Workshop Integral p-adic Hodge Theory

Bielefeld, April 10-12, 2017

In this workshop we will study integral p-adic Hodge theory following Bhatt-Morrow-Scholze. They construct a new cohomology theory for smooth proper formal schemes $\mathfrak{X}$ over the ring of integers of $\mathbb{C}_p$, denoted by $R\Gamma_{A_{\mathrm{inf}}}(\mathfrak{X})$, which interpolates the crystalline cohomology of the special fibre $\mathfrak{X}_k$, the de Rham cohomology of $\mathfrak{X}$, and the etale cohomology of the rigid analytic generic fibre X of $\mathfrak{X}$. As an application one gets new comparison theorems between the etale cohomology of X and the crystalline cohomology of $\mathfrak{X}_k$. For a more detailed overview we refer to the introduction of [BMS]; see also the surveys [B], [M], [Sch4] with emphasis on different aspects of the theory.

We will mainly follow [BMS], with some minor additions. The goal of the workshop is to delve deeply into technical details! The speakers are encouraged to explain interesting techniques thoroughly. All talks will be 90 minutes, which should allow to explain many aspects in detail. Most talks will just cover one section of [BMS]. The speakers are invited to consult additional literature if necessary. If in doubt which material to cover, please do not hesitate to contact the organizers for any questions. Please also contact other speakers to agree upon what you should cover, what has been explained previously, and what is needed in later talks.

In principle we assume that the participants are familiar with the notion of perfectoid spaces. Nevertheless it might be a good idea to recall the basic definitions and results, in particular in talks 1 and 3.

Talk 1. Perfectoid rings and algebras. [Sch1, §3,4,5] and [BMS, §3]. We suggest to focus on the different period rings and in particular A_{inf} that plays a central role (see also [Ke] for its algebraic properties). Explain the tilting equivalence and the maps θ_r and $\tilde{\theta}_r$, and if possible also [BMS, Prop 3.24 and Cor 3.29] on coherence.

Talk 2. Breuil-Kisin-Fargues modules. [BMS, §4], see the first lines of §4 for crucial topics. See also [M, Appendix A].

Talk 3. Perfectoid spaces and almost purity. [Sch1, §2,6,7], in particular Proposition 7.13. See also [Sch2, §7].

Talk 4. Rational p-adic Hodge theory. [BMS, §5] and [Sch3]. In particular explain the pro-étale site and its sheaves ([BMS, 5.4]). If possible explain how almost purity enters in the proof of [Sch3, Lemma 4.10].

Talk 5. The $L\eta$ operator. [BMS, §6]. This talk is pure homological algebra. Add [B, Le 5.12] and [BMS, Le 7.9], [BMS, Le 8.11], [BMS, Le 8.16].

Talk 6. The complex $\widetilde{\Omega}_{\mathfrak{X}}$. [BMS, §8]. See also [M, §2.2].

Talk 7. The complex $A\Omega_{\mathfrak{X}}$. [BMS, §9]. See [B, §1.2 (e)] for a simplification. Leave out Th 9.2 (iii) and Th 9.4 (iv) on the connection with the de Rham-Witt complex, which will be explaind in Talk 9. The q-de Rham complex of [BMS, Ex 7.7] appears in this talk for the first time.

Talk 8. The relative de Rham-Witt complex. [BMS, §10] and [LZ].

Talk 9. The comparison with de RhamWitt complexes. [BMS, §11]. The main result is a Cartier isomorphism for $A\Omega_X$ that coincides with Th 8.3 for $r = 1$. Note that 11.17 is not needed in the sequel although it is interesting in its own right.

Talk 10. The comparison with crystalline cohomology over A_{crys}. [BMS, §12].

Talk 11. Rational p-adic Hodge theory, revisited. [BMS, §13].

Talk 12. Proof of main theorems. [BMS, §14]. See also [BMS, §1], and [BMS, §2] for examples.

References

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[Ke] K. Kedlaya: Some Ring-Theoretic Properties of A_{inf}. arxiv:1602.09016

[LZ] A. Langer, T. Zink: De Rham-Witt cohomology for a proper and smooth morphism. J. Inst. Math. Jussieu **3**, (2004), 231-314

[M] M. Morrow: Notes on the $\mathbb{A}_{\text{inf}}$-cohomology of *Integral p-adic Hodge theory.*

[Sch1] P. Scholze: Perfectoid spaces. Publ. Math. IHES **116** (2012), 245-313

[Sch2] P. Scholze, J. Weinstein: p-adic geometry. Lecture notes from course at UC Berkeley in Fall 2014.

[Sch3] P. Scholze: p-Adic Hodge theory for rigid-analytic varieties. Forum of Mathematics, Pi (2013), Vol. 1, e1, 77 pages

[Sch4] Canonical p-deformations in arithmetic geometry. arxiv:1606.01796