

26. Let X be a topological space. Show that a set $A \subseteq X$ is open if and only if $A \cap \partial A = \emptyset$.

27. Let X be a topological space and $A, B \subseteq X$ such that $\partial A \cap \partial B = \emptyset$. Show that $(A \cup B)^\circ = A^\circ \cup B^\circ$.

28. Let X be a topological space, $A \subseteq X$ and $\chi_A : X \rightarrow \mathbb{R}$ be the *characteristic map* of A (i.e. $\chi_A(x) = 1$ if $x \in A$ and $\chi_A(x) = 0$ if $x \notin A$). Show that χ_A is continuous if and only if A is open and closed.

29. Let X, Y be two topological spaces and $f : X \rightarrow Y$ be a map. Show that f is continuous if and only if $\partial f^{-1}(B) \subseteq f^{-1}(\partial B)$ for every $B \subseteq Y$.

30. Let X, Y be two topological spaces and $f : X \rightarrow Y$ be an *open map*. Is it always true that $f(A^\circ) = (f(A))^\circ$ for every $A \subseteq X$?