Fourth International Workshop on Zeta Functions in Algebra and Geometry

29 May – 02 June 2017

Faculty of Mathematics
Bielefeld University

Abstracts

Monday, May 29th

08:30 – 08:55 **Registration** Also possible during first coffee break

08:55 – 09:00 **Opening**

09:00 – 10:00 **Mark Pollicott** (Warwick)
Part 1: Zeta functions and closed geodesics

We describe part of the theory of zeta functions for closed geodesics, beginning with the motivation from number theory, the classical harmonic analysis/trace formula approach to constant curvature surfaces and the dynamical approach to the more general setting of variable negative curvature.

Coffee Break & Registration

10:30 – 11:30 **Raf Cluckers** (Lille)
Uniform p-adic integration and applications

As a concrete variant of motivic integration, we will discuss uniform p-adic integration and constructive aspects of results involved. Uniformity is in the p-adic fields, and, for large primes p, in the fields $\mathbb{F}_p((t))$ and all their finite field extensions. Using real-valued Haar measures on such fields, one can study integrals, Fourier transforms, etc. We follow a line of research that Jan Denef started in the eighties, with in particular the use of model theory to study various questions related to p-adic integration. A form of uniform p-adic quantifier elimination is used. Using the notion of definable functions, one builds constructively a class of complex-valued functions which one can integrate (w.r.t. some of the variables) without leaving the class. One can also take Fourier transforms in the class. Recent applications in the Langlands program are based on Transfer Principles for uniform p-adic integrals, which allow one to get results for $\mathbb{F}_p((t))$ from results for $\mathbb{Q}_p$, once p is large, and vice versa. These Transfer Principles are obtained via the study of general kinds of loci, some of them being zero loci. More recently, these loci are playing a role in the uniform study of p-adic wave front sets for (uniformly definable) p-adic distributions, a tool often used in real analysis. This talk contains various joint works with Gordon, Hales, Halupczok, Loeser, Raibaut.

11:45 – 12:45 **Pierrette Cassou-Noguès** (Bordeaux)
Motivic Milnor fiber at infinity

In this talk, we shall give the main ideas for computing the motivic Milnor fiber at infinity for a polynomial in two variables, using the Newton algorithm. This is a joint work with Michel Raibaut.

Lunch Break

14:45 – 15:45 **Anke Pohl** (Jena)
Automorphic functions, resonances, and Selberg zeta functions via transfer operators

We report on the current status of a program to develop transfer operator approaches to automorphic functions, resonances, and Selberg zeta functions for non-compact hyperbolic surfaces of finite or infinite area and finite-dimensional representations.

Coffee Break

16:15 – 17:15 **Mark Pollicott** (Warwick)
Part 2: Zeta functions and higher Teichmüller theory

The Selberg Zeta function $Z(s)$ for Fuchsian groups can be described in terms of representations of (surface) groups in PSL(2,$\mathbb{R}$). The higher Teichmüller theory can be formulated in terms of representations in PSL(d,$\mathbb{R}$), for $d > 2$, and we consider how properties of the analogous zeta function compare with those of $Z(s)$.

Tuesday, May 30th

09:00 – 10:00 **Michel Raibaut** (Chambéry)
Motivic invariants at infinity of plane algebraic curves

Let f be a complex polynomial with isolated singularities. In this talk, we will start by recalling classical formulas of the Euler characteristic of a fiber of f in terms of Milnor numbers of the singularities of f and the defect of equisingularity at infinity in a compactification of f. Then, recalling some notions of motivic integration and Denef-Loeser, Guibert-Loeser-Merle theorems, we will consider some motivic zeta functions and define for each value a, a motivic invariant at infinity of the fiber of f at a. This invariant does not depend on the chosen compactification, it is generically equal to zero and, under isolated singularities assumptions, its Euler characteristic is equal to the defect of equisingularity at infinity of f for the value a.
In the last part of the talk, we will consider the case of plane curves, where computations of this invariant can be done in terms of Newton polygons at infinity, using an induction process based on Newton transformations and iterated Newton polygons.
This is a joint work with Pierrette Cassou-Noguès.

Coffee Break

10:30 – 11:30 **Lê Quy Thuong** (Hanoi)
On motivic multiple nearby cycles

We introduce a new product of two formal series with coefficients in distinct Grothendieck rings of algebraic varieties, which preserves the integrability and commutes with the limit of rational series. In the same context, we define a motivic multiple zeta function with respect to an ordered family of regular functions, which is integrable and connects closely to Denef-Loeser's motivic zeta functions. The limit of the motivic multiple zeta function is called the motivic multiple nearby cycles. We will present an explicit formula for the motivic double zeta functions and the motivic double nearby cycles using resolution of singularities. A version of the Euler reflexion formula for motivic double zeta functions will be also given, and by taking its limit the motivic Thom-Sebastiani theorem will be recovered.

11:45 – 12:45 **Chenyang Xu** (Beijing)
Birational models and zeta functions

The progress on birational geometry provides us new tools to study zeta functions via birational models. We will present some results along this line, including the solution of Veys' conjecture (jointly with Johannes Nicaise) and a construction of an alternative motivic zeta function.

Lunch Break

14:45 – 15:45 **Avraham Aizenbud** (Weizmann Institute of Science)
Counting representations of arithmetic groups and point of schemes

We will discuss the following question: How many irreducible representations of a given dimension n do groups like $\mathrm{SL}_d(\mathbb{Z})$ have?
We will see how this question is related to the number of $\mathbb{Z}/n\mathbb{Z}$-points of certain schemes. Those are related to singularities of moduli spaces, pushforward of smooth measures, commutators of random elements in finite groups, jet schemes and more.
As a result of those connections, we will show that the number of such representations is bounded by a polynomial in n whose degree is universally bounded for high rank arithmetic groups (by 40).
See slides on `http://www.wisdom.weizmann.ac.il/~aizenr/4Talks/Rep_count_tallk_Glob.pdf`
This is a joint project with Nir Avni.

Coffee Break

16:15 – 16:45 **Tobias Weich** (Paderborn)
Classical and quantum resonances on hyperbolic surfaces

It is a classical consequence of Selberg's trace formula, that for compact hyperbolic surfaces the zeros of the Selberg zeta-function are given by the Laplace eigenvalues and by topological zeros. Around 2000 this result has been extended to convex co-compact surfaces by Patterson-Perry and Bunke-Olbrich. In this talk we will see, that behind this correspondence of zeta-zeros and the Laplace spectrum, there is a deeper connection between so called classical and quantum resonant states.
This is joint work with C. Guillarmou (Orsay) and J. Hilgert (Paderborn).

16:50 – 17:10 **Miriam Bocardo Gaspar** (Mexico City)
String Amplitudes and Multivariate Local Zeta Functions

In this talk we will give some connections between local zeta functions and p-adic string amplitudes. A main connection is that the convergence of the p-adic Koba-Nielsen type string amplitudes strongly depends on the convergence of Igusa-type integrals with several complex parameters. String amplitudes are "essentially" local zeta functions, and thus, they are algebraic-geometric objects that can be studied over several ground fields, for instance $\mathbb{R}$, $\mathbb{C}$, $\mathbb{Q}_p$, $\mathbb{C}((t))$, and that on each of these fields these objects have similar mathematical properties.

Wednesday, May 31th

09:00 – 10:00 **Ann Lemahieu** (Nice)
On the monodromy conjecture for nondegenerate hypersurface singularities

The monodromy conjecture predicts a relationship between the poles of p-adic integrals associated to a complex polynomial f and the monodromies of the complex hypersurface defined by f. In this talk we will concentrate on the monodromy conjecture at the level of the topological zeta function for hypersurface singularities that are nondegenerate w.r.t. their Newton polyhedron. We explain some partial results in higher dimension and we give a proof of the monodromy conjecture for '0-convenient' singularities in dimension four. This is work in progress with Alexander Esterov (HSE, Moscow) and Kiyoshi Takeuchi (University of Tsukuba, Japan).

Coffee Break

10:30 – 11:30 **Sho Tanimoto** (Copenhagen)
The space of rational curves and height zeta functions

Manin's conjecture is a conjectural asymptotic formula for the counting function of rational points on a Fano variety after removing the contribution of an exceptional set from the counting function. Recently there are many developments regarding birational geometry of exceptional sets in Manin's conjecture due to Lehmann-Tanimoto-Tschinkel and Hacon-Jiang. In this talk I would like to explain some of applications of geometry of exceptional sets to the study of the space of rational curves on a Fano variety, and analytic properties of the height zeta function associated to the space of rational curves. This is joint work with Brian Lehmann.

11:45 – 12:45 **Daniel Loughran** (Manchester)
Brauer groups and height zeta functions

In this talk I present some results on a problem of Serre concerning specialisations of Brauer groups on algebraic varieties.

Lunch Break

Excursion

19:00 – **Workshop dinner**

Thursday, June 1st

09:00 – 10:00 **Wen-Ch'ing Li** (Pennsylvania State)

Group based combinatorial zeta functions

Similar to their counterparts in number theory, a combinatorial zeta function counts the geodesic closed cycles in a finite simplicial complex. In this survey talk we shall consider the complexes arising from finite quotients of buildings. The properties of such zeta functions will be discussed, and connection and comparison with number theoretical zeta functions will be mentioned.

Coffee Break

10:30 – 11:30 **Ming-Hsuan Kang** (Hsinchu)

Geometric zeta functions on reductive groups over non-archimedean local fields

Ihara zeta functions is a geometric zeta function associated to a finite quotient of the building of PGL_2 over a non-archimedean local field. In this talk, we will study geometric zeta functions on Tits buildings of split reductive groups of higher ranks via two different tools and the philosophy of the field with one element. For groups of adjoint type, we will first study the relation between Langlands L-functions and geometric zeta functions on a single apartment and then establish the result to the whole building. For simply connected groups, we will study zeta function using generalized Poincaré series associated to Iwahori Hecke algebra. Especially, our method can be applied to all groups of rank two including G_2.

11:45 – 12:15 **Shai Shechter** (Be'er Sheva)

On regular characters of classical groups

Regular characters of $G = \mathrm{GL}_n(\mathfrak{o})$, where $\mathfrak{o}$ is a discrete valuation ring, form a considerable subset of the set of irreducible complex continuous characters of G and the largest class currently amenable to explicit construction. The definition of regular characters goes back to Shintani and Hill, who proved a variety of favourable properties of such characters and completed their construction in several key cases. Recently, the construction of all regular characters of $\mathrm{GL}_n(\mathfrak{o})$ was completed by Stasinski and Stevens and, independently, by Krakowski, Onn and Singla, who also computed the regular representation zeta function of the special and unitary groups over $\mathfrak{o}$, i.e. the Dirichlet series $\zeta^{\mathrm{reg}}(s) = \sum \chi(1)^{-s}$, where χ ranges over all regular characters of G.
In my talk, I will report on a generalization of the definition and construction of regular characters to the classical groups over $\mathfrak{o}$.

Lunch Break

14:45 – 15:45 **Christian Bogner** (HU Berlin)
Part 1: Periods and Feynman integrals

In particle physics, many computations rely on the evaluation of so-called Feynman integrals. It is well-known that zeta values, multiple zeta values and generalizations of polylogarithms appear very frequently in these computations. Over the last decade, a new mathematical perspective on Feynman integrals was established in which these integrals are related to period integrals in the sense of algebraic geometry. In this talk I give a brief introduction to Feynman integrals and review some of their relations to multiple zeta values and periods in general.

Coffee Break

16:15 – 17:15 **Sylvie Paycha** (Potsdam)
Branched zeta functions and their renormalised values at poles

Multizeta functions generalise the well-known ordinary zeta function and branched zeta functions are discrete sums attached to trees that generalise multizeta functions which correspond to ladder trees. We view branched zeta functions as discrete sums of pseudodifferential symbols and accordingly, we decorate the trees with pseudodifferential symbols. We then implement a multivariate regularisation procedure in replacing the symbols in the decoration by holomorphic families of symbols. Using the universal property of trees, we then build the corresponding regularised branched zeta functions and show that they are *multivariate meromorphic functions with linear poles*.
In order to renormalise branched zeta functions at poles, we need a good control of the poles. For this purpose, we use a refined universal property for trees, which involves the notion of locality, reminiscent of locality in quantum field theory. In our framework locality is encoded in a binary symmetric relation, with which we equip sets that we call localised sets. We introduce the localised monoid of properly decorated forests, which serves as a model for what we call *partially operated localised monoids*. A refined universal property for partially operated localised monoids provides us with a good knowledge of the pole structure of the branched zeta functions. Branched zeta functions are then renormalised at poles using a multivariate minimal subtraction scheme, which generalises the (univariate) minimal subtraction scheme known to physicists. This talk is based on joint work with Pierre Clavier, Li Guo and Bin Zhang.

Friday, June 2nd

09:00 – 10:00 **Tobias Rossmann** (Auckland)
The average size of the kernel of a matrix and orbits of linear groups

Given a module M of matrices over a compact discrete valuation ring $\mathcal{O}$ of characteristic zero, we consider the generating function encoding the average sizes of the kernels of the elements of M over the finite quotients of $\mathcal{O}$. As we will see, special cases of these generating functions include conjugacy class and orbit-counting zeta functions of suitable groups.

Coffee Break

10:30 – 11:30 **Johannes Nicaise** (London)
A motivic Fubini theorem for the tropicalization map

This talk is based on joint work with Sam Payne. I will present a Fubini theorem for the tropicalization map in the context of Hrushovski and Kazhdan's theory of motivic integration. As an application, I will prove a conjectural description by Davison and Meinhardt of the motivic nearby fiber of a weighted homogeneous polynomial. This conjecture emerged in the theory of motivic Donaldson-Thomas invariants. The same method yields a short proof of the integral identity conjecture of Kontsevich and Soibelman, which was proven by Lê Quy Thuong.

11:45 – 12:45 **Steffen Kionke** (Düsseldorf)
Zeta functions associated to representations of compact p-adic Lie groups

To an admissible smooth representation of a profinite group we associate a zeta function defined via a Dirichlet series which encodes the multiplicities and degrees of the irreducible constituents. This provides a new perspective on the well-known 'representation zeta function' of (profinite) groups. We give a short introduction to the topic and explain how this approach can yield a more detailed understanding of problems in the area of representation growth. In particular, we discuss induced representations of compact p-adic Lie groups (e.g. the general linear group over the p-adic integers). In this case the zeta functions can be related to certain series of Igusa integrals and classical methods can be used to prove rationality and functional equations. This is based on joint work with Benjamin Klopsch.

Lunch Break

14:45 – 15:45 **Uri Onn** (Be'er Sheva)
Pro-isomorphic zeta functions of some D^*-groups

Pro-isomorphic zeta functions are Dirichlet series associated with finitely generated nilpotent groups that enumerate finite index subgroups having the same finite quotients as the parent group. They constitute one of the natural non-commutative analogues of the classical Dedekind zeta functions. While other analogues, such as (normal) subgroup zeta functions, have been studied intensively, the study of pro-isomorphic zeta functions is in a far less advanced state. A unique feature of the latter is that they are closely related to zeta functions of algebraic groups, studied by Weil, Igusa and others. In this talk I will describe the main tools that are used to study pro-isomorphic zeta functions and report on some recent results regarding zeta functions associated with members of a family of class-2 nilpotent groups called D^*-groups.
This is a joint work with Mark Berman and Benjamin Klopsch.

Coffee Break

16:15 – 17:15 **Christian Bogner** (HU Berlin)

Part 2: Algorithms for multiple polylogarithms

Multiple polylogarithms and multiple zeta values serve as a useful framework of functions and numbers for the computation of many Feynman integrals. They can be expressed in terms of a particular class of iterated integrals on moduli spaces of curves. In this talk I discuss algorithms for computations with these iterated integrals and present a resulting computer program whose main purpose is the evaluation of a certain class of Feynman integrals. I will also refer to other possible applications of this program.