Workshop
on
Dirichlet Spaces
and
Applications

The workshop was organized jointly by the Sonderforschungsbereich 256 "Nichtlineare partielle Differentialgleichungen", Bonn, and the Sonderforschungsbereich 343 "Diskrete Strukturen in der Mathematik", Bielefeld. It was held at Bielefeld University from August 10–14, 1998 with 41 participants from various countries.
The following areas of mathematics related to the Theory of Dirichlet Forms and their applications were represented in the talks given at the workshop: Analysis on Fractals; Control Theory; Infinite Dimensional Differential Geometry, in particular, on Path and Loop Spaces; Mathematical Statistical Mechanics; Parabolic and Generalized Dirichlet Forms; Pseudo–Differential Operators; Sobolev Inequalities and Spectral Theory; Stochastic (Partial) Differential Equations; Uniqueness of Solutions to Martingale Problems and Stochastic Equations.

Universität	**Bielefeld**
Fakultät für Mathematik	**Sonderforschungsbereich 343 Diskrete Strukturen in der Mathematik**

The SFB 343 (Bielefeld) together with the SFB 256 (Bonn) and the DFG–Schwerpunkt *Interagierende stochastische Systeme von hoher Komplexität* will organize and support at the University of Bielefeld a weeklong workshop on

Dirichlet Spaces and Applications

August 10 – 14, 1998

The workshop will cover a wide variety of topics related to Dirichlet forms and Stochastic Analysis and their applications (a program and complete list of participants will be available a few weeks before the workshop in the internet - see below).

Invited speakers will include:

Shigeki Aida (Sendai)
Masatoshi Fukushima (Osaka)
Yuri Kondratiev (Bonn)
Ma Zhi-Ming (Beijing)
Yoichi Oshima (Kumamoto)
Zhang Tu-Sheng (Haugesund)
Sergio Albeverio (Bonn)
Elton P. Hsu (Evanston)
Paul Lescot (St. Quentin)
Hirofumi Osada (Nagoya)
Masayoshi Takeda (Osaka)

Anyone who is interested in taking part in the workshop and/or giving a talk should contact the organizers via e-mail as soon as possible, but not later then **June 30, 1998**. Further information is provided on a web-page under URL

http://www.mathematik.uni-bielefeld.de/∼rgoschni/workshop.html

Organizers: **Michael Röckner** (Bielefeld) **Karl-Theodor Sturm** (Bonn)

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Workshop on Dirichlet Spaces and Applications

Bielefeld/Bonn, August 10–14

The lectures will be held at HS 8 in the main building of the University of Bielefeld and in the "Kleiner Hörsaal" at the Mathematical Institute of the University of Bonn at Wegelerstraße 10.

Program

Monday, 10th (Bielefeld)

9.00 – 9.50	N. Jacob	Fractional derivatives, the drift form and limits of non–symmetric Dirichlet forms
10.00 – 10.50	S. Fang	Ricci tensors in some infinite dimensional Lie algebras
		Coffee break
11.30 – 12.20	P. Lescot	Generalized Mehler semigroups on infinite–dimensional spaces
		Lunch
14.00 – 14.50	F.–Y. Wang	Sobolev–type inequalities
15.00 – 15.50	S. Assing	Comparison of solutions of stochastic partial differential equations
		Coffee break
16.30 – 17.20	F. Russo	Stochastic calculus for finite quadratic variation processes
17.30 – 18.20	J. Ren	An embedding theorem for fractional Sobolev spaces over Wiener spaces

Tuesday, 11th (Bonn)

11.00 – 11.50	M. Fukushima	On a singular control problem for the one–dimensional distorted Brownian motion
		Lunch
13.30 – 14.20	T.–S. Zhang	Probabilistic representation of semigroups and hyperbound estimates
14.30 – 15.20	S. Aida	On the short time asymptotics of transition probabilities in infinite dimensions
		Coffee break
16.00 – 16.50	W. Hoh	Some results on jump processes generated by pseudo–differential operators
Afternoon		**Visit of the Beethovenhaus**

Wednesday, 12th (Bonn)

9.00 – 9.50	Y. Oshima	Certain limit theorems for time inhomogeneous transition functions
10.00 – 10.50	A. Eberle	Markov uniqueness
		Coffee break
11.30 – 12.20	K. Kuwae	Dirichlet forms on Alexandrov spaces
		Lunch
Afternoon		**Boat trip on the river Rhein**

Thursday, 13th (Bielefeld)

11.00 – 11.50	M. Takeda	L^p–independence of the spectral radius of Markov processes and gaugeability
		Lunch
13.30 – 14.20	W. Stannat	On the validity of the log–Sobolev–inequality for symmetric Fleming–Viot operators
14.30 – 15.20	G. Trutnau	Stochastic differential equations in infinite dimensions – solutions via generalized Dirichlet forms
		Coffee break
16.00 – 16.50	V. Metz	The averaging effect of self–similar media

Friday, 14th (Bielefeld)

9.00 – 9.50	E. Hsu	Pathwise uniqueness of reflecting Brownian motion on euclidean domains
10.00 – 10.50	H. Osada	Anormal diffusion processes on $\mathbf{R}^d$ and carpets
		Coffee break
11.30 – 12.20	T. Kuna	Gibbs measures and correlation functions on manifolds
		Lunch
14.00 – 14.50	M. Yoshida	On non–linear (local) maps defined on Gaussian random fields of subspaces of $\mathcal{S}'(\mathbf{R}^d)$
15.00 – 15.50	A. Schied	A Rademacher–type theorem on configuration spaces and applications
		Coffee break
16.30 – 17.20	A. Bendikov	Ultracontractivity and off-diagonal behaviour of symmetric Markov semigroups

On the short time asymptotics of transition probability in infinite dimensional spaces

Shigeki Aida, Hiroshi Kawabi

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Let (E, H, μ) be an abstract Wiener space. Let us consider the following closable symmetric form on E.

$$\mathcal{E}_\sigma(u,u) := \int_E |Du(z)|_H^2 d\mu(z) + \int_E |\sigma(z)Du(z)|^2 d\mu(z), \tag{1}$$

where we assume that $\sigma(\cdot) \in \mathcal{D}^\infty_{\infty-}(E, L_2(H,H))$ and $\sup_z |\sigma(z)|_{L^2(H,H)} < \infty$. Let us consider the Dirichlet form $\mathcal{E}_\sigma$ which is the smallest closed extension of the above form. Let us denote the transition probability of the corresponding diffusion process by $P_t(x, dy)$ and denote for any Borel measurable A and B

$$P_t(A,B) := \int_A P_t(x,B) d\mu(y).$$

Our problem is the following. What is the asymptotics $P_t(A,B)$ when $t \to 0$? In the workshop, we discussed the asymptotics $\lim_{t\to 0} 4t \log P_t(A,B)$ introducing the distance $d_\sigma(A,B)$ between A and B. Of course if σ is smooth in the sense of Fréchet, then usual Large deviation theory of the solutions of stochastic differential equations implies that for open set A and measurable B,

$$\lim_{t\to 0} 4t \log P_t(A,B) = -d_\sigma(A,B)^2 \tag{2}$$

Our main interest is in the case that σ is not smooth in Fréchet but still regular in some sense. Let us assume that σ satisfies the following regularity

(Assumption) For any $\varepsilon > 0$, there exists a compact subset K_ε such that $\mu(E \setminus K_\varepsilon) < \varepsilon$ and the function

$$x (\in K_\varepsilon) \to \sigma(x + \cdot) \in C_b(H, L_2(H,H))$$

is continuous with respect to the topology of E where $C_b(H, L_2(H,H))$ is a Fréchet space with seminorms

$$\|f\|_r = \sup_{x\in H, |x|\le r} |f(x)|_{L^2(H,H)} \quad (r = 1, 2, \ldots)$$

Our main results are the following.

Theorem 1 *For any A and B,*

$$\overline{\lim}_{t\to 0} 4t \log P_t(A, B) \leq -d_\sigma(A, B)^2 \tag{3}$$

Theorem 2 *Assume that the Ricci curvature of the Dirichlet form of $\mathcal{E}_\sigma$ is bounded from below. Then for any open A and measurable B,*

$$\lim_{t\to 0} 4t \log P_t(A, B) = -d_\sigma(A, B)^2 \tag{4}$$

We can prove the above theorem using Wang's parabolic Harnack inequality. As a corollay of the above theorem, we can obtain the asymptotics (4) for σ where σ involves the multiple Wiener integral.

Remark 3 *Multiple Wiener integrals are not continuous in the sense of Fréchet but is regular in our sense. So this example is outside the application of the usual Large deviation theory.*

Comparison of solutions to stochastic differential equations

Sigurd Assing

We introduce a suitable partial ordering on $\mathcal{R}^n$ and study an n-dimensional system

$$\mathrm{d}\mathbf{u} \;=\; [\mathbf{A}\mathbf{u} + \mathbf{F}(t,\mathbf{u})]\,\mathrm{d}t + \sum_{j=1}^{m} \mathbf{M}(\mathbf{G}_j(t,\mathbf{u})) \circ B_j\,\mathrm{d}W_j(t)$$

of stochastic partial differential equations in $\mathcal{R}^d$ for different initial values and drift functions $\mathbf{F}$. The aim is to give conditions as weak as possible such that the corresponding solutions can be compared with respect to our partial ordering. We explain our complicated setting by examples.

ULTRACONTRACTIVITY AND OFF-DIAGONAL BEHAVIOR OF SYMMETRIC MARKOV SEMIGROUPS.

A. Bendikov (Erlangen University)
and
L. Saloff-Coste (Cornell University)

One of the aim of our investigation is to develop some analytical tools for studying symmetric Markov semigroups having a continuous kernel when the underlying space is infinite dimensional but locally compact (for example, a locally compact group). One of the specific problems addressed in our talk is whether or not, in some generality, the ultracontractivity property, that is, the on-diagonal behavior of the heat kernel implies an off-diagonal control of the heat kernel in terms of a distence associated to the corresponding Dirichlet form.

Markov uniqueness

Andreas Eberle

The Markov uniqueness problem is studied for singular diffusion operators defined on smooth test functions over some open subset of $\mathbf{R}^n$. It is shown that for every symmetric diffusion operator there exists a maximal Dirichlet extension, i.e., a maximal element in the set of all self-adjoint extensions of the operator that generate sub–Markovian semigroups. The explicit description of the maximal Dirichlet extension is used to prove a necessary and sufficient condition for Markov uniqueness of singular diffusion operators with non–degenerate second order part. In particular, it is shown that in the non–degenerate case conservativity of the semigroup generated by the Friedrichs extension of the operator implies Markov uniqueness. The same result is shown to be true up to a certain degree of degeneracy of the second order coefficients. On the other hand, we demonstrate that for "very degenerate" as well as for infinite–dimensional diffusion operators Markov uniqueness can break down although the Friedrichs semigroup is conservative.

Pathwise Uniqueness of Reflecting Brownian Motion
on Euclidean Domains

Richard F. Bass and Elton P. Hsu

For C^1 euclidean domains, we show by approximation and measurable selection that there is a strong solution to the Skorohod equation. If the domain is $C^{1,\alpha}$, then the weak uniqueness holds for the Skorohod equation. These two facts imply by a measure-theoretic argument the pathwise uniqueness for the Skorohod equation.

Ricci tensor on some infinite dimensional Lie algeras

Shizan Fang

Abstract: The Ricci tensor, which plays an important role in Stochastic geometric analysis, has been computed in severals infinite dimensional situations. In this work, we are interested in the case of central extensions of loop groups and in the asymptotic behaviour of the Ricci tensor as the Riemannian metric varies.

On a singular control problem for the one dimensional distorted Brownian motion

Masatoshi Fukushima
Kansai University, Osaka

A distorted Brownian motion on a topological measure space (X, m) is a symmetric diffusion process living on the support E of m associated with the Dirichlet form

$$\mathcal{E}^\rho(u,v) = \frac{1}{2}\int_X \nabla u(x)\cdot\nabla v(x) m(dx) \tag{1}$$

on $L^2(X;m)$ the so called energy form, which has been intensively studied since Albeverio, Hoegh Krohn and Streit introduced it in the latter half of 1970's both for finite and infinite dimensional underlying space X. The energy form $\mathcal{E}^\rho$ involves a non-negative locally m integrable function $\rho(x)$ on X and accordingly the sample path of the distorted Brownian motion is controlled by this function. Recently the author has investigated necessary and sufficient conditions (in terms of ρ) for a distorted Brownian motion to be a semimartigale. In particlur, a (modified) normally reflecting Brownian motion on the clusure of an open set D of a finite dimensional state space X is a distorted Brownian motion corresponding to $\rho = I_D$ the indicator function of D, and it is a semimartingale if and only if the set D is Caccioppoli.

The reflecting Brownian motions are interesting objects to be studied from divierse view points. Indeed, they arise in many other different contexts. In particlular, the reflecting Brownian motion on a one dimensional interval was characterized as a solution of a singlur control problem. More specifically, let (B_t, P) be a Brownian motion on R starting at the origin 0 and let

$$X_t = x + B_t + A_t^1 - A_t^2 \quad x \in R \tag{2}$$

for a pair $S = (A^1_t, A^2_t)$ of positive continuous additive functionals of the Brownian motion. S represents a strategy under which the cost function

$$k_S(x) = E\left(\int_0^\infty e^{-\alpha t} h(X_t)dt + \int_0^\infty e^{-\alpha t}(r dA^1_t + \ell dA^2_t)\right) \tag{3}$$

is to be minimized. Here, α, r, ℓ a preassigned positive constants and $h(x)$ is a given convex function taking its minimum at the origin. Taksar then showed the existence of the optimal strategy $\tilde{S}$ such that

$$w(x) = \min_S k_S(x) = k_{\tilde{S}}(x). \tag{4}$$

It was actually shown that $\tilde{S} = (\ell^a_t, \ell^b_t)$ where ℓ^a, ℓ^b are local times at points a, b of the Brownian motion for uniquely decided $a, b,\ a < 0 < b$ so that the corresponding optimal process (2) is the reflecting Brownian motion on the closed interval $[a, b]$.

In this report, we presents a similar result under a slightly different formulation by replacing the role of the strategy $S = (A^1_t, , A^2_t)$ in Taksar's one with the function ρ deciding the distorted Brownian motion. This is a joint work with T.Kawabata.

A function ρ on R is called *admissible* if
(i) ρ is non-negative and integrable,
(ii) the support E of $\rho(x)dx$ is a finite closed interval,
(iii)

$$\int_E \frac{d\xi}{\rho(\xi)} < \infty,$$

(iv) ρ is of bounded variation on E.

Let $m(x),\ x \in R$, be a positive continuous function on R and consider the measure $dm = m(x)dx$. For any admissible ρ, the symmetric form

$$\mathcal{E}^\rho(u, v) = \frac{1}{2}\int_R u'\ v'\ \rho\ dx \quad u, v \in C^1(R) \tag{5}$$

is closable on $L^2(R; \rho dm)$ and the closure $(\mathcal{E}^\rho, \mathcal{F}^\rho)$ becomes a regular local Dirichlet form on $L^2(E; \rho dm)$. Moreover $(\mathcal{E}^\rho_\alpha, \mathcal{F}^\rho)$ admits a reproducing kernel $g_\alpha(x, y),\ x, y \in E$, which is bounded continuous in y for each x and accordingly the α potential of a finite positive measure is bounded. Further each one point set has a positive capacity.

Denote by $\mathbf{M} = (X_t^\rho, P_x^\rho)$ the associated diffusion process on E. Let

$$\frac{1}{2}d\rho = \nu_1 - \nu_2 \tag{6}$$

be the Jordan decomposition of the finite signed measure $d\rho$. We denote by $A_t^{\rho 1}, A_t^{\rho 2}$ the positive continuous additive functionals with Revuz measures ν_1, ν_2. It then holds that

$$X_t^\rho - X_0^\rho = M_t^\rho + A_t^{\rho 1} - A_t^{\rho 2} \quad P_x - a.s.\forall x \in E, \tag{7}$$

where M_t^ρ is a continuous martingale additive functional with

$$\langle M^\rho \rangle_t = \int_0^t m(X_s^\rho)^{-1} ds. \tag{8}$$

Thus M_t^ρ is a time change of a Brownian motion and it is adapted to the filtration generated by the process X_t^ρ, but otherwise it is independent of the choice of the admissible ρ.

Let us given positive constants α, r, ℓ, and a convex function $h(x)$ on R such that $h'(0) = 0$, $h'(x)$ decreases to $-\infty$ as $x \downarrow -\infty$ and increses to ∞ as $x \uparrow \infty$, For any admissible function ρ, the cost function $k_\rho(x)$ is defined as (3) but E, X_t, A_t^1, A_t^2 being replaced by $E_x^\rho, X_t^\rho, A_t^{\rho 1}, A_t^{\rho 2}$ respectively when $x \in E$. Denoting E by $[\ell_1, \ell_2]$, $k_\rho(x)$ is extended outside E by

$$k_\rho(x) = \begin{cases} k_\rho(\ell_2) + \ell(x - \ell_2) & x > \ell_2 \\ k_\rho(\ell_1) - r(x - \ell_1) & x < \ell_1 \end{cases}$$

Theorem 1 *There exist a, b with $a < 0 < b$ such that, if we let*

$$\tilde{\rho}(x) = I_{[a,b]}(x) \quad x \in R,$$

then, for any admissible ρ,

$$k_{\tilde{\rho}}(x) \leq k_\rho(x) \quad x \in R. \tag{9}$$

The above $\tilde{\rho}$ is unique in the sense that, if the identity holds in the preceding inequality, then $\rho = \tilde{\rho}$ a.e.

The optimal diffuion process $\mathbf{M}^{\tilde{\rho}}$ is a reflecting (time changed) Brownian motion on the closed interval $[a, b]$. In the proof, a, b and the value function $w(x) = \min_\rho k_\rho(x)$ are constructed concretely by a solution of an associated stopping game problem for the one dimensional diffusion process with scale dm and speed measure dx. We can proceed along a similar line to Taksar. It seems however necessary to obtain a clearer analytic understanding of the solution of this stopping game problem even in the one-dimensional case before we can go over to analogous multidimensional singular control problems as have been investigated by Taksar and others.

Some results on jump processes generated by pseudo-differential operators

Walter Hoh

We consider pseudo-differential operators $p(x, D)$ with symbols $p(x, \xi)$ which are continuous negative definite functions with respect to the co-variable ξ. By a result of Ph. Courrège it is known that generators of Markov processes in $\mathbf{R}^n$ admit such representation, since they satisfy the so-called positive maximum principle. Starting with a given symbol of this type in the talk different approches are considered to construct a corresponding Markov process, Feller semigroup or Dirichlet form. Next the L^2-estimates obtained in this way are used to get further information about the corresponding process. In particular, we investigate ultra-contractivity and the strong Feller property of the transition semigroups. Finally, we consider a conservativeness criterion, which based on a condition purely in terms of the symbol implies non-explosion of the corresponding process.

Fractinal derivatives, non-symmetric and time-dependent Dirichlet forms and the drift form

Niels Jacob

Using fractional derivatives we show that the drift form $\int_{-\infty}^{\infty} u(x)\,\frac{dv(x)}{dx}\,dx$ can be approximated by non-symmetric Dirichlet forms. A similar result holds for the drift form in $\mathbf{R}^n$ with variable coefficients if the coefficient functions satisfy certain regularity and commutator conditions. Since time-dependent Dirichlet forms (in the sense of Y. Oshima) can be interpreted as sums of a drift form (in τ-direction) and a mixture of τ-parametrized Dirichlet forms over $\mathbf{R}^n$, our results show that time-dependent Dirichlet forms arise as limits of ordinary non-symmetric Dirichlet forms in $\mathbf{R}\times\mathbf{R}^n$-space. An abstract result on fractional powers of Markov generators allows to extend this observation to generalized Dirichlet forms. Another consequence is that the bilinear form induced by an arbitrary Lévy process is the limit of non-symmetric Dirichlet forms.

Gibbs measures and correlation functions on manifolds

Tobias Kuna

We prove that Gibbs measures, point processes which fulfills the Dobrushin-Lanford-Ruelle equation, are in one to one correspondence to correlation functions, which fulfill the Kirkwood-Salsburg equations, if we additionally assume that the correlation functions satisfy the "Ruelle bound"

$$|\rho(\gamma)| \leq C^{|\gamma|}.$$

Abstract conditions for existence of infinte volume correlation functions are given. As their state space X we take a locally compact metric space. The potential V has to be stable and integrable

$$\operatorname*{ess\,sup}_{x \in X} \int_X |e^{-\beta V(x,y)} - 1| dy.$$

All results can be extended to marked Gibbs measures.

Dirichlet forms on Alexandrov spaces

By
KAZUHIRO KUWAE

1 Abstract

An *Alexnadrov space* is defined to be a complete locally compact length metric space of curvature bounded below and of finite Hausdorff dimension. The following are examples of Alexandrov spaces:
- complete Riemannian manifolds of sectional curvature $\geq \kappa$ and with convex boundary.
- complete Riemannian orbifolds of sectional curvature $\geq \kappa$.
- The quatient M/Γ of a closed Riemannian manifold M by a group isometrically on M (not necessarily discrete nor free).
- The boundaries of convex bodies in $\boldsymbol{R}^n$.
- The (pointed) Gromov-Hausdorff limits of n-dimensional Alexandrov spaces of curvature $\geq \kappa$ for fixed constants $n \in \boldsymbol{N}$ and $\kappa \in \boldsymbol{R}$.

Let M be an Alexandrov space. Then the Hausdorff dimension and the topological dimension of M coincide and are a nonnegative integer $n \in \boldsymbol{N} \cup \{0\}$. For a given point $x \in M$, as $r \to \infty$ the pointed rescaled metric space (rM, x), where rM denotes the space M with distance function rd, necessarily converges to a pointed n-dimensional nonnegatively curved Alexandrov space (K_x, o_x), called the *tangent cone at* x. A singular point of M is defined to be a point $x \in M$ where the tangent cone is not isometric to $\boldsymbol{R}^n$. Denote by S_M the set of singular points in M and call it the *singular set of* M. Owing to Burago, Gromov, and Perelman[?], the Hausdorff dimension of S_M is $\leq n-1$ and there exists a closed subset $S_M^*(\subset S_M)$ of M such that $M \setminus S_M^*$ is a Lipschitz manifold. Note that $M \setminus S_M$ is not a manifold in general. Otsu and Shioya[?] gave a C^1-differentiable structure on $M \setminus S_M$, which means that we have an atlas on $M \setminus S_M^*$ whose charts transformation are all C^1 on $M \setminus S_M$, and gave a C^0-Riemannian metric g on $M \setminus S_M$ such that

1. the distance function induced from g coincides with the original one,
2. the volume measure induced from the Riemannian volume element $\sqrt{\det g}dx^1dx^2\cdots dx^n$ coincides with the n-dimensional Hausdorff measure $\mathcal{H}^n$ on $M \setminus S_M$.

Hence $M \setminus S_M^*$ is a(n incomplete) Lipschitz Riemannian manifold in the sense of Teleman[?]. The matrix function defined by g is locally uniform elliptic on $M \setminus S_M^*$. We can define the weak derivative ∇u of $u \in L^1_{loc}(\Omega; \mathcal{H}^n)$ and Sobolev spaces $W^{1,p}(\Omega), (p \geq 1)$. The canonical Dirichlet form on $L^2(\Omega)$ is defined by

$$\mathcal{E}(u,v) := \int_\Omega \langle \nabla u, \nabla v \rangle d\mathcal{H}^n \text{ and } \mathcal{E}_1(u,v) := \mathcal{E}(u,v) + (u,v)_{L^2(\Omega)},\ u,v \in W^{1,2}(\Omega).$$

Denote by $W_0^{1,2}(\Omega)$ the $\mathcal{E}_1^{1/2}$-closure of the set of $u \in W^{1,2}(\Omega)$ with compact support in Ω. Then we prove

Theorem 1.1 *$(\mathcal{E}, W_0^{1,2}(\Omega))$ is a strongly local regular Dirichlet form on $L^2(\Omega)$.*

Theorem 1.2 *Let Ω be a relatively compact open set subset, the embedding $W_0^{1,2}(\Omega)$ into $L^2(\Omega)$ is compact.*

Let Ω be an open subset and define $x, y \in \Omega$,

$$\rho_\Omega(x,y) := \sup\{u(x) - u(y) \mid u \in W^{1,2}_{0,loc}(\Omega) \cap C(\Omega), |\nabla u| \leq 1\ \mathcal{H}^n\text{-a.e.}\}.$$

Then $\rho_\Omega : \Omega \times \Omega \to [0, \infty]$ is a pseudo-distance function, called the *intrinsic metric induced from the Dirichlet form* $(\mathcal{E}, W_0^{1,2}(\Omega))$.

Theorem 1.3 *We have $\rho_M = d$ on $M \times M$.*

We can prove the weak Poincaré inequality without exceptional set and the doubling property of the measure $\mathcal{H}^n$ by use of the Bishop Gromov inequality. In particular, combining the above theorem and the works [**?**, **?**, **?**, **?**, **?**, **?**, **?**] of K.-T. Sturm, we have

Theorem 1.4 *1. Any local solution $u(t,x)$ of the heat equation $(\Delta_\Omega + \frac{\partial}{\partial t})u(t,x) = 0$ admits a locally Hölder continuous $dt \otimes d\mathcal{H}^n$-version on $\boldsymbol{R} \times \Omega$.*

2. Any eigenfunction of Δ_Ω admits a locally Hölder continuous $\mathcal{H}^n$-version on Ω.

Here Δ_Ω is the generator on $L^2(\Omega)$ with respect to $(\mathcal{E}, W_0^{1,2}(\Omega))$. Finally we have the existence of the locally Hölder continuous heat kernel $p_t^\Omega(x,y)$ associated with $(\mathcal{E}, W_0^{1,2}(\Omega))$. We also have the strong Feller property of the $L^2(\Omega)$-semi-group associated with $(\mathcal{E}, W_0^{1,2}(\Omega))$. When Ω is relatively compact, we have that the heat kernel $p_t^\Omega(x,y)$ admits the expansion by eigenfunctions and eigenvalues:

$$p_t^\Omega(x,y) = \sum_{i=1}^{\infty} e^{-\lambda_i t} u_i(x) u_i(y)$$

for any $t > 0$ and $x, y \in \Omega$, where the convergence is uniform on any compact subset of $(0,\infty) \times \Omega \times \Omega$. Here λ_i (resp. u_i) is an eigenvalue (resp. an eigenfunction in $L^2(\Omega)$ with $\|u_i\|_{L^2(\Omega)} = 1$ corresponding to λ_i).

The boundary ∂M of M defined by induction(see [**?**]).

Theorem 1.5 *Suppose that $\partial M = \emptyset$. Let* Cap *be the* 1*-capacity with respect to $(\mathcal{E}, W_0^{1,2}(M))$. Then the Hausdorff dimension of S_M is $\leq n-2$([**?**]). We have*

$$\mathrm{Cap}(S_M) = 0.$$

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SUMMARY OF MY TALK (AUGUST 10TH, 1998)

Paul Lescot

October 7th, 1998

It is well–known that the Ornstein–Uhlenbeck semigroup $(P_t)_{t\geq 0}$ on the Wiener space (X, H, μ) is given by *Mehler's formula* :

$$\forall f \in L^2(X) \;\; P_t f(x) = \int_X f(e^{-t}x + \sqrt{1-e^{-2t}}y)\mu(dy) \;\; .$$

This may be rewritten as :

$$P_t f(x) = \int_X f(T_t x + y)\mu_t(dy) \qquad (* \;\; ,)$$

where

$$T_t : X \to X \;\; ,$$

defined by :

$$\forall x \in X \quad T_t(x) = e^{-t}x$$

is (obviously) a strongly continuous semigroup on X, and $(\mu_t)_{t\geq 0}$ is a family of measures on X.

Following Bogachev, Roeckner and Schmuland (**PTRF** 105(1996), pp. 193–225) as well as Fuhrman and Roeckner (preprint, Bielefeld 1997), Michael Roeckner and I have, in an as yet unpublished paper, studied more general semigroups defined by formula (*), X being replaced by a general Banach space. Roughly speaking, from the semigroup property for $(P_t)_{t\geq 0}$ it follows that the Fourier transform of μ_t is given by :

$$\forall \xi \in E' \;\; \hat{\mu}_t(\xi) = e^{-\int_0^t \lambda(T_s^*(\xi))ds}$$

for some negative definite $\lambda : E' \to \mathbf{R}$.

A very plausible, but purely formal, argument now suggests the following expression for the generator $\mathcal{A}$ of $(P_t)_{t\geq 0}$ in terms of λ and the generator A of T_t :

$$\mathcal{A}\phi(x) = \int_{E'} (i < x, A^*\xi > -\lambda(\xi))e^{i<x,\xi>}\mathcal{F}^{-1}(\phi)(d\xi) \qquad (**)$$

($\mathcal{F}$ denoting the Fourier transform).

The main result of our work is a rigorous derivation, under various additional hypotheses, of formula (**). For the first time, pseudodifferential operators on an

Typeset by $\mathcal{A}\mathcal{M}\mathcal{S}$-TEX infinite–dimensional space of similar shape to those considered by Bogachev, Lescot and Roeckner (**Nagoya Mathematical Journal** , to appear) arise *naturally* in a probabilistic context.

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The averaging effect of self-similar media

Volker Metz (University of Bielefeld)

Many fractal sets are constructed by recursive self-similar cutouts like, for example, the Cantor set. This can be viewed as polluting a homogeneous material by recursively inserting self-similar inclusions. The properties of the resulting composite material are studied among others in the theory of homogenization. One of its approaches is to avoid the complicated local structure in considering only global (or effective) properties hoping that the effects of the local impurities will be averaged. This idea will be made mathematically precise: We use Dirichlet forms to specify the materials; the global structure is the trace of a Dirichlet form on a fixed finite set; the averaging effect comes from strict convex combinations of harmonic kernels provided we have some a priori boundedness; the convergence is with respect to Hilbert's projective metric on the cone of positive semidefinite forms.

Anormal diffusion processes on $\mathbf{R}^d$ and carpets

Hirofumi Osada

We present a new method to construct diffusion on multi-dimensional spaces by using Dirichlet form theory. We consider distorted Brownian motion with singular measures. We prove the degeneracy of diffusion by estimating the isoperimetric constant for these singular measures. As examples of applications of our method, we give "exotic Brownian motion" on R^d $(d > 1)$ and the fractals Sierpinski carpets. These diffusions are self-similar and non-degenerate but different from the Brownian motions.

Certain limit theorem for time inhomogeneous transition functions

Yoichi Oshima

Let $\{p_i^j(x,dy)\}_{0\le i\le j}$ be a family of transition functions on a locally compact separable metric space X such that

$$\begin{aligned} p_i^i(x,dy) &= \delta_{\{x\}}(dy) \\ p_i^j(x,dy) &= p_i^{i+1}p_{i+1}^{i+2}\cdots p_{j-1}^j(x,dy). \end{aligned}$$

(1)

We assume that there exists a family of positive Radon measures $\{m_i\}$ with full support satisfying $m_i p_i^j = m_j,\ \forall i \le j$. The purpose of this talk is to give a certain ratio limit theorem related to p_i^j and m_i. For the families of non-negative m_i-integrable functions $f = (f_i) and g=(g_i)$, put

$$B_{i,n}(g,f) = \bigcup_{i\le k\le n}\left\{x : \sum_{i\le j\le k} p_i^j g_j(x) < \sum_{i\le j\le k} p_i^j f_j(x)\right\}. \tag{2}$$

The next basic lemma is a version of Hopf's maximal ergodic lemma.

Lemma 1 *If* $\lim_{n\to\infty}\frac{1}{k_n}\sum_{i=1}^n\int_X f_i^{(n)}dm_i = 0$ *for some increasing sequence* $k_n\uparrow\infty$*, then*

$$\overline{\lim_{n\to\infty}}\frac{1}{k_n}\sum_{i=1}^n\int_X I_{B_{i,n}(g,f^{(n)})}(x)g_i(x)dm_i(x) = 0. \tag{3}$$

Define the kernels ${}_FG_i^{(n)}$ and ${}_FG_i$ by

$${}_FG_i^{(n)}g = I_F g_i + \sum_{j=i+1}^n I_F p_i^{i+1} I_{F^c}\,{}^F p_{i+1}^j(I_{F^c}g_j) \tag{4}$$

and ${}_FG_i g = \lim_{n\to\infty}{}_FG_i^{(n)}g$. In the next lemma, we use the condition that

$$\lim_{n\to\infty}\frac{\sum_{i\le k\le n}{}^F p_i^k(I_{F^c}g_k)}{\sum_{i\le k\le n} p_i^k g_k} = 0. \tag{5}$$

For a family $f=(f_i)$ of functions $f_i \in L^1(X;m_i)$, put

$$\alpha_n(f)=\sum_{k=0}^{n}\int_X f_k dm_k. \tag{6}$$

Lemma 2 *Suppose that (4) holds and* $\lim_{n\to\infty}\frac{1}{n}\alpha_n(g)=\bar{\alpha}(g)$ *exists. Then*

$$\lim_{n\to\infty}\frac{\sum_{k\leq n} p_i^k {}_F G_k^{(n)} g}{\sum_{k\leq n} p_i^k g_k}=\lim_{n\to\infty}\frac{\alpha_n({}_F G^{(n)} g)}{\alpha_n(g)}=1.$$

Let $\mathcal{L}_F(g)=\{f=(f_i): f_i\in L^1_+(m_i),\ \lim_{n\to\infty}\frac{1}{n}\alpha_n(|f-{}_F G^{(n)} g|)=0\}$ and $\mathcal{L}(g)$ be the family of functions $f=(f_i)$ for which there exists a sequence of functions $g^{(p)}=(g_i^{(p)})$ of the form $g_i^{(p)}=\sum_{k=1}^p c_{l\,F_k} G_i g$ with compact sets F_k and constants c_k such that $\lim_{p\to\infty}\lim_{n\to\infty}\frac{1}{n}\alpha_n(|f-g^{(p,n)}|)=0$, where $g_i^{(p,n)}=\sum_{k=1}^p c_{k\,F_k} G_i^{(n)} g$. For the next theorem, we introduce a stronger condition than (4): The limit

$$\lim_{n\to\infty}\frac{\sum_{i\leq k\leq i+n} {}^F p_i^k (I_{F^c} g_k)}{\sum_{i\leq k\leq i+n} p_i^k g_k}=0, \tag{7}$$

converges uniformly in i. We then have the following theorem.

Theorem 1 *Let $g=(g_i)$, be a family of functions such that* $\lim_{n\to\infty}\frac{1}{n}\alpha_n(g)=\bar{\alpha}(g)$ *exists and (6) holds. Suppose that there exist a positive constant γ and a positive finite measure μ such that $\frac{1}{\gamma}\mu\leq g_i\cdot m_i\leq\gamma\mu$ for all i. Then for all $f\in\mathcal{L}(g)$ and $\varepsilon>0$,*

$$\lim_{n\to\infty}\frac{1}{n}\int\sum_{i=1}^{n} I_{\Gamma_{i,n}^{\varepsilon}(f,g)}(x)d\mu(x)=0, \tag{8}$$

where

$$\Gamma_{i,n}^{\varepsilon}=\bigcup_{k=i}^{n}\left\{x:\left|\frac{\sum_{i\leq j\leq k} p_i^j f_j}{\sum_{i\leq j\leq k} p_i^j g_j}-\frac{\bar{\alpha}(f)}{\bar{\alpha}(g)}\right|>\varepsilon\right\}.$$

In particular, if $\lim_{n\to\infty}\frac{1}{n}\sum_{i=1}^n {}_F G_i^{(n)} g$ *converges to some non-trivial function a.e., then the result holds for any $f_i\equiv f\in L^1(\mu)$.*

If p_i^j is periodic, that is $p_i^{i+1}=p_{i+d}^{j+d+1}$ for some d, then it follows easily from Theorem 1 that

$$\lim_{n\to\infty}\frac{\sum_{j=i}^n p_i^j f}{\sum_{j=i}^n p_i^j g}=\frac{\bar{\alpha}(f)}{\bar{\alpha}(g)}\quad a.e.$$

for any non-negative bounded integrable functions f and g.

To get further result, assume that X is compact and there exists a density $p_i^{i+1}(x,y)$ of $p_i^{i+1}(x,dy)$ with respect to m_{i+1} satisfying

$$0 < \delta \le p_i^{i+1}(x,y) \le M < \infty, \quad \forall i \tag{9}$$

for some positive constants δ and M. Put

$$c_{i,j} = \sup_x ||p_j^{j+1}(x,\cdot) - p_i^{i+1}(x,\cdot)|| \text{ and } D_i^k = \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} \sum_{\ell=1}^{k-1} c_{i+\ell,j+\ell}.$$

Then the following theorem holds.

Theorem 2 *Suppose, for any $\eta < 1$, there exists a sequence $m(i) \to \infty$ such that*

$$\sum_{i=1}^{\infty} \left(\eta^{m(i)} + \sum_{k=1}^{m(i)} D_i^k \eta^k \right) < \infty.$$

Then, for all non-negative bounded measurable functions f, positive number ε and μ-a.e. x,

$$\limsup_{n\to\infty} \left| \frac{\sum_{j=i}^n p_i^j f(x)}{\sum_{j=i}^n p_i^j g(x)} - \frac{\bar{\alpha}(f)}{\bar{\alpha}(g)} \right| < \varepsilon$$

except finitely many i. In particular

$$\lim_{i\to\infty} \lim_{n\to\infty} \frac{\sum_{j=i}^n p_i^j f(x)}{\sum_{j=i}^n p_i^j g(x)} = \frac{\bar{\alpha}(f)}{\bar{\alpha}(g)} \quad \mu - a.e.x.$$

The conditions of Theorem 2 hold if $c_{ij} \le \xi^{i^\alpha}$ $(i < j)$ for some $\xi < 1$ and $\alpha > 0$.

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An embedding theorem for fractional Sobolev spaces over Wiener spaces and its applications

Jiagang Ren

We prove that for a given Cameron-Martin direction h, any Wiener functional in the fractional Sobolev space $D_{p,r}$ with $pr > 1$ is ray Hölder continuous along this direction. We then applied this result to prove the closedness of some bilinear forms which include, in particular, the bilinear form associated with the 2-dimensional polymer measure.

Stochastic calculus with respect to a finite quadratic variation process

Francesco Russo

The quadratic variation of a continuous process (when it exists) is defined through a regularization procedure. A large class of finite quadratic variation processes is provided, with a particular emphasis on Gaussian processes. For such processes a calculus is developed with application to the study of some stochastic differential equations.

A Rademacher-type theorem on configuration spaces and applications

Alexander Schied

This is joint work with M. Röckner. We consider an L^2-Wasserstein type distance ρ on the configuration space Γ_X over a Riemannian manifold X, and we prove that ρ-Lipschitz functions are contained in a Dirichlet space associated with a measure on Γ_X satisfying certain natural assumptions. These assumptions are in particular fulfilled by the classical Poisson measures and by a large class of tempered grandcanonical Gibbs measures with respect to a superstable lower regular pair potential. As an application we prove a criterion in terms of ρ for a set to be exceptional. This result immediately implies, for instance, a quasi-sure version of the spatial ergodic theorem. We also show that ρ is optimal in the sense that it is the intrinsic metric of our Dirichlet form.

ON THE VALIDITY OF THE LOG–SOBOLEV INEQUALITY FOR SYMMETRIC FLEMING–VIOT OPERATORS

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Abstract. It is a remarkable fact that in the class of measure–valued diffusions there is up to now not one single nontrivial example in which a logarithmic Sobolev inequality has been verified. In our talk we will discuss this question for generators of Fleming–Viot processes with parent–independent mutation which are defined as follows: Let S be a compact space which is interpreted as a space of types of a given population. Let $E := \mathcal{M}_1(S)$ be the space of all probability measures on S (i.e., all possible distributions of types within the given population). Define

$$Af(x) = \frac{\theta}{2}\int_S (f(y) - f(x))\,\nu_0(dy)\,, f \in C(S)\ ,$$

where $\theta > 0$ and $\nu_0 \in E$ such that $\mathrm{supp}\,(\nu_0) = S$. The Fleming–Viot process associated with mutation operator A (with no recombination and no selection) is defined as the unique solution of the $C_E[0,\infty)$–martingale problem $(L_{\theta,\nu_0}, \mathcal{F}C^\infty)$, where
$\mathcal{F}C^\infty := \{F = \varphi(\langle f_1,\cdot\rangle,\ldots,\langle f_d,\cdot\rangle) | f_i \in C(S), \varphi \in C^\infty(\mathbb{R}^d), d \in \mathbb{N}\}$ and

$$\begin{aligned} L_{\theta,\nu_0}F(\mu) &:= \frac{1}{2}\sum_{i,j=1}^d (\partial_{x_i}\partial_{x_j}\varphi)(\langle f_1,\mu\rangle,\ldots,\langle f_d,\mu\rangle)\mathrm{cov}_\mu(f_i,f_j) \\ &+ \quad \frac{1}{2}\sum_{i=1}^d (\partial_{x_i}\varphi)(\langle f_1,\mu\rangle,\ldots,\langle f_d,\mu\rangle)\langle Af_i,\mu\rangle\ . \end{aligned}$$

Here $\langle f,\mu\rangle := \int f\,d\mu$. This process has a unique symmetrizing probability measure m_{θ,ν_0}. The associated Dirichlet form $(\mathcal{E}_{\theta,\nu_0}, H^{1,2}(m_{\theta,\nu_0}))$ is obtained as the closure of $(-L_{\theta,\nu_0}F, G)_{L^2(m_{\theta,\nu_0})}$, $F, G \in \mathcal{F}C^\infty$, in $L^2(m_{\theta,\nu_0})$. In our talk we will prove that $\mathcal{E}_{\theta,\nu_0}$ determines a Poincaré inequality with constant $\frac{2}{\theta}$ whereas it determines a logarithmic Sobolev inequality if and only if $|S| < +\infty$. Moreover, we will show that the set

$$\{F^2 | F \in H^{1,2}(m_{\theta,\nu_0}), \mathcal{E}_{\theta,\nu_0}(F,F) + \|F\|^2_{L^2(m_{\theta,\nu_0})} \leq 1\}$$

is uniformly integrable if and only if $|S| < +\infty$ which implies that no reasonable substitute for the logarithmic Sobolev inequality (which could serve as an infinite dimensional substitute for compactness) can be formulated in terms of the Dirichlet form $\mathcal{E}_{\theta,\nu_0}$. All the results are contained in [1].

[1] Stannat, W., On the Validity of the Log-Sobolev Inequality for Symmetric Fleming-Viot Operators, SFB–343 Preprint 98-063, Bielefeld 1998.

L^p-independence of spectral radius for symmetric Markov processes and gaugeability

Takeda Masayoshi (Tohoku University)

Let X be a locally compact separable metric space and m a positive Radon measure on X with full support. Let $\mathbf{M} = (\Omega, X_t, P_x, \zeta)$ be an m-symmetric Borel right process on X. Let us denote by $\{p_t\}_{t\geq 0}$ and $\{R_\alpha\}_{\alpha>0}$ the semigroup and resolvent of $\mathbf{M}$, i.e., $p_t f(x) = E_x(f(X_t))$ and $R_\alpha f(x) = E_x(\int_0^\infty e^{-\alpha t} f(X_t)dt)$. We make following assumptions:

I (Irreducibility) If a Borel set A is p_t-invarient, i.e., $p_t(\chi_A f)(x) = \chi_A p_t f(x)$ m-a.e. x for any $f \in L^2(X;m) \cap \mathcal{B}(X)$ and $t > 0$, then A satisfies either $m(A) = 0$ or $m(X \setminus A) = 0$. Here $\mathcal{B}(X)$ is the space of Borel functions on X.
II (Absolute continuity condition) The transition probability $p_t(x, dy)$ of $\mathbf{M}$ is absolutely continuous with respect to m for each $t > 0$ and $x \in X$.
III $p_t(C_\infty(X)) \subset C_\infty(X)$, where $C_\infty(X)$ is the space of continuous functions vanishing at infinity.
IV $R_1 1(x) \in C_\infty(X)$.

Denote by $(\mathcal{E}, \mathcal{F})$ the Dirichlet space on $L^2(X;m)$ generated by $\mathbf{M}$. We then obtain

Theorem 1. Assume I $\sim$ IV. Then,

$$\lim_{t\to\infty} \frac{1}{t} \log \sup_{x\in X} P_x(t < \zeta) = -\inf\{\mathcal{E}(u,u) : u \in \mathcal{F},\ \int_X u^2 dm = 1\}. \qquad (1)$$

Let us denote by $\|p_t\|_p$ the operator norm of p_t from $L^p(X;m)$ to $L^p(X;m)$ and put

$$\lambda_p = -\lim_{t\to\infty} \frac{1}{t} \log \|p_t\|_p, \quad 1 \leq p \leq \infty.$$

Noting that $\sup_{x\in X} P_x(t < \zeta) = \|p_t\|_\infty$ and that the right hand side of (1) is equal to $-\lambda_2$, we see from Theorem 1 that

$$\lambda_\infty = \lambda_2, \qquad (2)$$

which implies p-independence of λ_p because $\|p_t\|_2 \leq \|p_t\|_p \leq \|p_t\|_\infty$ for any p.

Theorem 2. Assume I $\sim$ IV.
(i) $\lambda_2 > 0$.
(ii) $\sup\{\lambda \geq 0 : \sup_{x\in X} E_x(\exp(\lambda\zeta)) < \infty\} = \lambda_2$.
(iii) $\sup_{x\in X} E_x(\exp(\lambda\zeta)) < \infty$ if and only if $\lambda < \lambda_2$.

Corollary 1. Assume I $\sim$ IV. Then

$$\sup_{x\in X} E_x(\exp(\zeta)) < \infty \quad \text{if and only if} \quad \lambda_2 > 1.$$

Let (B_t, P_x^W) be the Brownian motion on R^d. Let μ be a finite positive measure in the Kato class and A_t^μ the additive functional associated with μ. We can show, as an application of Corollary 1, that for a regular domain D of R^d

$$\sup_{x\in D} E_x^W(\exp(A_{\tau_D}^\mu)) < \infty$$

if and only if

$$\inf\{\frac{1}{2}\mathbf{D}(u,u) : u \in C_0^\infty(D),\ \int_D u^2 d\mu = 1\} > 1,$$

where τ_D is the exit time from D and $\mathbf{D}$ is the Dirichlet integral.

Stochastic differential equations in infinite dimensions - Solutions via generalized Dirichlet forms

Gerald Trutnau

Using a further developed stochastic calculus for generalized Dirichlet forms we construct weak solutions to SDE's in infinite dimensions of type

$$dX_t \;\; = \;\; dW_t + \frac{1}{2}\beta_H^{\mu}(X_t)dt + \overline{\beta}(X_t)dt \ , \ X_0 = z. \tag{1}$$

Here $(X_t)_{t\geq 0}$ takes values in some real separable Banach space E, $z \in E$, $(W_t)_{t\geq 0}$ is an E-valued Brownian motion, $\overline{\beta}$ is some square integrable vector field on E taking values the (generalized) Cameron-Martin space $H \subset E$ and $\beta_H^{\mu} : E \to E$ is the logarithmic derivative of μ associated with H. We also present an application of equation (**??**) for more explicit maps β_H^{μ} and $\overline{\beta}$.

Sobolev type inequalities

Feng-Yu Wang

A general version of Sobolev type inequality, including both of the classical Sobolev inequality and the logarithmic Sobolev one, is studied for general symmetric forms by using isoperimetric constants. As results, some sufficient conditions and necessary ones are presented. It is illustrated by some birth-death processes that our conditions can be optimal.

Nonlinear local maps defined on Gaussian random fields of subspaces of $\mathcal{S}'(R^d)$

Minoru W. Yoshida*

August 14, 1998

Abstract

This talk is an introduction of results derived in a preprint [Y]. The main subjects are considerations of $H - C^1$ maps on abstract Wiener spaces on subspaces of measures. Let $\mathcal{B}(\boldsymbol{R}^d)$ be the topological Borel σ-field of $\boldsymbol{R}^d$ and λ^d be the Lebesgue measure on $\boldsymbol{R}^d$. On a complete probability space $(\Omega, \mathcal{F}, P)$ we can define a family of random variables $\{W(A), A \in \mathcal{B}(\boldsymbol{R}^d),\ \lambda^d(A) < \infty\}$ which satisfies the following: The probability law of $W(A)$ is $N(0, \lambda^d(A))$; $W(A_1)$ and $W(A_2)$ are independent random variables when $A_1 \cap A_2 = \emptyset$. W is called as an $L^2(\Omega; P)$-valued Brownian measure on $(\boldsymbol{R}^d; \mathcal{B}(\boldsymbol{R}^d))$ or the white noise based on λ^d. We do not formulate our problems on an abstract Wiener space that is given *a priori*, but rather adopt above $(\Omega, \mathcal{F}, P)$ as our underlining (canonical) probability space. We define a sequence of $C_0(\boldsymbol{R}^d \to \boldsymbol{R})$-valued random variables $\{{}_k\phi_\omega\}$, of which element is defined as a multiple stochastic integral with respect to W, so that it has a limit in a Banach space B ($B = B_d^{a,b}$ defined below), included in the space of tempered distributions $\mathcal{S}'(\boldsymbol{R}^d)$. Also $\{{}_k\phi_\omega\}$ is chosen to satisfy that the probability space (B, μ) becomes an abstract Wiener space, where μ is the probability law of $\phi_\omega = \lim_{k\to\infty} {}_k\phi_\omega$. Abstract Wiener spaces constructed by our method has the so called free Euclidean field as their particular case. We have to emphasise that various abstract Wiener spaces are able to be discussed on the common probability space $(\Omega, \mathcal{F}, P)$. The main reson why we start ourselves from a general probability space $(\Omega, \mathcal{F}, P)$ is that by making use of the expression of ϕ_ω by means of multiple stochastic integral these calculus as derivatives and Skorohod integrals becomes clear (cf. Chapter 4 in [Nu]). Our objectives are considerations of some non-linear measurable functions τ on (B, μ) and of non-linear transformations T which are defined by making use of τ.

A little bit precisely, for each $a, b > 0$ let $B_d^{a,b}$ be a Banach space (in fact it is a separable Hilbert space)

$$B_d^{a,b} = \{(|x|^2 + 1)^{\frac{b}{4}}((-\Delta + 1)^{\frac{a}{2}} f) \mid f \in L^2\} \subset \mathcal{S}'(\boldsymbol{R}^d),$$

where Δ denots the Laplace operator on $\boldsymbol{R}^d$. Some properties of topological Borel σ-field of $B_d^{a,b}$, such as every $C_0(\boldsymbol{R}^d \to \boldsymbol{R})$-valued measurable functions defined on some measurable space being able to be regarded as a $B_d^{a,b}\,(a, b > 0)$ valued measurable functions, can be shown. Next, for some given mass $m > 0$ assume that positive integer p and positive numbers a, b and α satisfy

$$\min(1, \frac{2a}{d}) + p \times \min(1, \frac{2\alpha}{d}) > p, \qquad b > d. \tag{0. 1}$$

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Then a $B_d^{a,b}$-valued random variable $:\phi^p_{\alpha,\omega}:$ can be defined as a $a.s.$ convergent limit of a sequence $\{:_k \phi^p_{\alpha,\omega}:\}$ of $C_0(\boldsymbol{R}^d \to \boldsymbol{R})$-valued random variables. For each k the random variable $:_k \phi^p_{\alpha,\omega}:$ has been defined as some multiple stochastic integral. In case when $p \geq 2$ the random variable $:\phi^p_{\alpha,\omega}:$ is not Gaussian, and $\phi_{\alpha,\omega} =: \phi^1_{\alpha,\omega}:$ is a Gaussian random variable such that

$$\int_\Omega e^{\sqrt{-1}<\phi_{\alpha,\omega},\varphi>_{\mathcal{S}',\mathcal{S}}} P(d\omega) = \exp\left(-\frac{1}{2}\|(-\Delta+m^2)^{-\frac{\alpha}{2}}\varphi\|^2_{L^2}\right). \qquad (0.\ 2)$$

In particular, when $d=2$ and $\alpha=1$, then it is shown that $\phi_{1,\omega}$ and $:\phi^p_{1,\omega}:$ are the free Euclidean field on $\boldsymbol{R}^2$ and its p-th Wick power respectively. (Support properties of the free Euclidean field have been studied by many authors (cf. for eg. [Re], also cf. [Rö1] and [Rö2] for studies of its trace properties), in examples discussed in [AR1] and [AR2] the fact that the p-thWick power of the free Euclidean field lives in some Banach space has been used to construct an infinite dimensional diffusion process of which invariant measure is $P(\phi)_2$-Euclidean field (stochastic quantization).) Also, some measurability properties of $:\phi^p_{\alpha,\omega}:$ with respect to $\phi_{\alpha,\omega}$ can be discussed. Roughly speaking, it is possible to show that if D is an open set of $\boldsymbol{R}^d$ and $\varphi \in C_0^\infty(\boldsymbol{R}^d \to \boldsymbol{R})$ satisfying supp$[\varphi] \subset D$ then the random variable $<\varphi, :\phi^p_{\alpha,\omega}:>_{\mathcal{S},\mathcal{S}'}$ is measurable with respect to the σ-field $\sigma\{<\psi,\phi_{\alpha,\omega}>_{\mathcal{S},\mathcal{S}'} \mid \psi \in C_0^\infty(\boldsymbol{R}^d \to \boldsymbol{R})$ such that supp$[\psi] \subset D\}$. Lastly, we present properties of non-linear maps of $\phi_{\alpha,\omega}$. If

$$\min(1,\frac{2a}{d}) + \min(1,\frac{2\alpha}{d}), \qquad b > d,$$

then the random variable $\phi_{\alpha,\omega}$ is a $B_d^{a,b}$-valued random variable (cf. (0.1)), and the probability law of $\phi_{\alpha,\omega}$ can be defined on $B_d^{a,b}$, we denote this by μ_α. It can be seen that $(B_d^{a,b},\mu_\alpha)$ is endowed with the structure of abstract Wiener space (cf. [Rö2] and [ARK]). Assuming that γ, p and β satisfy

$$\min(1,\frac{2\gamma}{d}) + p \times \min(1,\frac{2\beta}{d}) > p \qquad (0.\ 3)$$

the existence of a measurable function $\tau = \tau_{\alpha,(\beta,p)} : B_d^{a,b} \longrightarrow B_d^{\gamma,b}$ satisfying

$$\tau(\phi_{\alpha,\omega}) =: \phi^p_{\beta,\omega}: \qquad P-a.s.\ \omega\in\Omega$$

can be proved. And then giving a cut-off function $\eta \in C_0^\infty(\boldsymbol{R}^d)$, $\eta \geq 0$, a non-linear map $F : B_d^{a,b} \longrightarrow L^2$ is defined by

$$F(\phi)(x) = (-\Delta+m^2)^{-\frac{\gamma}{2}}\left(\eta\tau_{\alpha,(\beta,p)}(\phi)\right)(x).$$

As a result we see that if γ, p $(p \geq 2)$ and β satisfy an assumption, that is stronger than (0.3), such that

$$(2+p*)(p-p*)-1 < \min(1,\frac{2\gamma}{d}) + \{(2+p*)(p-p*)-1\}\min(1,\frac{2\beta}{d}), \qquad (0.\ 4)$$

$$p* = \text{the greatest non-negative integer that is not greater than } \tfrac{p}{2}-1,$$

then the map F is $H-C^1$ on the abstract Wiener space $(B_d^{a,b},\mu_\alpha)$. (0.3) is exactly a sufficient condition under which F becomes $\mathbf{L}^{1,2}$, but it may be not sufficient in order that F becomes $H-C^1$. By making use of the non-linear map F, for some given real λ a non-linear transform $T : B_d^{a,b} \longrightarrow B_d^{a,b}$ can be defined by

$$T(\phi) = \phi + \lambda((-\Delta+m^2)^{-\frac{\alpha}{2}}F)(\phi) \qquad \phi \in B_d^{a,b}.$$

To the transform T we apply the change of variable formula on abstract Wiener spaces given by [ÜZ1], which are the generalizations of the results derived by [Kus1], and then consider the absolute continuities of $\mu_\alpha \circ T^{-1}$ and $\mu_\alpha \circ T$ with respect to μ_α. One important result derived here is the concrete formulae of Radon-Nikodym densities $\frac{d\mu_\alpha \circ T^{-1}}{d\mu_\alpha}$ and $\frac{d\mu_\alpha \circ T}{d\mu_\alpha}$ (restricting the definition domain of T, and taking λ so that $|\lambda|$ is small). The free Euclidean field case can be discussed as an application.

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Probabilistic Representations and Hyperbound Estimates for Semigroups

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Abstract

In this paper, we study lower order perturbations of a symmetric second order differential operator generating a hypercontractive semigroup. We give a probabilistic representation of the in general not sub-Markovian semigroup associated with the perturbed operator and prove that the perturbed semigroup is also hypercontractive under some exponential integrability conditions on the coefficients.