16. Let $\mathbb{R}$ endowed with the topology with subbase

$$\mathcal{S} = \{(-\infty, b]\,,\,(a, +\infty)\,:\,a, b \in \mathbb{R}\,\}.$$

(1) Show that in this topology the sets $(a, +\infty)$, $(-\infty, b]$ and $(a, b]$ are open and closed.

(2) Show that $\mathbb{R}$ with this topology is a first countable space but it is not second countable.

17. Let (X, d) be a metric space and $A \subseteq X$. Show that

$$\overline{A} = \{x \in X\,:\,\text{there exists } \{x_n\}_{n\in\mathbb{N}} \subseteq A \text{ such that } x_n \to x\}.$$

18. Let X be a topological space and $A \subseteq X$ be a closed set. Show that

$$A^o = (\,\overline{A^o}\,)^o.$$

19. Let X be a topological space and $A \subseteq X$. Show that A is open if and only if $A \cap \overline{B} \subseteq \overline{A \cap B}$ for every $B \subseteq X$.

20. Let X be a topological space and $\{A_i\,:\,i \in I\,\}$ be a family of subsets of X. Show that if the union $\bigcup_{i\in I} \overline{A_i}$ is a closed set then $\overline{\bigcup_{i\in I} A_i} = \bigcup_{i\in I} \overline{A_i}$.