Workshop Stable homotopy theory and algebraic K-theory

Bielefeld, February 22-24, 1999

The workshop was held at the Department of Mathematics of the University of Bielefeld and was hosted and funded by the SFB 343 "Diskrete Strukturen in der Mathematik".

Program:

Monday, February 22:

09:00	Welcome	
09:05	S. Bauer (Bielefeld)	Seiberg-Witten theory and stable homotopy
10:15	T. Pirashvili (Tbilisi)	Higher order Hochschild homology
11:45	M. Ando (U. Virginia/Johns Hopkins U.)	Calculating E_∞-maps out of Bott periodicity spectra and into complex-orientable spectra
14:30	W. Lück (Münster)	Isomorphism Conjectures in K- and L-theory and computations for certain crystallographic groups
16:00	B. Dundas (Trondheim)	Nerves of n-categories

Tuesday, February 23:

09:00	S. Betley (Warszawa)	Twisted homology of symmetric groups
10:15	B. Shipley (Purdue U.)	New models for rational equivariant stable homotopy
11:45	T. Goodwillie (Brown U.)	On Voevodsky's h-topology
14:30	J. Rognes (Oslo)	Two-primary algebraic K-theory of pointed spaces
16:00	J. Klein (Wayne State U.)	The dualizing spectrum of a topological group

Wednesday, February 24:

09:00	R. McCarthy (Urbana-Champaign)	Universal degree n versus universal n-excisive: a tale of two calculus towers and the analyticity that brought them together
10:15	M. Lydakis (Bielefeld)	Topological Cyclic Homology and Γ-spaces
11:45	J. Greenlees (Sheffield)	Equivariant connective K-theory and formal groups
14:30	J. Smith (Purdue U.)	Periodic homotopy types
16:00	M. Mahowald (Northwestern U.)	The topological modular forms spectrum associated to the subgroup $\Gamma_0(3)$ of $SL_2(\mathbb{Z})$

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CALCULATING E_∞-MAPS OUT OF BOTT PERIODICITY SPECTRA AND INTO COMPLEX ORIENTABLE SPECTRA

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For $t \geq 1$ the let $BU\langle 2t\rangle$ be the indicated connective cover of BU, and let $MU\langle 2t\rangle$ be its Thom spectrum. Then $MU\langle 2t\rangle$ and the spectrum $\Sigma^\infty BU\langle 2t\rangle_+$ are E_∞-ring spectra. The category of E_∞-ring spectra is a topological category, and I shall discuss the calculation of $\pi_* E_\infty[M, E]$, where M is $\Sigma^\infty BU\langle 2t\rangle_+$ or $MU\langle 2t\rangle$, and E is an E_∞-spectrum whose underlying ring spectrum is complex-orientable in the traditional sense.

In order to calculate $\pi_* E_\infty[M, E]$ one expects to have to understand what might be called the E-Dyer-Lashof algebra. It turns out that this algebra has a natural interpretation in terms of the isogeny structure of the formal group of E. M. Hopkins, N. Strickland, and I have developed a fairly elaborate program for analyzing this algebra, and we have made some progress in calculating $\pi_* E_\infty[M, E]$ when E is $K(1)$ or $K(2)$-local.

In my talk I shall concentrate on the case that E is p-adic K-theory. In that case the algebra which arises from our analysis is fairly tractable, but a useful residue remains of at least some of the subtleties we have encountered. Some points which I may touch on, time permitting, are:

1. The natural map $\pi_0 E_\infty[MU, K] \to H_\infty[MU, K]$ is an isomorphism. (The target set is known).
2. The calculation of $\pi_0 E_\infty[MU, K]$ gives a calculation of $\pi_0 E_\infty[\Sigma^\infty BU_+, K]$.
3. An explicit description of $\pi_{s\geq 1} E_\infty[\Sigma^\infty BU_+, K]$.
4. Bott periodicty gives a weak equivalence
$$E_\infty[\Sigma^\infty BU\langle 2t+2\rangle_+, K] \cong \Omega^2 E_\infty[\Sigma^\infty BU\langle 2t\rangle_+, K].$$

SEIBERG-WITTEN THEORY AND STABLE HOMOTOPY

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For a closed Riemannian four-manifold (X, g) the choice of both an orientation $or(X)$ and a $spin^c$-structure $\mathfrak{s}$ gives rise to an equivariant monopole map $\Psi = \Psi_{g,or(X),\mathfrak{s}}$ between certain geometrically defined affine Hilbert spaces. To define Seiberg-Witten invariants for four-dimensional manifolds, one considers the zero-sets of such monopole maps. More precisely, one considers such moduli spaces up to a certain bordism relation. If the relevant Hilbert spaces were finite dimensional, the Pontrijagin-Thom construction would relate the corresponding bordism groups to stable homotopy classes of maps. This analogue suggests to interpret Seiberg-Witten invariants as invariants of some kind of homotopy class of the monopole map Ψ.

Theorem 1: *The monopole map Ψ defines an element in an equivariant stable cohomotopy group $\pi^b_{U(1)}(Pic^0(X); ind(\mathcal{D}))$. The choice of a homology orientation determines a homomorphism $t : \pi^b_{U(1)}(Pic^0(X); ind(\mathcal{D})) \to \mathbb{Z}$, which maps Ψ to the Seiberg-Witten invariant $SW_X(or(X), \mathfrak{s})$.*

Here $Pic^0(X)$ denotes the Picard torus $H^1(X; \mathbb{R})/H^1(X; \mathbb{Z})$, the Dirac operator associated to the chosen $spin^c$-structure defines a virtual complex index bundle $ind(\mathcal{D})$ over the Picard torus, and $b = b_+(X)$ denotes the rank of the positive part of the intersection form on X. This cohomotopy group element is independent of the Riemannian metric g.

The main emphasis of the talk lies in the fact that this refined version actually contains more information than the integer valued Seiberg-Witten invariants. The main reason lies in the fact that there is torsion in some of the relevant cohomotopy groups. The main technical step is to show that the stable cohomotopy invariant defined by the monopole map behaves nicely when taking connected sums.

Theorem 2: *For $X = X_1 \# X_2$ a connected sum of 4-manifolds, the invariant $\Psi_X = \Psi_{X_1} \wedge \Psi_{X_2}$ is the smash product of the invariants of its summands.*

Here are two sample applications:

Corollary 3: *Let K denote the K3-surface and suppose there is an oriented diffeomorphism $X_1 \# K \cong X_2 \# K$, where the X_i are manifolds with $b^+ = 3 \bmod 4$. Then the integer Seiberg-Witten invariants of X_1 and X_2 are the same mod 2.*

Corollary 4: *Suppose the connected sum $\#_{i=1}^m E_i$ of simply connected minimal elliptic surfaces of odd geometric genus is diffeomorphic to a connected sum $\#_{j=1}^n F_j$ of elliptic surfaces. If $m < 4$, then $n = m$ and – up to permutation – the F_j and the E_i are diffeomorphic.*

Taking a more general point of view, the monopole map defines an element in some stable equivariant cohomotopy of the classifying space of the total gauge groupoid of four manifolds with $spin^c$-structure.

TWISTED HOMOLOGY OF SYMMETRIC GROUPS

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Let Γ be the category of finite pointed sets and let $T : \Gamma \to Ab$ be a functor. The main purpose of this talk is to study the stable behaviour of the groups $H_*(\Sigma_n; T([n]))$. The symbol $[n]$ denotes here the finite set $\{0, 1, ..., n\}$ and we are really interested in groups

$$\mathrm{colim}_n H_*(\Sigma_n; T([n]))$$

which will be denoted $H_*(\Sigma; T)$.

Let $\mathbf{R}$ denotes the category of finitely generated free modules over a given ring R. Let $F : \mathbf{R} \to Ab$ be a functor of finite degree. We proved in [1] that in such case

$$H_*(Gl(R); F) = 0$$

In the spirit of the further paper [2] we can spell this result out saying that stable K-theory groups for F , $K^s_*(R, F)$ are trivial for $0 \leq *$. Now we are going to calculate $K^s_*(\Sigma, T)$ for any functor $T \in Ab^\Gamma$. We show that these groups are highly nontrivial and the situation is completely different from the linear case. This is one of the reasons for our study. The calculations which we are going to present show that the belief that Σ_n is almost linear (over "the field with one element" see [3, p.272]) has its limits.

In the more general words all this is a starting point for the study of the abelian category Ab^Γ and the homological algebra over it. The importance of this category comes, among others things, from the stable homotopy theory. If $X_.$ is a pointed simplicial set then applying $T \in Ab^\Gamma$ to it degreewise we obtain a simplicial abelian group whose stable (co)homotopy is expressable in terms of Ext and Tor groups in Ab^Γ, compare [4, Prop.2.2]. As a side effect of our study we get a new proof of the stability for $H_*(\Sigma_n; T([n]))$.

[1] S. Betley. Homology of Gl(R) with coefficients in a functor of finite degree. *J. Algebra* **150** (1992) 73–86.

[2] S. Betley and T. Pirashvili. Stable K-theory as a derived functor. *J. of Pure and Appl. Alg.* **96** (1994), 245–258.

[3] W. van der Kallen. Homology stability for linear groups. *Inventiones Math.* **60** (1980) 269-295.

[4] T. Pirashvili. Hodge decomposition for higher order Hochschild homology. *Bielefeld preprint 1998.*

NERVES OF n-CATEGORIES

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The nerve functor from the category Cat of small categories to the category of simplicial sets $\mathcal{S}$ gives the interpretation of Cat as the linear functors from Δ^o to sets, that is, the functors that preserve the pullbacks that exist in Δ^o. The notion of an n-category is perhaps easiest understood in a similar manner as a special type of multi linear functor from $\Delta^o \times \ldots \times \Delta^o$ to sets. But there is a general sentiment that nerves should be spaces, and I will try to address the question of how to encode n-categories as spaces without loosing structure.

The answer I will give came as a complete surprise to me, but people with more fundamental insight into classical algebraic topology may well feel that there is nothing new under the sun. After all, in special cases the construction I offer has been with us for many years, but until now, I for one thought the explicit formulas in the literature were just unfortunate consequences of a rare occurence of uglyness in algebraic topology.

EQUIVARIANT CONNECTIVE K-THEORY AND FORMAL GROUP LAWS

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In my talk I will discuss the following circle of ideas. Some of the papers are available from my webpage at `http://www.shef.ac.uk/~pm1jg/` .

In the non-equivariant world, connective K-theory is an extremely useful cohomology theory. It is not much harder to calculate than periodic K-theory, but can be a lot more powerful. Here is a list of desirable properties for an G-equivariant form of connective K-theory.

Consider the following properties of a G-spectrum E.

1. E is a split ring spectrum, and nonequivariantly ku.
2. Periodic equivariant K-theory arises from E by inverting the Bott element: $E[1/v] \simeq K$
3. E is complex orientable (equivariantly).
4. E^*_G is a Noetherian ring.

Theorem 1: [3] *If G is of prime order p, there is a G-spectrum ku with all these properties. It has coefficient ring*

$$ku_G^* = R(G)[v, e]/([p](e))$$

where v is the Bott element of degree 2, the element e is the Euler class of a one dimensional representation, and $[p](x) = (1 - (1 - ev)^p)/v$.

The notion of an G-equivariant formal group law for a compact abelian Lie group G was introduced in [2] to study complex oriented A-equivariant formal group laws, but has some intrinsic algebraic interest. The theorem that the coefficient ring of equivariant complex bordism is the universal ring for equivariant formal group laws [5] establishes that the definition is the correct one. We are concerned with multiplicative equivariant formal group laws. It turns out [4] that there is a universal ring L_G^m for multiplicative formal group laws, and this may be identified explicitly.

Proposition 2: *If G is of prime order $ku_G^* = L_G^*$.*

Unfortunately the situation is more complicated for other groups. Thus if there is an G-equivariant form of connective K-theory, its coefficient ring cannot be in even degrees in general, and hence cannot equal L_G^m. Some clues to its behaviour come from its completion [1].

[1] R. R. Bruner and J. P. C. Greenlees "Varieties and local cohomology for the connective K-theory of finite groups." (In preparation)

[2] M. M. Cole, J. P. C. Greenlees and I. Kriz "Equivariant formal group laws." (Submitted for publication).

[3] J. P. C. Greenlees "Equivariant forms of connective K-theory." Topology (to appear)

[4] J. P. C. Greenlees "Multiplicative equivariant formal group laws." Preprint (1998)

[5] J. P. C. Greenlees "The coefficient ring of equivariant bordism is the universal ring for equivariant formal group laws." Preprint (1998)

THE DUALIZING SPECTRUM OF A TOPOLOGICAL GROUP

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Let $G = |G.|$ be the geometric realization of a simplicial group. One then has the "group ring over the sphere spectrum" $S^0[G]$, which is the spectrum with G-action whose j-th space is $S^j \wedge G_+$, where G acts by left translation. The *normal spectrum* D_G is obtained by taking the homotopy fixed points of G acting on $S^0[G]$.

If X is a connected based space, we may identify X with BG up to homotopy for some G (using a suitable group model for the loop space of X). In this way, we can associate a D_G to any given connected based space $X \simeq BG$. One reason for considering D_G is that it detects Poincaré duality:

Fact 1: *Assume X is homotopy finite. Then X is a Poincaré duality space of dimension n if and only if D_G is the $(-n)$-sphere spectrum.*

This can be applied to show that the assignment $X \mapsto D_G$ (with $BG \simeq X$) is multiplicative for fibrations of connected homotopy finite spaces. As a corollary, one has a homotopy theoretic solution to a problem posed by C. T. C. Wall:

Theorem 2: *If $F \to E \to B$ is a fibration of connected homotopy finite spaces, then E satisfies Poincaré duality if and only if F and B do.*

This last statement can be used in turn to give a new proof (actually a slight strengthening) of a result of W. Browder:

Theorem 3: *Any connected finite H-space is a Poincaré duality space.*

Other applications. This circle of ideas has connections with the transfer and Tate cohomology: right translation together with inversion provides another G-action on $S^0[G]$ which induces a G-action on D_G. If E is any spectrum with G-action, then it is possible to concoct in an *elementary* way a "norm" map

$$D_{G \wedge hG} E \to E^{hG}$$

for *any* G. This map is a weak equivalence whenever E is a homotopy finite spectrum with G-action, or whenever BG is homotopy finite. It is not a weak equivalence in general.

If $G = |G.|$ is such that $G.$ is the total singular complex of a compact Lie group, then it turns out that D_G is equivariantly weakly equivalent to the suspension spectrum of S^{ad_G} = the one point compactification of the adjoint representation of G. In this case, one obtains the usual norm map whose cofiber is the *Tate construction* for G acting on E.

ISOMORPHISM CONJECTURES IN K- AND L-THEORY AND COMPUTATIONS FOR CERTAIN CRYSTALLOGRAPHIC GROUPS

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In the first part of the lecture we give a survey about the isomorphism-conjectures of Farrell and Jones for the algebraic K-theory and L-theory of integral group rings and the Baum-Connes conjecture for the topological K-theory of reduced group C^*-algebras. We present a unified approach to these conjectures using spectra over a category and give a common universal property of the relevant assembly maps. We discuss some of the main important consequences of these conjectures like the Borel-conjecture or the stable Gromov-Lawson-conjecture. In some sense one may think of these isomorphism conjectures as generalized induction theorems (like Artin's induction theorem) where the relevant K- and L-groups are expressed in terms of the finite or virtually cyclic subgroups.

In the second part we apply these conjectures to certain classes of virtually abelian groups which allow to express their K- and L-groups in terms of the corresponding groups associated to the certain finite groups. These computations exploid the homology theory aspect of the constructions.

The first part is joint work with Jim Davis, the second with Roland Stamm.

TOPOLOGICAL CYCLIC HOMOLOGY AND Γ-SPACES

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If $*$ is a symmetric monoidal product in a category $\mathcal{C}$ then every monoid M under $*$ gives a simplicial object in $\mathcal{C}$, the *cyclic nerve* $N^{cy}(*, M)$ of M. If $*$ is the tensor product of abelian groups, so that the monoids under $*$ are the rings, then the homotopy groups of $N^{cy}(\otimes, R)$ are the Hochschild homology groups of R.

In case $\mathcal{C}$ is a model for the category of spectra, such that $*$ corresponds to the smash product, we write $\wedge$ instead of $*$, and say that $\wedge$ is a symmetric monoidal smash product of spectra. A monoid F under $\wedge$ is a ring spectrum which has enough extra structure so that the K-theory $K(F)$ of F is defined. In the early 80's it was realized that $N^{cy}(\wedge, F)$, the *topological Hochschild homology* of F, would be useful in studying $K(F)$. Although no symmetric monoidal smash product of spectra was known at that time, Bökstedt was able to give a definition of "ring spectrum with extra structure" (he called these objects FSPs), as well as a definition $THH(F)$ of the topological Hochschild homology of the FSP F. Later on, the *topological cyclic homology* of an FSP F was discovered. It is related to $K(F)$ in a stronger way than $THH(F)$ is.

Recently quite a few symmetric monoidal smash products of spectra were discovered and, if F is a monoid in one of these smash products, then one knows that F "corresponds" to an FSP F' and that $N^{cy}(\wedge, F)$ "corresponds" to $THH(F')$ (for "reasonable", i.e., cofibrant F).

In this talk we will consider the smash product of Γ-spaces, which has the property that the FSP F' above can be taken to equal F, i.e., the monoids *are* FSPs. We will describe an alternative way to define topological cyclic homology, which is as simpler than the original definition as $N^{cy}(\wedge, F)$ is simpler than $THH(F)$.

THE TOPOLOGICAL MODULAR FORMS SPECTRUM ASSOCIATED TO THE SUBGROUP $\Gamma_0(3)$ OF $SL_2(\mathbb{Z})$

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Let S_2 be the Morava stabilizer group for the prime 2. There is a finite subgroup of order 24, G_{24}. A connected version of the spectrum $E_2^{hG_{24}}$, eo_2 has been identified with modular forms over $SL_2(\mathbb{Z})$. The subgroup of order 6, $\mathbb{Z}/6$, defines a spectrum $E_2^{h\mathbb{Z}/6}$. A connected version of this spectrum defines the spectrum $tmf(\Gamma_0(3))$. Several connected versions show up in other contexts. First, the beginning of an eo_2-resolution has $tmf(\Gamma_0(3))$ as summand. Second, $MO\langle 8\rangle$ has the cohomology of an extended eo_2 module. The second piece which includes the homology from 16 to 30 when smashed with eo_2 gives another version of $tmf(\Gamma_0(3))$. This is a report of joint work with Charles Rezk.

UNIVERSAL DEGREE n VERSUS UNIVERSAL n-EXCISIVE: A TALE OF TWO CALCULUS TOWERS AND THE ANALYTICITY THAT BROUGH THEM TOGETHER

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Tom Goodwillie defined a calculus for homotopy functors from spaces to spaces which provides a Taylor tower $\{P_n\}$ of universal n-excisive approximations for homotopy functors which satisfy a mild stability condition. He also observed that these constructions worked equally well in similar situations like functors from spectra to spectra or chain complexes to chain complexes. Brenda Johnson and Randy McCarthy felt that a detailed examination of these ideas for functors between chain complexes would help to expose these new methods to a wider audience and perhaps even provide some new insights, examples and practical tools for the study of homotopy functors in general.

For a functor F from an additive category to chain complexes, one can form a new functor from chain complexes to chain complexes by applying F degreewise to the simplicial replacement of a chain complex and then taking the total complex of the normalized simplicial chain complex this yields. Using an extension of Mac Lane's Q–construction one can write down explicit models which, up to natural quasi-isomorphism, are $\{P_nF\}$. Upon further examination, one sees that these models for $\{P_nF\}$ are exactly the complexes that arise from the augmented simplicial complex of a cotriple which is defined from an adjoint pair of functors: (Δ^*, cr_{n+1}). The functor cr_{n+1} is the Eilenberg-Mac Lane $(n+1)$st cross effect functor which takes a functors and produces a $(n+1)$–multifunctor which is reduced in each variable. The functor Δ^* takes a multifunctor reduced in each variable to its diagonal.

Using this cotriple idea, one can define a tower $\{P_n^\Gamma F\}$ for any functor of chain complexes. However, much to our initial surprise, this tower, though formally very similar, is not Goodwillie's $\{P_nF\}$ in general. The difference is that the $\{P_n^\Gamma F\}$ are universal degree n–approximations to F (the $(n+1)$st cross effect of P_n is weakly equivalent to a point) while the $\{P_nF\}$ are universal n–excisive approximations to F. For $n=1$, the difference is like asking a functor to be additive (preserve direct sums) or to be exact (preserve arbitrary short exact sequences). In general, n–excisive implies degree n and the expected factorization $F \to P_n^\Gamma F \to P_nF$ exists. For X within the analytic range of F, the map $P_n^\Gamma F(X) \to P_nF(X)$ is a quasi-isomorphism. In other words, there are two natural towers which are not equal in general but do agree for objects X for which Goodwillie's original tower is generally known to be effective (converge). One of the interesting differences between these two towers is that $P_n^\Gamma F$ is always degree n, while P_nF is known to be n–excisive only when F satisfies some stability properties. Another difference between the theories is that $\{P_n^\Gamma\}$ can be defined for functors from any pointed category with finite sums (like pointed sets) while $\{P_n\}$ needs the domain category to have additional structure.

When one wants to extend the above algebraic setting to homotopy functors of spaces, one first has to observe that even though (Δ^*, cr_{n+1}) is no longer an adjoint pair of functors, one can still (by choosing appropriate models) form a strict cotriple on homotopy functors by using the composite $\perp_{n+1} = \Delta^* \circ cr_{n+1}$. For functors whose values are in spectra, everything can now be done as it was for functors of chain complexes. In general, for functors of spaces one has an obstruction to P_n^Γ existing because realizations of simplicial objects no longer commute with fibrations. With mild assumptions, however, one can deloop an appropriate map to once again obtain a theory $\{P_n^\Gamma\}$ of universal degree n approximations. There is a natural transformation from $P_n^\Gamma F \to P_nF(X)$ which is once again an equivalence for X in the analytic range of F.

Theorem: *For F a ρ–analytic functor and X a ρ–connected space, there is a fibration up to homotopy*

$$\perp_{n+1}^* F(X) \longrightarrow F \longrightarrow P_nF(X) \ .$$

When $F(X)$ is simply connected (one can do better) there is a convergent first quadrant spectral sequence

$$E^2_{p,q} \ = \ \mathrm{H}_q\,(P_n^\Gamma[\pi_pF(X \wedge *)]) \quad \Longrightarrow \quad \pi_{p+q}F(X)$$

where $\pi_pF(X \wedge *)$ is considered as a functor from pointed sets to abelian groups.

HIGHER ORDER HOCHSCHILD HOMOLOGY

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Higher order Hochschild homology relates to the cohomology of mapping spaces in the same way as Hochschild homology relates to the cohomology of free loop spaces. This theory has so far only been constructed for commutative algebras.

The definition of higher order Hochschild homology is as follows. Let A be a commutative algebra and M be an A-module. There is a functor $\mathcal{L}(A,M):\Gamma\to Vect$ from the category Γ of finite pointed sets to the category of vector spaces given by

$$\{0,1,...,n\}\mapsto M\otimes A^{\otimes n}.$$

We let $H_*^{[d]}(A,M)$ denote the homology of the chain complex associated to the simplicial vector space

$$\Delta^{op}\xrightarrow{S^d}\Gamma\xrightarrow{\mathcal{L}(A,M)}Vect.$$

Here $S^d:\Delta^{op}\to\Gamma$ is a simplicial model of the sphere of dimension $d\geq 1$. For $d=1$ one recovers the usual Hochschild homology of A with coefficients in M. Because of this fact, we call the groups $H_*^{[d]}(A,M)$ *Hochschild homology of order* d of A with coefficient in M. We will show that in the characteristic zero case Hochschild homology of order d of the DeRham cochain algebra of a d-connected space X is isomorphic to the cohomology of the mapping space X^{S^d}.

The main result is to show that the higher order Hochschild homology groups have a natural decomposition which for $d=1$ coincides with the classical Hodge decomposition for Hochschild homology obtained by Quillen. More generally, for any functor F from Γ to the category of vector spaces over a characteristic zero field we prove that $\pi_nF(S^d)$ has a natural decomposition which for $d=1$ coincides with Loday's decomposition

$$\pi_nF(S^1)\cong\bigoplus_{i=0}^{n}H_n^{(i)}(F),\ n\geq 0.$$

The striking point is the fact that the knowledge of the decomposition for $\pi_nF(S^1)$ (resp. $\pi_nF(S^2)$) completely determines the decomposition of $\pi_nF(S^d)$ for any odd (resp. even) d. Thus the Hodge decomposition of $H^*(X^{S^1})$ and $H^*(X^{S^2})$ determines all groups $H^*(X^{S^d})$.

TWO-PRIMARY ALGEBRAIC K-THEORY OF POINTED SPACES

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Let $A(X)$ be Waldhausen's algebraic K-theory of spaces functor evaluated on the space X. When X is a manifold, $A(X)$ provides the fundamental link between algebraic K-theory and the geometric topology of X – in particular with the concordance space, the h-cobordism space and the automorphism space of X. We are therefore interested in evaluating its homotopy type. It is the aim of this talk to compute the 2-primary homotopy type of $A(X)$ in the case when $X = *$ is the one-point space. We achieve this by computing the mod 2 spectrum cohomology of $A(*)$ as a module over the mod 2 Steenrod algebra. The result is a complete calculation valid in all dimensions. Using an Adams spectral sequence we also compute the 2-primary homotopy groups of $A(*)$ in dimensions $* \leq 18$, and up to extensions in dimensions $19 \leq * \leq 21$.

The homotopy type of $A(*)$ is made accessible by means of a comparison of algebraic K-theory with the topological cyclic homology theory (TC) of Bökstedt, Hsiang and Madsen, relying on a theorem of Dundas. The identification of the resulting homotopy type uses 2-primary results of Voevodsky, Bloch and Lichtenbaum from motivic algebraic geometry to identify Quillen's algebraic K-theory of the integers, $K(\mathbb{Z})$, as well as calculations of the topological cyclic homology of $X = *$ and of the integers, by Bökstedt, Hsiang, Madsen and myself. These lead to a spectrum level description of the 2-primary homotopy fiber of the cyclotomic trace map from $A(X)$ to $TC(X)$ for every 1-connected space X. In the case $X = *$ the cohomological analysis of $TC(*)$ is easy, while the homotopical analysis is equivalent to that of the truncated complex projective space $\mathbb{C}P^{\infty}_{-1}$, which we perform in dimensions $* \leq 20$. This leads to the claimed homotopical calculations for $A(*)$.

As a sample application, we determine the connectivity of the 2-completed rigid tubes map from a Grassmannian manifold to a space of handlebody decompositions, or equivalently of the Hatcher-Waldhausen map from G/O to $\Omega Wh^{Diff}(*)$. If time allows, I will conclude with some speculations on how to interpret these results along the lines of Waldhausen's paper on $A(*)$ and the chromatic filtration of stable homotopy theory.

NEW MODELS FOR RATIONAL EQUIVARIANT STABLE HOMOTOPY

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In this talk I will describe a new model for equivariant stable homotopy over a compact Lie group based on symmetric spectra. This new category is a symmetric monoidal model category which is Quillen equivalent to the model category of equivariant S-modules due to Elmendorf, Kriz, Mandell and May. I will then describe an algebraic model for rational equivariant stable homotopy. In the case of finite groups this model simplifies considerably. For the circle, the model can also be given explicitly. This uses work due to Greenlees, but arrives at a different model. This talk is based on work in progress.

This new model for equivariant stable homotopy is constructed using "A classification of stable model categories" which is joint work with S. Schwede. The model for G-spaces as diagrams of fixed point spaces over the unstable topological orbit category is similar to this model of G-spectra. Here an equivariant spectrum is modeled by an enriched diagram over the stable orbit category which can be thought of as a diagram of fixed point spectra. That is, a G-spectrum is an enriched functor from the enriched stable orbit category to the category of symmetric spectra. The enrichment is over symmetric spectra instead of the usual notions of simplicial or topological enrichment.

The rationalization of an equivariant spectrum corresponds in this model to rationalizing each of the associated fixed point spectra. Since the rational stable homotopy category is equivalent to the homotopy category of $H\mathbb{Q}$-modules, the model for rational equivariant spectra here is as diagrams of $H\mathbb{Q}$-modules over a rationalized stable orbit category. Here the enrichment, and everything in sight, is over $H\mathbb{Q}$-modules where instead of spectra or S-modules.

To make this an algebraic model, we first note that the homotopy category of $H\mathbb{Q}$-modules is equivalent to the homotopy category of integer graded chain complexes of rational vector spaces, referred to as rational chain complexes. Although there are several technicalities involved in this step, one can then replace the above model by a model for rational equivariant stable homotopy as diagrams of rational chain complexes over a rationalized stable orbit category which is also modeled by chain complexes.

The last step towards a useful algebraic model is then to simplify the chain complex model of the rationalized stable orbit category. In the case of finite groups, this diagram can be replaced by a single differential graded algebra which is concentrated in degree zero. Hence, this recovers for finite groups the equivalence of G-spectra with the derived category of Mackey functors. For the case of a circle the model is still somewhat complicated, but an explicit model will be given. The comparison of this model with the one due to Greenlees is work in progress.

PERIODIC HOMOTOPY TYPES

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By a theorem of Swann, a finite group has periodic cohomology if and only if it acts freely on finite CW-complex that is homotopy equivalent to a sphere. In joint work with Adem this result is generalized in two directions. First we show that finite groups be replaced by topological groups: the classifying space of a topological group has "periodic" cohomology if and only if there is a finite free G-CW complex that is weak equivalent to a sphere. Second one can consider finite groups of higher rank. We show that many finite groups of rank 2 act freely on a CW-complex that is homotopy equivalent to a two fold product of spheres.