GL_n(k),\ Y \in M_n(k) \right\}$, and $G(n)$ acts on $\mathcal{V}(n)$ by conjugacy, then $\mathcal{V}(n)$ is the n-dimension module variety of $\mathcal{C}$. We prove that $\mu(\mathcal{V}(n)) = 2n$ and $\mu(\mathcal{V}(n)_{\text{indec}}) = n + 1$.

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A Combinatorial Description of Coils

Bin Zhu

Coils appear as components of AR-quivers of tame algebras, which are first formed by Assem and Skowroński in [1] [2]. They defined six admissible operations on a translation quiver, generalizing an idea of d'Este and Ringel in [3].

We will give some descriptions about complete coils, a special kind of coils in some sense, namely coils with property (P): if a vertex x is an injective(or projective), then x is either a projective-injective vertex, or an injective (or projective respectively) middle term of a mesh with three middle terms. Some special complete coils appear as connected components in the AR-quivers of selfinjective algebras, and as realizations of Frobenius funtions on a stable tube.

Let Γ be a translation quiver without multiple arrows.

We consider the conditions:

(a).There is a stable tube T, a cofinite subquiver T' of T, a cofinite subquiver Γ' of Γ, such that $\Gamma' = T'$, and $T \backslash T'$ is a set of non-intersecting wings.

(a'). There is a stable tube T, a cofinite subquiver T' of T, a cofinite subquiver Γ' of Γ, such that $\Gamma' = T'$, and $T \backslash T'$ is a wing without selfintersection.

(b).There is a length function f for Γ.

(c). Γ has property (P).

Theorem. Let Γ be a translation quiver without multiple arrows Then the following statements are equivalent.

1. Γ is a complete coil.
2. Γ satisfies conditions (a), $(b), (c)$.
3. Γ satisfies conditions (a'), $(b), (c)$.

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Derived equivalences, orders and group rings

Alexander Zimmermann

J. Rickard showed in his work [6] that two bounded derived module categories $D^b(A)$ and $D^b(B)$ for two rings A and B are equivalent if and only if there is a particular object, a so-called tilting complex T, in $D^b(A)$ such that B is the endomorphism ring of T. The functor linking A and B cannot be derived by the knowledge of T only.

In case A and B are R-algebras for a commutative ring R and if A is flat as R-module, then Keller [2] proved that there is a complex X in $D^b(A \otimes_R B^{op})$ such that $X \otimes_B^{\mathbb{L}} -$ induces an equivalence. Here, $T \simeq X \otimes_B B$.

We discuss the case of an R-order A and an R-algebra B over a Dedekind domain R. Examples [3] prove that even if A is an R-order, B need not to be so. We shall give a short proof that in case A is a symmetric R-torsion free R-algebra, also B is a symmetric R-torsion free R-algebra. Hence, the phenomenon in [3] cannot occur for symmetric orders. In case one knows already that B is R-flat, then this is a result of Rickard. We shall deduce that B is R-torsion free and hence flat.

In case A is a symmetric R-order and T is a tilting complex without torsion homology satisfying a further technical vanishing condition then one can describe B in terms of the homology of T only [4]. This gives an a priori determination of B. Under one more technical vanishing condition it is possible to construct the complex X explicitely [7].

We shall illustrate the above mentioned constructions first abstractly, and then on the example for a group ring of a special linear group in natural characteristic, following [1].

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Tree of reduction algorithms.

Rita Zuazua

This is a joint work with Raymundo Bautista.

Let $\mathcal{A}$ be a triangular free bocs of tame representation type. Then for each given dimension $\underline{d}$, there exists a finite tree with only one sink. To each vertex corresponds a bocs and to each arrow a reduction algorithm.

The sink corresponds to the bocs $\mathcal{A}$. The bocses corresponding to the sources are minimal.

Let h be a fixed natural number. We consider the family $\mathcal{F}$ consisting of the indecomposable representations M of $\mathcal{A}$ with $\underline{dim}M = \underline{d}$ and $dim_k End_{\mathcal{A}}(M) = h$.

We prove that $\mathcal{F}$ is given up to isomorphism by $\mu_{\mathcal{F}}$ one paremeter families, $\mu_{\mathcal{F}} \leq$number of sources.

A bound for $\mu_{\mathcal{F}}$ is given in terms of h, $\underline{d}$ and the number of a certain class of representations in $\mathcal{F}$ (singular representations).

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Classical groups adjoint actions invariants

Alexandr N. Zubkov

Let G be an orthogonal or symplectic group over some algebraicalyy closed field K. In the orthogonal case we presuppose that the characteristic of the ground field is not even. Let G is canonically embedded into the general linear group $GL(n, K) = GL(n)$. The group G acts on the space of m $n \times n$ matrices $M(n)^m = \underbrace{M(n) \times \ldots \times M(n)}_{m}$ diagonally by conjugations. The following problem was formulated by Procesi in [1]: What are the generators of the invariant algebra $K[M(n)^m]^G$? Procesi proved that in the characteristic zero case this invariant algebra is generated by values of trace on any products of "coordinate" matrices or transposed of ones (symplectically transposed in the symplectic group case). Not long ago professor Donkin has proved that in the case $G = GL(n)$ the invariant algebra $K[M(n)^m]^G$ is generated by the elements $\sigma_j(A_{i_1} \ldots A_{i_k})$, where $A_1, \ldots, A_m$

are the "matrix coordinates" on the space $M(n)^m$, σ_j – j-th coefficient of the characteristic polynomial [2]. This result generalized Procesi's theorem [1]. A few years later the author proved more general result than Donkin's one [3]. Not only generators but all relations between ones were found on the language of "mixed" or "trace" identities.

In this talk I would like to announce the following

Theorem 1 *Let G be a classical group as above. Then the algebra $K[M(n)^m]^G$ is generated by the elements $\sigma_j(B_{i_1}\ldots B_{i_k})$, where B_j is equal to either A_j or its transposed one (symplectically transposed).*

Moreover, if n is an add number then this theorem remains the same for the case when G is a special orthogonal group too.

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