ERRATUM: ON THE QUATERNIONIC *p*-ADIC *L*-FUNCTIONS ASSOCIATED TO HILBERT MODULAR EIGENFORMS

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This erratum provides a correction to the main theorem given in [8], most notably the discussion of μ -invariant in [8, Theorem 1.1 (iii)], plus a list of other corrections.

To begin with, let us clarify two conventions in [8]. First, we define ring class characters of K in the more general sense of [1], so that the restriction to the finite adeles of F must be everywhere unramified (rather than trivial). At the same time, we later consider only those ring class characters which factor through the quotient $G_{\mathfrak{p}^{\infty}} = \lim_{n} G_{\mathfrak{p}^{n}} \approx \mathbb{Z}_{p}^{\delta}$, whose restrictions to the finite adeles of F are necessarily trivial. Second, the word "supersingular" is used abusively to refer exclusively to the special setting where $a_{\mathfrak{p}} = 0$ (and is defined this way in [8, p. 1006, line 14]).

The interpolation formulae of the main result [8, Theorem 1.1] should include certain normalization factors, as well as extra factors in the so-called supersingular case, plus a correction to the characterization of the μ -invariants. Let us for ease of reading first summarize these main corrections (with an indication of proof) in the following revised statement of [8, Theorem 1.1]. Given an integer $n \geq 1$, let $m(\mathcal{O}_{\mathfrak{p}^n})$ denote the volume of $\widehat{\mathcal{O}}_{\mathfrak{p}^n}^{\times}$ in the space $K^{\times} \setminus \mathbf{A}_K^{\times} / \mathbf{A}_F^{\times}$ with respect to our fixed choice of Haar measure. Let $h(\mathcal{O}_F)$ denote the class number of F. Let us also write $G_0 = G[\mathfrak{p}^{\infty}]_{\text{tors}}$ to denote the torsion subgroup of $G[\mathfrak{p}^{\infty}] = \lim_{n \to \infty} G[\mathfrak{p}^n]$, and $\epsilon = \operatorname{ord}_{\mathfrak{P}}(|G_0|)$ the order of \mathfrak{P} dividing the order of G_0 . Recall that we normalize the quaternionic eigenform Φ to take values in the \mathcal{O} , including the value $1 \in \mathcal{O}$, and that this choice of normalization determines Φ uniquely up to multiplication by a unit in \mathcal{O} . Let us then define $\mu = \mu(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})) \geq 0$ to be the largest integer for which $\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}}) \in \mathfrak{P}^{\mu}\mathcal{O}[[G[\mathfrak{p}^{\infty}]]]$. Recall that we fix an embedding $\overline{\mathbf{Q}} \to \overline{\mathbf{Q}_p}$.

Theorem 0.1. Let $\mathbf{f} \in S_2(\mathfrak{N})$ be a cuspidal Hilbert eigenform of parallel weight 2, trivial nebentype character, and level $\mathfrak{N} \subset \mathcal{O}_F$ for which $\operatorname{ord}_{\mathfrak{p}}(\mathfrak{N}) = 1$, as in [8].

 (i) If f is p-ordinary, then there is a nontrivial element L_p(f, K_{p∞}) ∈ O[[G[p∞]]] such that the following formula holds in Q_p: Given ρ a primitive ring class character of conductor pⁿ (factoring through G_{p∞}) for n sufficiently large,

$$\rho\left(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})\right) = \frac{\alpha_{\mathfrak{p}}^{-2n} \cdot \zeta_{F}(2)}{2 \cdot L(\pi, \mathrm{ad}, 1)} \cdot \left(\frac{h(\mathcal{O}_{F})}{m(\mathcal{O}_{\mathfrak{p}^{n}})}\right)^{2} \\ \times \left[L(\pi, \rho, 1/2) \cdot L(\pi, \rho^{-1}, 1/2) \cdot \prod_{v \nmid \infty} \alpha(\Phi_{v}, \rho_{v}) \cdot \alpha(\Phi_{v}, \rho_{v}^{-1})\right]^{\frac{1}{2}}.$$

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(ii) If f is p-supersingular (as above), then there are nontrivial elements L_p(f, K_{p∞})[±] ∈ O[[G[p∞]]] such that the following formulae hold in Q_p: Given ρ a primitive ring class character of conductor pⁿ (factoring through G_{p∞}) for n sufficiently large and for which -ε = (-1)ⁿ (for ε = ± the sign),

$$\rho\left(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})^{\pm}\right) = C_{n}(\rho)^{2} \cdot \frac{\zeta_{F}(2)}{2 \cdot L(\pi, \mathrm{ad}, 1)} \cdot \left(\frac{h(\mathcal{O}_{F})}{m(\mathcal{O}_{\mathfrak{p}^{n}})}\right)^{2} \\ \times \left[L(\pi, \rho, 1/2) \cdot L(\pi, \rho^{-1}, 1/2) \cdot \prod_{v \nmid \infty} \alpha(\Phi_{v}, \rho_{v}) \cdot \alpha(\Phi_{v}, \rho_{v}^{-1})\right]^{\frac{1}{2}}.$$

Here, the fudge factor $C_n(\rho)$ is defined as follows: Fixing a set of topological generators $\gamma_1, \ldots, \gamma_{\delta}$ of $G_{\mathfrak{p}^{\infty}} \approx \mathbf{Z}_p^{\delta}$, and writing $\Sigma_{p^n}(T)$ to denote the p^n -th cyclotomic polynomial in T+1 (whose roots take the form $\zeta -1$ for $\zeta \in \mu_{p^n}$),

$$C_{n}(\rho) = \begin{cases} \prod_{\substack{j=0 \ j=1 \\ j\equiv 0(2)}}^{n} \prod_{i=1}^{\delta} \Sigma_{p^{n}}(\rho(\gamma_{i})-1) & \text{if } n \text{ is even} \\ \prod_{\substack{j=1 \\ j\equiv 1(2)}}^{n} \prod_{i=1}^{\delta} \Sigma_{p^{n}}(\rho(\gamma_{i})-1) & \text{if } n \text{ is odd.} \end{cases}$$

(iii) Let $\nu = \nu_{\Phi} \geq 0$ denote the largest integer for which the quaternionic form Φ associated to \mathbf{f} is congruent to a constant modulo \mathfrak{P}^{ν} , i.e. so that $\Phi \equiv \vartheta \mod \mathfrak{P}^{\nu}$ for some constant function $\vartheta \in \mathcal{S}_2^B(H; \mathcal{O})$. We have that $\mu(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})) \geq 2\nu - \epsilon$ in the \mathfrak{p} -ordinary case, and similarly that $\mu(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})^{\pm}) \geq 2\nu - \epsilon$ in the so-called \mathfrak{p} -supersingular case.

Indication of proof. Note that constructions of the elements are as given in [8], at least up to the corrections described below, as so we consider only the interpolation formulae and μ -invariants for the moment. The normalization factors defined above should be added as follows in the proof of [8, Theorem 4.10], specifically for the identification with the period integral on the first displayed line of [8, p. 1035] and the last two displayed lines of [8, p. 1033] (cf. [10, (5)]). To be more precise, let us assume that n is sufficiently large that the natural map $\operatorname{Pic}(\mathcal{O}_F) \to \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n})$ is an injection. If ρ is a primitive ring class character of conductor \mathfrak{p}^n factoring through $G_{\mathfrak{p}^{\infty}}$, then it is easy to see that it factors through the finite adelic quotient

$$\widehat{K}^{\times}/\widehat{F}^{\times}K^{\times}\widehat{O}_{\mathfrak{p}^n}^{\times} = \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n})/\operatorname{Pic}(\mathcal{O}_F).$$

Since ρ is invariant under $\widehat{\mathcal{O}}_{\mathfrak{p}^n}^{\times}$, we can write the period $l(\Phi, \rho)$ as a finite sum:

On the other hand, we can decompose our sum over $X_n = \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n})/\operatorname{Pic}(\mathcal{O}_F)$ into

$$\sum_{A \in X_n} \rho(A) \Phi(A) = \sum_{\tau \in \operatorname{Pic}(\mathcal{O}_F)} \sum_{t \in \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n}) / \operatorname{Pic}(\mathcal{O}_F)} \rho(t) \Phi(\tau t),$$

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using that ρ is trivial on $\operatorname{Pic}(\mathcal{O}_F)$. Observe now that for any choice of $\tau \in \operatorname{Pic}(\mathcal{O}_F)$, the inner sum satisfies the property that

$$\sum_{t \in \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n})/\operatorname{Pic}(\mathcal{O}_F)} \rho(t)\Phi(\tau t)$$

=
$$\sum_{\tau^{-1}t \in \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n})/\operatorname{Pic}(\mathcal{O}_F)} \rho(\tau^{-1}t)\Phi(t) = \sum_{t \in \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n})/\operatorname{Pic}(\mathcal{O}_F)} \rho(t)\Phi(t).$$

from which it follows that

$$\sum_{A \in X_n} \rho(A) \Phi(A) = |\operatorname{Pic}(\mathcal{O}_F)| \sum_{t \in \operatorname{Pic}(\mathcal{O}_{\mathfrak{p}^n}) / \operatorname{Pic}(\mathcal{O}_F)} \rho(t) \Phi(t).$$

Hence, we have shown that

$$\sum_{A \in X_n} \rho(A) \Phi(A) = \frac{h(\mathcal{O}_F)}{m(\mathcal{O}_{\mathfrak{p}^n})} \cdot l(\Phi, \rho),$$

from which the claimed formula follows after dividing out by α_{p}^{n} where needed in the ordinary case (i). In the so-called supersingular case (ii), we make an extra argument following those Pollack [5, §5.2] or Kobayashi [4, (3.4), (3.5)] to derive the interpolation formula with extra factors. To be more precise, recall that we define in the notations of [8, Proposition 4.6] (after Darmon-Iovita [2]) an element

$$\mathcal{L}_{\mathfrak{p}}(\Phi, K)^{\pm} = \vartheta_{\Phi}^{\pm} (\vartheta_{\Phi}^{\pm})^{*} = \begin{cases} (-1)^{\frac{n}{2}} \Theta_{\Phi}^{+} ((-1)^{\frac{n}{2}} \Theta_{\Phi}^{+})^{*} & \text{if } n \text{ is even} \\ (-1)^{\frac{n+1}{2}} \Theta_{\Phi}^{-} ((-1)^{\frac{n+1}{2}} \Theta_{\Phi}^{-})^{*} & \text{if } n \text{ is odd.} \end{cases}$$

Here, $\Theta_{\Phi}^{\pm} \in \Lambda/\Omega_n^{\pm}$ is the unique element such that $\vartheta_{\Phi,n} = \widetilde{\Omega}_n^{\pm} \Theta_{\Phi}^{\pm}$, where

$$\widetilde{\Omega}_n^+ = \widetilde{\Omega}_n^+(T_1, \dots, T_{\delta}) = \prod_{\substack{j=1\\ j\equiv 0(2)}}^n \xi_j(T_1, \dots, T_{\delta})$$
$$\widetilde{\Omega}_n^- = \widetilde{\Omega}_n^-(T_1, \dots, T_{\delta}) = \prod_{\substack{j=1\\ j\equiv 1(2)}}^n \xi_j(T_1, \dots, T_{\delta})$$

and

$$\xi_n(T_1,\ldots,T_\delta) = \prod_{i=1}^{\delta} \Sigma_{p^n}(T_i), \quad \Sigma_{p^n}(T) \text{ the } p^n \text{-th cyclotomic polynomial in } T+1.$$

Granted such a description, we see that the specialization to a primitive ring class character ρ of conductor \mathfrak{p}^n factoring through $G_{\mathfrak{p}^{\infty}}$ introduces extra factors coming from the $\widetilde{\Omega}_n(T_1, \ldots, T_{\delta})|_{(T_1, \ldots, T_{\delta}) = (\rho(\gamma_1) - 1, \ldots, \rho(\gamma_{\delta}) - 1)}$, i.e. for $\gamma_1, \ldots, \gamma_{\delta}$ the fixed set of topological generators of $G_{\mathfrak{p}^{\infty}} \approx \mathbf{Z}_p^{\delta}$. To be more precise, we argue that we obtain the stated extra extra factor $C_n(\rho)^2$, where

$$C_n(\rho) = \rho(\widetilde{\Omega}_n^{\pm}) = \begin{cases} \prod_{\substack{j=0\\j\equiv 0(2)}}^{n} \prod_{i=1}^{\delta} \Sigma_{p^n}(\rho(\gamma_i) - 1) & \text{if } n \text{ is even} \\ \prod_{\substack{j=1\\j\equiv 1(2)}}^{n} \prod_{i=1}^{\delta} \Sigma_{p^n}(\rho(\gamma_i) - 1) & \text{if } n \text{ is odd.} \end{cases}$$

For (iii), let us first note that the proof given in [8, Theorem 1.1 (iii)] for the stated characterization of $\mu = \mu(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}}))$ is not valid, as the constant term in the power series expansion of the theta element θ_{Φ} is in fact not given by the expression $c_{\infty}(\mathbf{1})$ appearing in the first displayed line of [8, p. 1036]. However,

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the lower bound $\mu \geq 2\nu - \epsilon$ can be justified via the following argument. We first show that it will suffice to establish the divisibility by $\mathfrak{P}^{2\nu-\epsilon}$ of the specializations $\rho(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})) = \rho(\theta_{\Phi} \theta_{\Phi}^*)$ for ρ any ring class character ρ of $G[\mathfrak{p}^{\infty}]$, i.e. assuming for simplicity that we are in the \mathfrak{p} -ordinary case. Keep all of the same notations and conventions with power series as given on p. 1029. Let $g = g(T_1, ..., T_{\delta})$ be a nonzero power series in $\mathcal{O}[[T_1, \ldots, T_{\delta}]]$. If for all finite order characters ρ of $G_{\mathfrak{p}^{\infty}} \approx \mathbf{Z}_p^{\delta}$ it is true that $\operatorname{ord}_p(\rho(g))$ is greater than or equal to some integer $m \geq 0$, then we claim it is also true that the μ -invariant $\mu(g)$ is greater than or equal to m. To see why this is true, let us write A to denote the \mathcal{O} -Iwasawa algebra $A = \mathcal{O}[[T_1, \ldots, T_{\delta-1}]]$. Since A is a complete local ring, we can apply the Weierstrass preparation theorem to argue that $g \in A[[T_{\delta}]]$ is expressible uniquely in the form

$$g(T_1,\ldots,T_{\delta}) = \mathfrak{P}^{\mu(g)} f(T_1,\ldots,T_{\delta}) u(T_1,\ldots,T_{\delta})$$

for $f(T_1, \ldots, T_{\delta}) \in A[[T_{\delta}]]$ a distinguished polynomial and $u(T_1, \ldots, T_{\delta}) \in A[[T_{\delta}]]$ a unit. Let us now write Y to denote the set of all finite order characters of Γ which are trivial on the subgroup of Γ generated by $\gamma_1, \ldots, \gamma_{\delta-1}$. If ψ is a character in Y, then the specialization $\psi(g)$ of g to ψ is equal to $\psi(g) = \mathfrak{P}^{\mu(g)}\psi(f)\psi(u)$, where $\psi(u)$ is a p-adic unit. On the other hand, the specialization $\psi(f)$ of f to ψ takes the form

$$\psi(f) = b_0 + b_1 \psi(T_{\delta}) + \ldots + b_{k-1} \psi(T_{\delta})^{k-1} + \psi(T_{\delta})^k,$$

where each of the coefficients b_0, \ldots, b_{k-1} lies in \mathcal{O} . Hence, when the order of ψ is sufficiently large, we see that $\operatorname{ord}_p(\psi(f)) = k \cdot \operatorname{ord}_p(\psi(T_{\delta}))$. Thus, we have shown that $\operatorname{ord}_p(\psi(f)) = \mu(g) + \sigma(\psi)$ for some $\sigma(\psi)$ which tends to zero as the order of ψ tends to infinity. In other words, we have shown that $\mu(g) \geq m$, as claimed.

We now claim that that the specialization $\rho(\theta_{\Phi})$ is divisible by \mathfrak{P}^{ν} for ρ any ring class character. To see why this is, let ρ be a ring class character factoring through some quotient X_n as above. Since we know that $\rho(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})) = \rho(\theta_{\Phi})\rho^{-1}(\theta_{\Phi})$, it will suffice to show that $\rho(\theta_{\Phi}), \rho^{-1}(\theta_{\Phi}) \geq \nu$. This latter assertion follows easily from the definition of the specialization of $\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})$ to characters of X_n , using orthogonality relations. To be more precise, we have for the elements constructed in case (i) the relation

$$\rho(\theta_{\Phi}) = \alpha_{\mathfrak{p}}^{-n} \sum_{A \in X_n} \rho(A) \Phi(A) \equiv \alpha_{\mathfrak{p}}^{-n} \sum_{A \in X_n} \rho(A) \vartheta(A) \equiv 0 \mod \mathfrak{P}^{\nu}.$$

Here, the last congruence follows from the fact that the values $\vartheta(A)$ do not depend on the classes A. The same argument shows that $\rho^{-1}(\theta_{\Phi}) \equiv 0 \mod \mathfrak{P}^{\nu}$, as well as the analogous assertions for the elements constructed in (ii). To deduce the final part of the claim, we consider the image of $\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}})$ under the canonical injection of completed group rings

(1)
$$\mathcal{O}[[G[\mathfrak{p}^{\infty}]]] \longrightarrow \bigoplus_{\rho_0 \in G_0^{\vee}} \mathcal{O}[[G_{\mathfrak{p}^{\infty}}]], \ \mathcal{L} \longmapsto (\rho_0(\mathcal{L}))_{\rho_0},$$

and then for each component under the non-canonical isomorphism of completed group rings $\mathcal{O}[[G_{\mathfrak{p}^{\infty}}]] \approx \mathcal{O}[[T_1, \ldots, T_{\delta}]]$, and then apply the observation about power series given above. If p does not divide the order of the torsion subgroup G_0 , then the canonical injection (1) is a bijection, making stated property (iii) easy to deduce. In general, the cokernel of (1) is annihilated by the order $|G_0|$ of the torsion subgroup G_0 of $G[\mathfrak{p}^{\infty}]$, and hence by \mathfrak{P}^{ϵ} for $\epsilon = \operatorname{ord}_{\mathfrak{P}}(|G_0|)$. In this way, we deduce

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the claimed bound $\mu(\mathcal{L}_{\mathfrak{p}}^{(\delta)}(\Phi, K)) \geq 2\nu - \epsilon$. The lower bounds can be established in a similar way for the invariants $\mu(\mathcal{L}_{\mathfrak{p}}^{(\delta)}(\Phi, K)^{\pm})$ in the so-called supersingular case, as is consistent with the descriptions given by [6, Theorem 1.1 (2)] and also [3]. \Box

Now, here is a list of other errors and corrections:

p. 1008, [8, Theorem 1.1 (iii)]. As noted above, the characterization of $\mu(\mathcal{L}_{\mathfrak{p}}(\mathbf{f}, K_{\mathfrak{p}^{\infty}}))$ is not proven in Theorem 4.14, as the constant term in the power series expansion of θ_{Φ} is not in fact given by the expression $c_{\infty}(1)$ appearing in the first displayed line of p. 1036. However, the lower bound $\mu \geq 2\nu - \epsilon$ can be justified via the line of argument given above.

p. 1009, Proof of Theorem 1.3: The argument is only sketched, and the reference to [9] should be replaced by a reference to [10].

p. 1009, line 20: The j should be a k.

p. 1012, line 4: The space is treated as zero if π_v is a discrete series.

p. 1012, line 8: The discussion here means that we choose the quaternion algebra B so that its ramification set Ram(B) equals Σ .

p. 1013, Corollary 2.5: The statement of requires some modification if the residue degree $\delta = [F_{\mathfrak{p}} : \mathbf{Q}_p]$ is greater than one. More precisely, if $\delta = 1$, then the stated result is easy to deduce from either the cited algebraicity theorem of Shimura [7] or else Weierstrass preparation (after Cornut-Vatsal [1]). If on the other hand δ is greater than one, then all that can be deduced is the nonvanishing of each of the Galois conjugate twists. Thus if $L(\pi, \rho, 1/2)$ does not vanish for some ring class character ρ , then the same is true for each $L(\pi, \rho^{\sigma}, 1/2)$, where σ runs over the automorphisms of \mathbf{C} which fix the Hecke field $\mathbf{Q}(\pi)$. In other words, if ρ has exact order p^x say, then $L(\pi, \rho', 1/2) \neq 0$ for each of the $\varphi(p^x)$ many ring class characters ρ' of exact order p^x . Thus, for each of the good $\rho \in P(n, \rho_0)$ in the statement of Theorem 2.4, nonvanishing can be deduced for each $L(\pi, \rho', 1/2)$ for ρ' in the Galois orbit of ρ . The stronger assertion that nonvanishing occurs for all but finitely many $\rho \in Y$ can then be deduced in the special case where the residue degree δ equals one.

p. 1015, Lemma 3.1: The double cosets are not necessarily disjoint. Also, the union should be over classes $F_+^{\times} \setminus \widehat{F}^{\times} / F_{\mathfrak{p}}^{\times} \widehat{\mathcal{O}}_F^{\times}$, i.e. over the quotient of the narrow class group by the subgroup generated by the class of \mathfrak{p} .

p. 1017, line 11. The description of the Hecke operator $U_{\mathfrak{p}}$ on c_{Φ} requires more justification if the prime \mathfrak{p} is not principal. The general case is treated in [9].

p. 1017, line -5. The orientation s, t is chosen arbitrarily.

p. 1018, line 14. The $\operatorname{End}_{F_{\mathfrak{p}}}(L)$ should be $\operatorname{End}_{\mathcal{O}_{F_{\mathfrak{p}}}}(L)$.

p. 1019, Proposition 3.7. Though not given explicitly, the case of $\mathfrak{p} \mid \mathfrak{M}$ is proven in a similar way.

p. 1023, line 22. The line should read "... use the Yuan-Zhang-Zhang generalization of Waldspurger's theorem ..."

p. 1023, final line: We should in fact mod out by $\mathcal{O}_{F_{\mathfrak{p}}}^{\times}$ in the domain of r_K , and this affects all subsequent definitions in the obvious way.

p. 1026, line 12. The claim that the functions $\Phi_{K,j}$ descend to functions on the quotients \mathcal{H}_j requires justification, as the subsequent clauses in lines 13-15 are nonsense. See the construction given in [9] for a corrected discussion of these issues.

p. 1028, final line. The H_{∞} should be replaced by a \mathcal{H}_{∞} .

p. 1029, lines 6-7. A more explicit account of how the construction of θ_{Φ} depends on the choice of directed edgeset (as would be appropriate here) is given in [9, §4].

p. 1031, statement of Lemma 4.7. The subscript should be $n \equiv (-1)^n$.

p. 1035, lines 16-18. To define the anticyclotomic μ -invariant, we normalize the eigenform Φ to take values in \mathcal{O} , and in such a way that it takes value $1 \in \mathcal{O}$. Cf. the discussions in [6] and [9, §4] for more details.

Finally, let us note that a more general construction has now been given in the preprint [9] using purely representation theoretic language, although this latter work only treats the ordinary case, and does not include any discussion of the analytic μ -invariant (or Howard's criterion). Still, it should be possible to use this latter approach to give a more general and uniform treatment of all of these ideas.

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