CONVERGENT GEOMETRIC INTEGRATOR FOR THE LANDAU-LIFSHITZ-GILBERT EQUATION WITH MAGNETOSTRICTION

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The understanding and development of magnetic materials and devices is of utter relevance for example in magnetic sensors, recording heads, and magneto-resistive storage devices. In the literature it is well-accepted that dynamic micromagnetic phenomena are described best by the Landau-Lifshitz-Gilbert equation (LLG). This non-linear parabolic equation reads

$$\begin{aligned}\mathbf{m}_t &= -\alpha \mathbf{m} \times \big(\mathbf{m} \times \mathbf{H}(\mathbf{m})\big) + \mathbf{m} \times \mathbf{H}(\mathbf{m})\\ \mathbf{m}(0) &= \mathbf{m}_0 \quad \text{in } H^1(\Omega; \mathbb{S}^2)\\ \partial_n \mathbf{m} &= 0 \quad \text{on } (0,\tau) \times \partial\Omega\\ |\mathbf{m}| &= 1 \quad \text{a.e. in } (0,\tau) \times \Omega,\end{aligned}$$

where $\mathbf{H}(\mathbf{m})$ denotes the total magnetic field which is given by

$$\mathbf{H}(\mathbf{m}) := \Delta\mathbf{m} + D\Phi(\mathbf{m}) + \mathcal{P}(\mathbf{m}) - \mathbf{f} + \mathbf{h}^{\boldsymbol{\sigma}}(\mathbf{m})$$

and consists of the exchange field, the anisotropy field, the stray-field, the applied field, as well as the contribution of the non-linear magnetostrictive field. In this equation, the magnetic behavior of the ferromagnetic body is characterized by the vector valued magnetization $\mathbf{m} : (0,\tau) \times \Omega \to \mathbb{R}^3$. In addition, the magnetostrictive field $\mathbf{h}^{\boldsymbol{\sigma}}(\mathbf{m})$ depends on the so-called stress tensor $\boldsymbol{\sigma}$ and is thus coupled to the equation of elastodynamics

$$\varrho \mathbf{u}_{tt} - \nabla \cdot \boldsymbol{\sigma} = 0 \qquad \text{on } (0,\tau) \times \Omega.$$

Numerical challenges for the time integration arise from the strong non-linearity, the non-convex side constraint $|\mathbf{m}| = 1$, the non-local dependence of the demagnetization field $\mathcal{P}(\mathbf{m})$ from the magnetization $\mathbf{m}$ as well as from the coupling of the two equations.

The great number of applications as well as the amount of numerical issues makes LLG of equal interest for both, physicists and mathematicians, and thus the scientific community benefits in many ways.

Recently there has been a huge progress in the mathematical literature for the so-called small-particle limit. In this model all energy terms but the exchange energy are neglected, i.e. the magnetic field is simplified to $\mathbf{H}(\mathbf{m}) = \Delta\mathbf{m}$, cf. [Alouges/Jaisson, Math. Models Methods Appl. Sci., 16 (2006)], [Bartels/Prohl, SIAM J. Numer. Anal., 44 (2006)], and [Alouges, Discrete Contin. Dyn. Syst. Ser. S, 1 (2008)]. In our contribution, we generalize the approach of Alouges to the total magnetic field stated above, i.e. including all five energy terms and combine it with the approach from Banas/Slodicka, cf. [Banas/Slodicka, Appl. Numer. Math., 56 (2006)] for the discretization of the second equation. Since the computation of the demagnetization field is very time and memory consuming, the proposed time integrator is split into an implicit part and an explicit part. The first one deals with the higher-order term $\Delta\mathbf{m}$ stemming from the exchange energy, whereas all other terms are treated explicitly. In addition, the two equations can be decoupled. As the original algorithm, our extension guarantees the side constraint $|\mathbf{m}(t,\mathbf{x})| = 1$ for all nodes $\mathbf{x}$ and all time-steps t to be fulfilled as well as unconditional convergence if spatial mesh-size h and time-step size k tend to zero. In contrast to previous works, another benefit of our scheme is the fact that it requires only to solve two linear systems per time-step. Finally, our analysis allows to replace the operator $\mathcal{P}$ which maps $\mathbf{m}$ onto the corresponding demagnetization field, by a discrete operator $\mathcal{P}_h$. Possible choices for $\mathcal{P}_h$ are given by an extended convolution operator, the FEM-BEM coupling, or the hybrid FEM-BEM approach proposed in [Fredkin/Koehler, IEEE Transactions on Magnetics 26 (1990)] which is mostly used in the Physics literature.