46. Show that:

(1) The n-dimensional sphere $S^n := \{x \in \mathbb{R}^{n+1} : \|x\| = 1\}$ is connected.

(2) The spaces S^1 and S^n, $n > 1$ are *not* homeomorphic.

47. Let $\{A_i : i \in I\}$ be a family of connected subsets of a topological space X. Show that if there exists a connected set $A \subseteq X$ such that $A \cap A_i \neq \emptyset$ for every $i \in I$, then the set $A \cup \bigcup_{i \in I} A_i$ is connected.

48. Let $\{A_n : n \in \mathbb{N}\}$ be a sequence of connected subsets of a topological space X such that $A_n \cap A_{n+1} \neq \emptyset$ for every $n \in \mathbb{N}$. Show that the set $\bigcup_{n=1}^{+\infty} A_n$ is connected.

49. Let X be a topological space and $A \subseteq X$. Show that if L is a connected subset of X such that $L \cap A \neq \emptyset$ and $L \cap (X \setminus A) \neq \emptyset$ then $L \cap \partial A \neq \emptyset$.

50. Let $\{X_i : i \in I\}$ be a family of topological spaces and $x = (x_i)_{i \in I} \in \prod_{i \in I} X_i$. Show that $C(x) = \prod_{i \in I} C(x_i)$, where $C(x)$ and $C(x_i)$ denote the connected components of x and x_i respectively.